



Unit 1: Whole Number Place Value & Operations; Volume



Unit 2: Understanding Multiplication & Division of Fractions

5th Grade

Big Rocks for 2014-2015

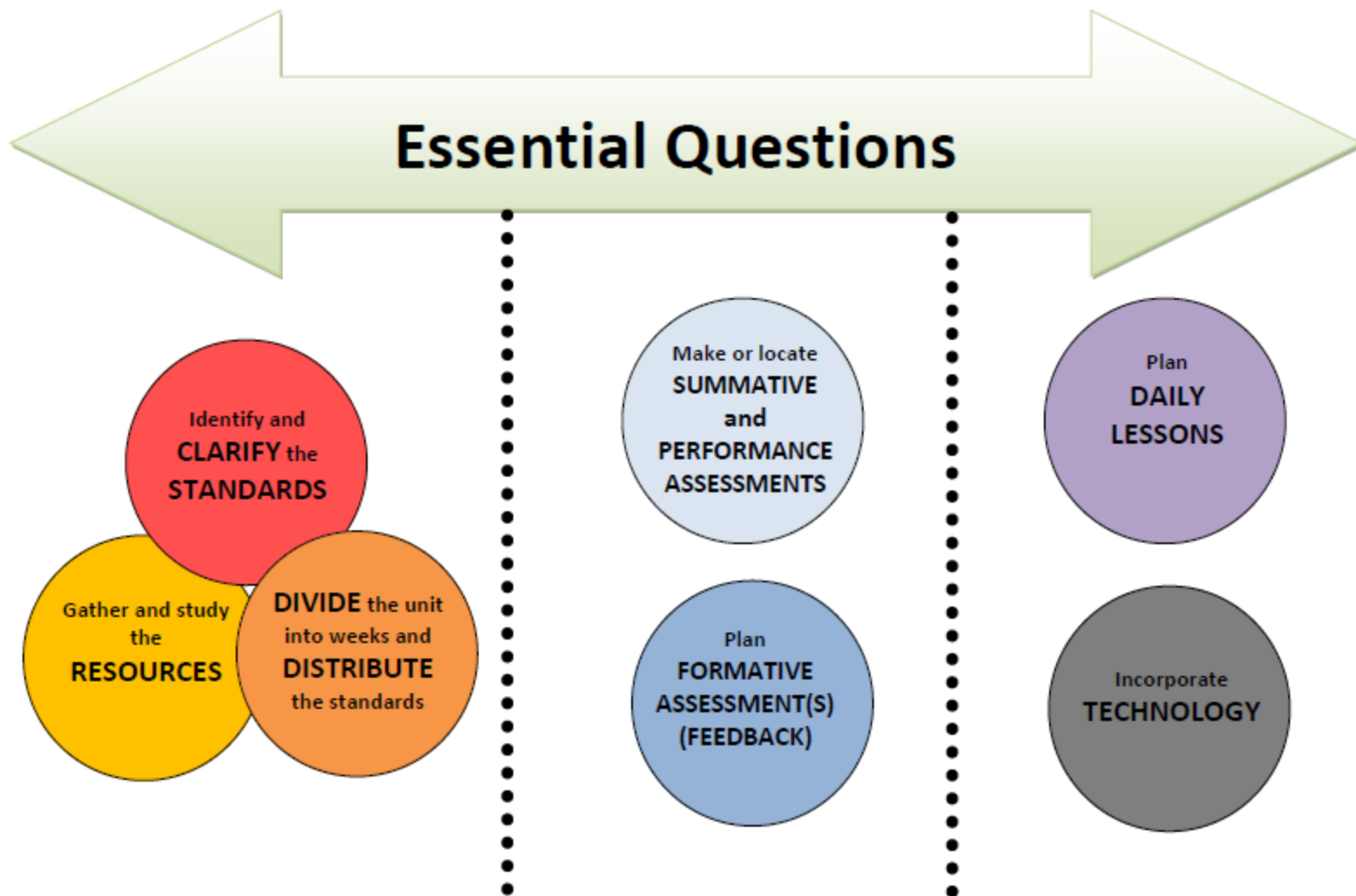
**Understanding the
Standards
Scoring to the
Standards
Vocabulary**

