Theory of Measurement

ToM

The Theory of Measurement (ToM) construct describes how children come to understand foundations of measurement, such as the nature of units and how scales are constructed to aid measurement. The construct focuses primarily on linear measure, although the foundations of linear measure have their counterparts in other forms of measurement as well. Learning about measurement involves a fusion of practical activity (e.g., how to use tools) and the conceptual underpinnings of unit and scale (e.g., units should be identical, the origin of the scale is labeled as zero). Sound knowledge of measurement serves as a resource for the Model Measure approach to data and chance. **ToM** is not intended to portray every nuance of learning about the mathematics of measurement, but instead, to highlight conceptual attainments that would position students to understand statistics as measures or to develop novel measures in new circumstances when modeling data.

At the initial point of learning, level **ToM1**, the student selects potentially measurable attributes of an object, ideally motivated by an authentic question. At level **ToM2** the attribute is defined in a way that would make its measure feasible. At this initial point, the definition of the attribute allows the student to reliably distinguish it from other attributes. The chief conceptual attainment in **ToM1** and **ToM2** is coming to see an object as a set of potential attributes, each of which must be differentiated in order to be measured. For example, attributes of a pencil might include length and circumference, and each attribute needs to be operationally defined. (Should the length of the pencil include the length of its eraser?) Students at the second level can employ direct comparison to order or otherwise relate two or more objects with respect to a common attribute.

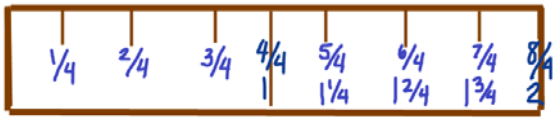
At level **ToM3,** students consider the nature of a unit. Units enable indirect measurements, because lengths can be compared via accumulation of units. Students can explain the role of identical units, and they demonstrate iteration as repeated application of a unit, even when one has “run out” of literal copies of a unit. Students also begin to consider properties of scale, especially the important notion that a measurement scale

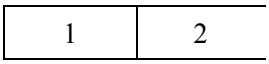
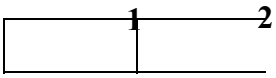
begins at zero. As students learn about the nature of unit, they relate (**ToM4)** these units to the goals of the measurement. For example, students evaluate the suitability of a unit: Inches might be a good choice for measuring a short distance but a poor choice for measuring a long distance. Or, they notice that standard units enable comparison among measurements. They consider, too, how units should be partitioned and how partitions can be composed to accomplish measurement that does not involve whole numbers of units. These partitions begin with factors of two; students understand that the effects of splitting a unit by 2, and then by 2 again, results in 4 partitions.

At level **ToM5,** students coordinate and organize units to form a scale of measure. Students symbolically express relations between units, including more complex partitions (e.g., compositions of splits involving 3 or 5) and compositions of units, and develop strategies to treat arbitrary points as if they were zero. At first, these strategies are confined to whole numbers, but students progress to coordinate any arbitrary starting point with any ending point to obtain a measure.

At level **ToM6**, students predict the effects of changes in unit on measure or scale. Students use relations among units to quantify the effects of a change in unit on the resulting measure and evaluate tradeoffs when selecting measurement tools.

At level **ToM7,** students flexibly employ ideas about measurement to invent measures as needed or to decide upon reasonable surrogates, such as time as a stand-in for distance, given knowledge of a particular rate.

