Pumpkins, Pumpkins! Foundations of Measure

Unit 1

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Mathematical Concepts

- Objects are described by their attributes, such as length and weight.
- Attributes must be defined if they are to be used for comparisons.
- Attributes of objects, such as height, can be compared directly.
- Attributes of objects can be compared via representations of attributes.
- Objects can be ordered by the magnitude of one or more attributes.
- Agreements about methods of comparison facilitate object comparisons.

Unit Overview

This unit introduces students to foundations of measurement. Students look at a few pumpkins and tell what they notice about them. Students typically notice attributes such as texture (smooth, bumpy), size (big, little), girth (fat-around), color, and height. What children notice establishes that an object like a pumpkin can be described by multiple attributes. The teacher asks which of these attributes or qualities will help determine the "biggest" pumpkin. Different senses of "bigness" are elicited from children, and the teacher guides consideration of two attributes: height and girth (circumference). What is meant by height? By "fatness?" Because girths are difficult to compare directly, representations are introduced to facilitate indirect comparisons. Using the representations, children order the pumpkins, first by girth (one order) and then by height (a different order). They learn that agreeing on common methods of measure is necessary to reduce variability from student to student in the representations of girth or of height of the same pumpkin, and that agreed-upon means and methods of representation facilitate reliable orderings of pumpkins by girth or height. The unit concludes with a formative assessment involving comparison of a pumpkin that students can see to another hidden behind a screen.

Which is Biggest? (First Attempt)

The teacher asks students to determine which pumpkin is biggest by comparing girth or height. Although students are permitted to make comparisons directly (by moving pumpkins), the teacher introduces the representation of streamers to support indirect comparisons, especially of

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girth. Using streamers, small groups of students develop methods to order the pumpkins either by girth or by height, but it is likely that their methods will vary from group to group. Students post their streamers on chart paper to permit public inspection of the results.

Looking at the posters, students propose reasons for the variability in their representations of the length or girth of the same pumpkin. They refine their methods to produce greater agreement.

Which is Biggest? (Second Attempt)

Students work in small groups with their refined methods and again post their representations, along with their orderings of the pumpkins' girth or height.

Formative Assessment

The lesson concludes with a formative assessment that requires comparing a visible pumpkin to a mystery pumpkin that is screened from view.

Extension Activity

Using a pan balance, children order the pumpkins from lightest to heaviest.

Materials & Preparation

Read

- ☐ **Theory of Length Measure (ToM) Construct Map**
- ☐ **Unit 1**

Start by reading the unit to learn the content and become familiar with the activities.

Academic Vocabulary

Height
Circumference
Weight

Gather

- ☐ 4 pumpkins (different sizes and shapes, so that ordering by girth is not the same as ordering by height)
- ☐ Chart paper
- ☐ Streamers (4 colors, one for each pumpkin)
- ☐ Tape
- ☐ Student journals
- ☐ Teacher math journal and/or flip camera to document the lesson

Prepare

The pumpkin measurement requires a large open space. Identify a space (playground, field, gym, a part of the classroom) large enough so that students can measure the pumpkins. Groups should be spread apart to avoid sharing information; each group explores in its own way. This is likely to produce variability in the streamer lengths for the same pumpkin. Accounting for and proposing ways to reduce this variability are important, so it is important to preserve the independence of the groups.

Mathematical Background

The big ideas of this unit are (a) defining attributes of an object, (b) using representations—streamers that stand in for height and girth of a pumpkin—to allow direct comparison of pumpkins via their representations, and (c) employing common methods of representation to facilitate comparison. The mathematical foundations are described in the Theory of Measure construct map.

Mathematical Literacy

Children develop academic vocabulary: height, circumference, and weight. They write about their findings in their journals. They engage in conversation about methods for comparing pumpkin girth and height.

Which is Biggest? 1
Which is Biggest? 2
Formative Assessment
Extension Activity

Which is Biggest? (First Attempt)

Students notice attributes of four different pumpkins labeled A, B, C, and D (the "pumpkin patch"), and establish a way to use a representation to stand in for girth and height.

Whole Group

1. **With a KWL chart, the streamers, and four pumpkins, ask students: "What do you notice about the pumpkins?"**

   Students will typically notice a variety of qualities of the pumpkins, such as color, size (e.g., big, small, skinny), shape (e.g., round, oval), and texture (e.g., bumpiness).

2. **Which of these (what students noticed) can help us find out which pumpkin is biggest? What could we mean by biggest?**

   Students typically suggest that the biggest pumpkin is the tallest (height) or the fattest (circumference). When students suggest ideas like these, teachers suggest academic language: height and circumference.

3. **How can we find out which one is the tallest?**

   Students often want to directly compare the pumpkins. Allow them to do so. Then introduce tape or streamer and ask if the streamer could be used to find the tallest. What would we have to do?

4. **How can we find out which one is the fattest? Could we use the streamers here, too? How?**

   Students will typically use a streamer to wrap around the pumpkin but they may choose different senses of "fat." Some may look for the longest circumference, but others may be inconsistent in where they wrap each pumpkin.

Construct: ToML 1A, 1B and ToML 2A, 2B, 2C
This task engages students in thinking at the early levels of the Theory of Measure construct.