**GANAG/CGI &
ECM**

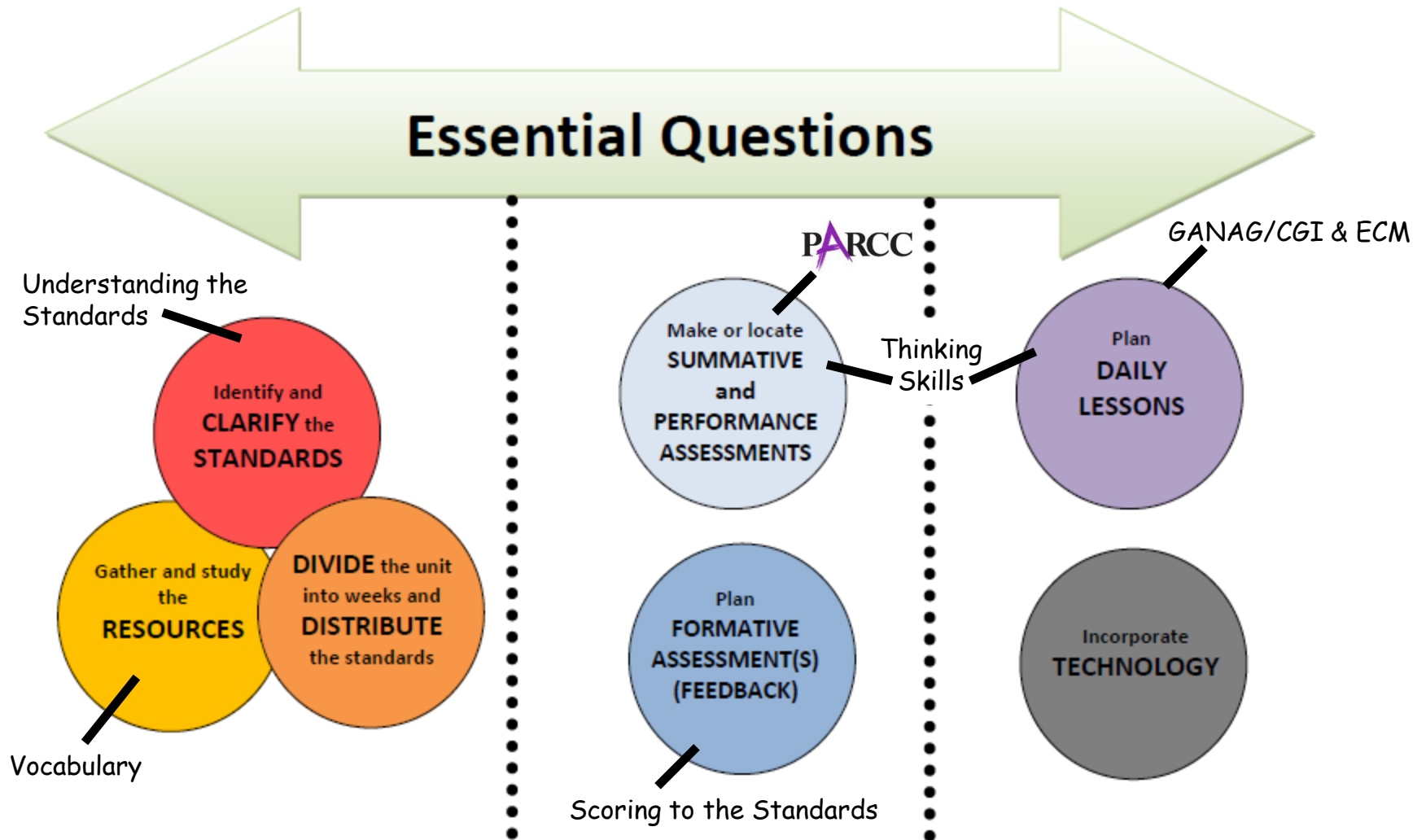
Thinking Skills



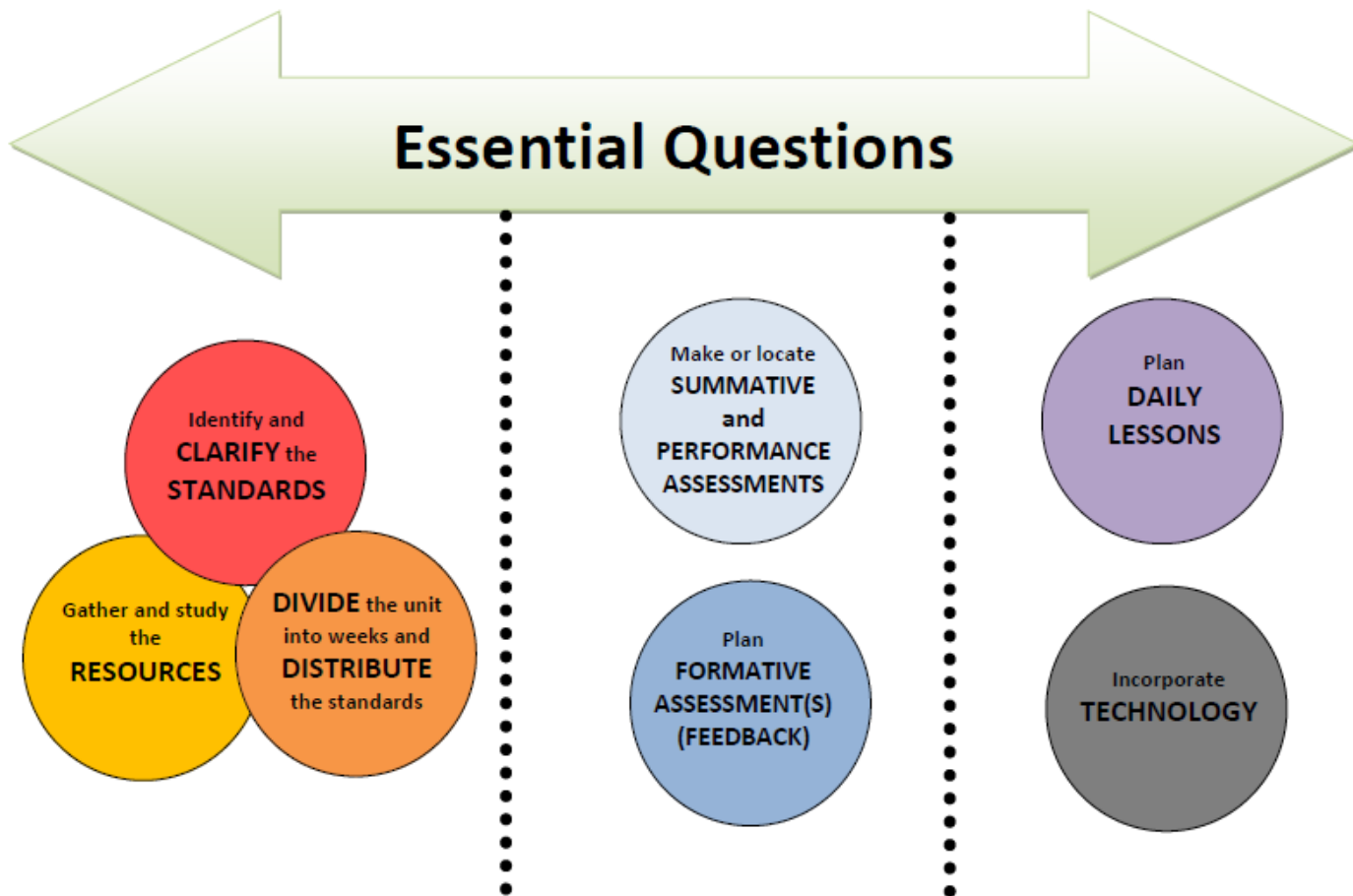
Backward Unit Planning 1.0



Backward Unit Planning 1.0



Backward Unit Planning 1.0



**The next 3 slides are
for use at the
beginning of the
presentation on
August 13.**

BIG 4

Well Articulated Curriculum

Plan for Instruction

Varied Assessment

Standards-based Feedback

Teaching Closet



The Big Four

Well Articulated Curriculum

Plan for Instruction

Varied Assessment

Standards-based Feedback

Common Core Standards
ELD Standards
Arkansas Curriculum Frameworks
Curriculum Documents

GANAG
Lesson Plan Book
SIOP
C.G.I./ECM
Smartboard
Co-teaching
Kagan Cooperative Learning

MAP DIBELS
DRA CRT/NRT Tests
Formative/Summative Assessment Scoring Guides
Performance Assessments

Progress Report
Daily scores & grade book
Student Conferences Report Card
Parent/Teacher Conferences