Level		Performances		Examples
ToM 7	**Use theory of measurement to solve novel problems.**	*ToM7* **B**	Invent and justify measure.	▪ "This bushiness index (extent of branching) tells me how much the elodea plant (its total length) grew in the water."
		ToM7 **A**	Use derived units to solve measurement problems.	▪ "We can use the time it takes as a measure of distance, because we can assume a constant rate."
ToM 6	**Predict the effects of changes in unit on measure or on scale.**	*ToM6* **C**	Evaluate tradeoffs when selecting methods and tools for measurement.	▪ We could measure the width of the classroom with our 15 cm. ruler, but it would take forever. And we don't need to be so accurate to decide how to re-arrange our desks *(not an actual student response)*.
		ToM6 **B**	Derive relations among units, given expression of the same attribute in different scales of measure.	▪ "If the measure of the height of the plant is about 10 cm or about 4 inches, then an inch is about 2 ½ cm."
		ToM6 **A**	Use relations among units to quantify results of changes in unit.	▪ "The measure of the height of the plant is 14 cm. If a cm. is 10 times as long as a mm, then the measure is 140 mm." ▪ "If I change the unit so that it is half as long as the original unit, the measure doubles."
ToM 5	**Coordinate units to constitute a scale.**	*ToM5* **D**	Account for change of origin when measurement does not start at zero. *(Zero-Point: Any number can serve as zero.)*	▪ "I can start to measure from the 3 on the inches ruler, and take off 3 inches from the result." (Note: Starting at 2 or 3 is generally more difficult than starting at 1. Starting at a non-whole number is more difficult still). ▪ "If I start at 3 and go to 7 ¼, the measure is 4 1/4." (Note: Shifts involving fractions are usually more difficult than those with whole numbers.)
		ToM5 **C**	Compose units and represent the composition on a scale.	▪ "It was 15 inches or 1 ¼ feet." *Note:* These are 2-level units, or units-of-units.  ▪ "Its 6 quarters long, or we can say 1 ½ or 1 2/4 Anyia long (the name of the nonstandard unit).
		ToM5 **B**	Generate and compose splits of units involving odd factors, such as 3 or 5.	▪ "I made 12th's like this. First I split the unit into 3; then I split it by 2 and then 2 again. That makes 12th's." ▪ "A meter is 100 times as long as a cm." ▪ "A cm is 1/100 times as long as a m."
		ToM5 **A**	Symbolize relation between origin and partitioned units on scale.	▪ "You don't write 1/4 in the middle. . .  ▪ . . .Put it at the end of the part of the unit, so you can see how far you have traveled." 

ToM4	**Consider properties of unit in relation to goals of measurement.**	*ToM4* **D**	Partition and compose partitions by factors of 2, and use the partitions as a unit when an object cannot be measured in whole units.	▪ "It takes 2 and a half units to measure this notebook."
		ToM4 **C**	Qualitatively predict inverse relation between size of unit and measure.	▪ "If we use small steps, the measure is larger than if we use large steps."
		ToM4 **B**	Consider suitability of unit.	▪ "To measure that distance, a clipboard (unit) works a lot better than a pencil (unit)."
		ToM4 **A**	Use and justify standard (including conventional) unit.	▪ "If we all agree to use mm, then we can compare our measurements of the heights of these different plants."
ToM3	**Explain/Justify/ Demonstrate use of particular properties of a unit of measure.**	*ToM3* **D**	Zero serves as the origin of measure.	▪ "You don't write a 1 in the middle of the unit like this one,  because the unit starts at zero and ends at 1-- that's how far you have traveled! When you put it in the middle, it's not easy to see where the unit ends!" 
		ToM3 **C**	Re-use (iterate) a unit to measure.	▪ "We used all our books, but it was not enough; so we took the first book and put it at the end. The book case is 11 science books long."
		ToM3 **B**	Use identical units and explain why.	▪ "It is better to measure with a tool with all the same units because then you can just count the number of units."
		ToM3 **A**	Tile and explain why.	▪ "The units should touch so there is no empty space. Otherwise, you miss some of the space you are trying to measure."

TOM 2	**Identify and characterize the attribute of the object to be measured. Direct comparison of attributes.**	*ToM2* **C**	Associate measure with count.	▪ "This book is 4 (Student reads number off a ruler.)" ▪ "The pencil is 5 paper-clips long (the paper clips need not be identical lengths).
		ToM2 **B**	Distinguish (e.g., equal, not equal) or order (e.g., greater, lesser) quantities of an attribute by direct comparison.	▪ "This book is taller than that one (Student aligns the books and compares)."
		ToM2 **A**	Define the attribute being measured.	▪ "Fat means how far it is around the caterpillar" (analogy to circumference of wrist). ▪ "Big means the one that weighs the most."
TOM 1	**Identify the object/event to be measured.**	*ToM1* **B**	Identify measurable attributes (qualities).	▪ "We could find out how long the caterpillar is or how fat it is."
		ToM1 **A**	Pose a question or make statements about a potentially measurable object of interest.	▪ "How big is the pumpkin? ▪ "Which rocket flies the best?" ▪ "This pumpkin is big."