Which is Biggest? 1
Which is Biggest? 2
Formative Assessment
Extension Activity

Small Groups

5. Students are divided into four groups. Each group measures the height of circumference of four different pumpkins to order them from smallest to biggest.

a. Review KWL chart. Ask: How can we use the streamer to measure the pumpkins? Students will typically respond "wrap around" (circumference) or "hold it up" (height). Teacher records answers on the "W" of the KWL chart.

b. Explain job titles:

- pumpkin holder (holds pumpkin steady while streamer holder measures)
- streamer holder (measures pumpkin)
- streamer cutter (cuts streamer after the pumpkin has been measured)
- streamer taper (tapes the streamer onto the data chart)
- measurement drawer (draws a line on the data chart to show which way the pumpkin was measured—i.e., a horizontal line for circumference and a vertical line for height)

c. Explain that you expect productive group work and what you think that will look and sound like. Each group will rotate to each pumpkin and perform their job duties.

d. Each group will use the streamers to propose a pumpkin order from smallest to largest. If students ask whether big means height or circumference, tell them it is up to them. Each group records its findings by pasting the streamers on a chart labeled A, B, C, or D, so that someone else will know which pumpkin was biggest, which was smallest, and which were in-between.

Whole Group

6. Display the data posters, one from each group.

Construct: ToML 2C, 2D

Choose a poster that provides opportunities to consider ideas such as the need to compare all the pumpkins on a common attribute (i.e., height or circumference) or the need to compare congruency of the streamers from a common starting point. Some posters may

Which is Biggest? 1
Which is Biggest? 2
Formative Assessment
Extension Activity

mix attributes or fail to align the streamers for purposes of comparison.

7. **The teacher asks students to tell about the poster, and using response from students:**

   a. Streamer placement: How does this affect our findings? (If the streamers don't all start at the same place on the poster, how does that affect our comparisons?

   b. If students did not compare all of the pumpkins on the same attribute (mixed height and circumference): How does this affect our findings?

8. **For each pumpkin (A-D), transfer all the groups' streamers that represent height onto another data poster, using a common starting point. Do the same thing for all the streamers representing circumference.**

   a. For each attribute, ask:
   What do you notice? *Students should notice differences in lengths.*
   What could we do differently next time so our streamers for height will be more alike?
   What could we do differently next time so our streamers for circumference will be more alike?

   b. Teacher records response from students onto the "L" of the KWL chart.

Which is Biggest? 1
Which is Biggest? 2
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Which is Biggest? (Second Attempt)

Students re-represent the pumpkins, labeled A, B, C, and D, and order them from smallest to biggest.

Whole Group

Construct: ToML 2C, 2D

1. **Review the KWL chart, job titles, and expectations for productive group work. Each group will measure each pumpkin and record the findings.**

2. **Display posters with streamers representing height or circumference.**

   Do all the groups agree about the ordering of the pumpkins by height? By circumference?

3. **For each pumpkin, transfer all the streamers representing height onto another data poster, using a common starting point. Do the same thing for the streamers representing circumference.**

   Q: Do the lengths of our streamers for **height** tend to agree more this time? Why or why not? Were the streamers for some pumpkins more the same than for others?
   Q: Did the lengths of the streamers for **circumference** tend to agree more this time? Why or why not? Were the streamers for some pumpkins more the same than for others?

4. **Record additional findings on the "L" of the KWL chart and review KWL chart.**

Which is Biggest? 1
Which is Biggest? 2
Formative Assessment
Extension Activity

Formative Assessment

Students compare one of the patch pumpkins to a hidden pumpkin.

Whole Group

1. **Display the streamers representing the height and circumference of the patch pumpkins.**

    a. Remind students which streamers represent height and which represent circumference.
    b. Say: We have a hidden pumpkin. Your job is to find out if the hidden pumpkin is bigger than this one (show class one of the patch pumpkins). You can take whatever tools you like with you to help you make that decision (provide ruler, streamers, etc.).

Individual

2. **One at a time, students will go to the hidden pumpkin.**

    a. Students will choose a measurement tool and measure the pumpkin.
    b. Students will tell the teacher which pumpkin is bigger and say how they know.
    c. Teacher will record the answers on the Formative Assessment record provided (teacher looks for use of tools, whether or not child compares the class pumpkin and the hidden pumpkin by aligning the streamers, use of attribute words of height or circumference).
    d. Students write in their journals about what they learned about comparing pumpkins.

Which is Biggest? 1
Which is Biggest? 2
Formative Assessment
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Extension Activity

Students order pumpkins by weight with a pan balance.

Whole Group

1. **Introduce students to the pan balance.**

   Place an object in one pan. Ask students to predict what will happen if a second object is placed in the other pan and why they think so.

2. **After establishing that the heavier object goes down, ask students how they would record that one object is heavier than another, and how they could write that.**

3. **Ask students to predict the order of the pumpkins in the pumpkin patch, from lightest to heaviest.**

4. **Use the pan balance to establish the ordering of pumpkins by weight. Compare this ordering to those of height and girth (circumference).**

   Ask students: If we know that A is heavier than B, and B is heavier than C, do we need to compare A and C with the pan balance, or can we already tell which is heavier? This transitive inference may be challenging to students, but it is worth trying to get students to think about it. If they say that the pan balance must be used, then go ahead and use it.

Formative Assessment

Student _____ Date _____

Indicate the levels of mastery demonstrated by circling those for which there is clear evidence:

Level	Description	Notes
ToM2A	Identifies attribute for comparing pumpkins. *Circle those that apply:* Height | Circumference | Other _____	
ToM2C	Child compares class and hidden pumpkins streamers by aligning the ends of the streamers to test congruency and drawing appropriate conclusion.	
ToM2D	Child's method using streamer is that developed in class, so the definition of the attribute is the same for both class and hidden pumpkins.	
ToM1B	Uses streamers but does not align them to ensure adequate comparison.	
NL	Does not use streamers. Makes claim by perception.	

Academic Language: Indicate academic words the student is familiar with by recording them here.