The BIG FOUR

Well Articulated Curriculum

Identify and
CLARIFY the
STANDARDS

Plan for Instruction

Gather and study the
RESOURCES

DIVIDE the unit
into weeks and
DISTRIBUTE
the standards

Plan
**DAILY
LESSONS**

Incorporate
TECHNOLOGY

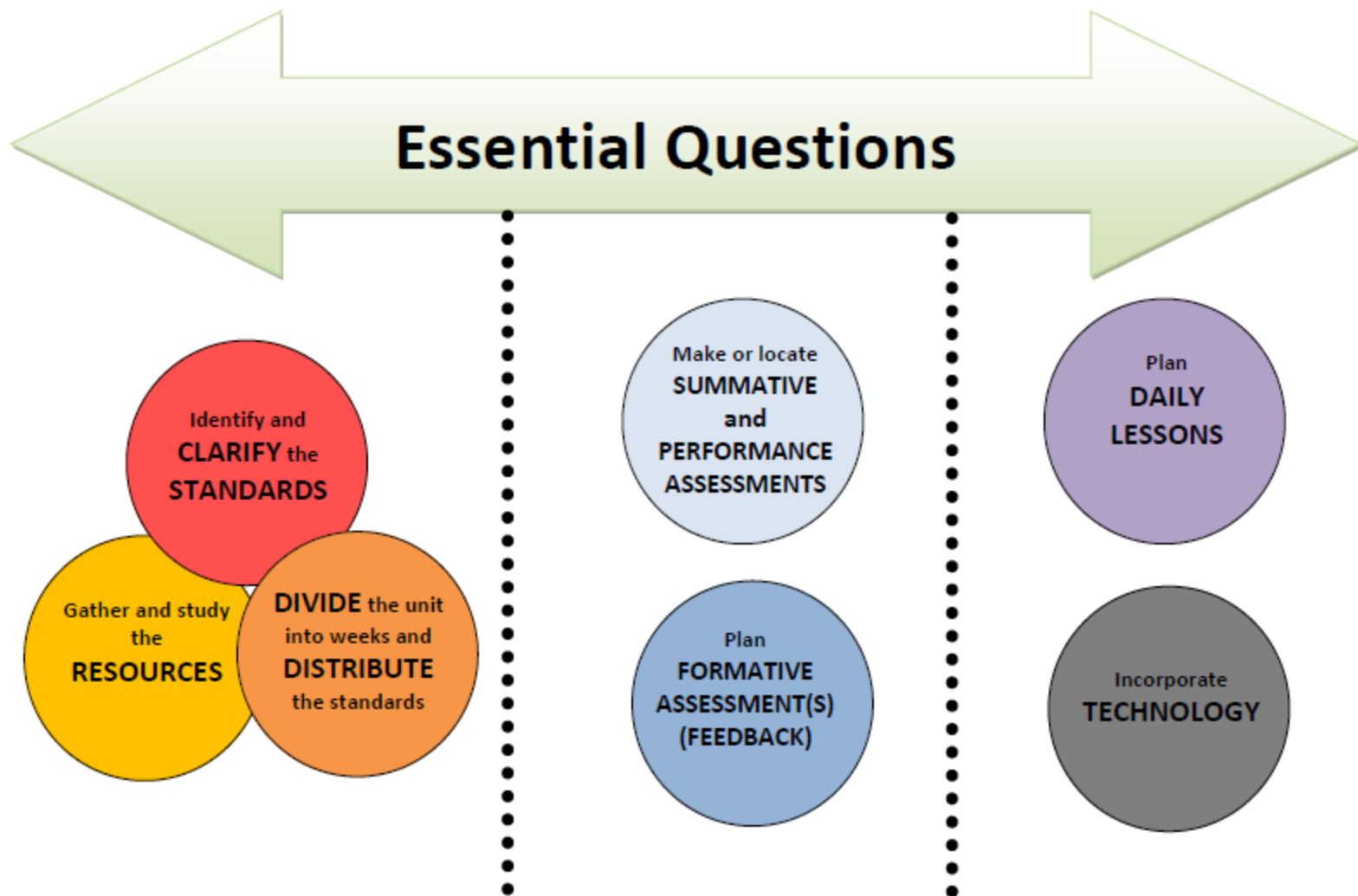
Varied Assessment

Locate or make
**SUMMATIVE
and
PERFORMANCE
ASSESSMENTS**

Standards-based Feedback

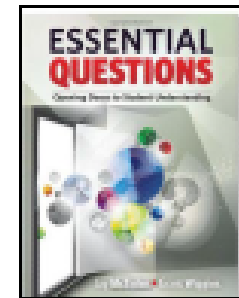
Plan
**FORMATIVE
ASSESSMENT(S)
(FEEDBACK)**

Backward Unit Planning 1.0



7

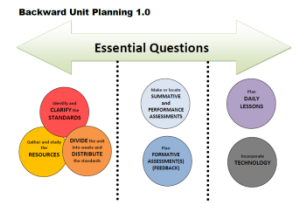
Defining Characteristics



A good essential question:

1. Is *open-ended*; that is, it typically will not have a single, final, and correct answer.
2. Is *thought-provoking* and *intellectually engaging*, often sparking discussion and debate.
3. Calls for *higher-order thinking* such as analysis, inference, evaluation, prediction. It cannot be effectively answered by recall alone.
4. Points toward *important, transferable ideas* within (and sometimes across) disciplines.
5. Raises *additional questions* and sparks further inquiry.
6. Requires *support* and *justification*, not just an answer.
7. *Recur*s over time; that is, the questions can and should be revisited again and again.

Mathematical Practices



1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.



Unit 1 – Whole Number Place Value & Operations; Volume

3 W

Students will build on their work from Fourth grade using various strategies based on place value to multiply and divide multi-digit numbers. Students will only be scored on four digits by one digit in first quarter. They will continue to use these different area model, base ten model, array, etc.) throughout the year to solidify their understanding until the standard is mastered in fourth quarter. Students will experience finding volume of rectangular prisms and understand concepts of volume. Finding volume will develop from these experiences.

Essential Questions:

How are place value patterns repeated in numbers?

How can place value help me multiply and divide?

How do I solve real-world problems involving volume?

Number and Operations in Base Ten**Understand the place value system.**

- 5.NBT.1** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as the digit in the place to its right and 1/10 of what it represents in the place to its left.
- 5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10; explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

- 5.NBT.5** Fluently multiply multi-digit whole numbers using the standard algorithm.
- Minimum expectation: Standard algorithm is not expected to be applied until 4th quarter.*
- 5.NBT.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Measurement and Data**Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.**

- 5.MD.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
- 5.MD.4** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
- 5.MD.5** Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
- a.** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
- b.** Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems.
- c.** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Operations and Algebraic Thinking**Write and interpret numerical expressions.**

- 5.OA.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
- 5.OA.2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.

Essential Questions

1. How are place value patterns repeated in numbers?
2. How can place value help me multiply and divide?
3. How do I solve real-world problems involving volume?

Essential Questions



Unit 1 – Whole Number Place Value & Operations; Volume

3 Weeks

Students will build on their work from Fourth grade using various strategies based on place value to multiply and divide multi-digit whole numbers. Students will only be scored on four digits by one digit in first quarter. They will continue to use these different strategies (i.e. area model, base ten model, array, etc.) throughout the year to solidify their understanding until the standard algorithm is applied in the fourth quarter. Students will experience finding volume of rectangular prisms and understand concepts related to volume. Notation for finding volume will develop from these experiences.

Essential Questions:

How are place value patterns repeated in numbers?

How can place value help me multiply and divide?

How do I solve real-world problems involving volume?

Number and Operations in Base Ten

Understand the place value system.

- 5.NBT.1** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $1/10$ of what it represents in the place to its left.
- 5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

- 5.NBT.5** Fluently multiply multi-digit whole numbers using the standard algorithm.
Minimum expectation: Standard algorithm is not expected to be applied until 4th quarter.
- 5.NBT.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Measurement and Data

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

- Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
- 5.MD.3**
- A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
 - A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.
- 5.MD.4** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
- Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
- 5.MD.5**
- Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
 - Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems.
 - Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Operations and Algebraic Thinking

Write and interpret numerical expressions.

- 5.OA.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
- 5.OA.2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.

Identify and CLARIFY the STANDARDS

Whole numbers only ...not decimals this quarter

Focus on strategies based on place value /fluent by 4th quarter

4th grade develops ideas for area and perimeter. Volume is the focus for 5th grade.
Students need to develop the ideas of volume using models and equations

This can be brought out during # talks and posing of problems. In this unit you would pose multiplication and measurement division problems with multiples of 10 in each group.

Build order of operations through discussions (connecting strategies to notation)

Essential Questions

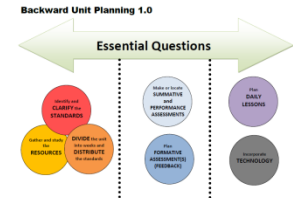


Factor x Factor = Product	Efficient Multiplication
Relational Thinking – Building Up Factor • Adding and/or multiplying to build up to factor $25 \times 43 =$ $\begin{array}{r} 25 \\ \times 43 \\ \hline 75 \\ 1000 \\ \hline 1075 \end{array}$	Relational Thinking – Complex Doubling • Double groups of factor repeatedly. $25 \times 43 =$ $\begin{array}{r} 43 (1) \\ \times 25 \\ \hline 215 \\ \times 43 \\ \hline 1075 \end{array}$
Relational Thinking – Partitioning using Tens and Ones (Distributive Property) • Break apart factor into tens and ones. $25 \times 43 =$ $\begin{array}{r} 25 \\ \times 43 \\ \hline 75 \\ 1000 \\ \hline 1075 \end{array}$	Relational Thinking – Partitioning Both Factors (Distributive Property) • Break apart both numbers into place values (expanded form). $25 \times 43 =$ $\begin{array}{r} 25 \\ \times 43 \\ \hline 75 \\ 1000 \\ \hline 1075 \end{array}$

2013 D. Chubbard & Humble - Van Dyke-Wade (3rd Edition) and BEM

Multiplication Strategy Level	Specific Strategy Within Each Level	Students who are using that strategy
Non-Yield Strategy		
Direct Modeling	Direct Modeling by 1s Direct Modeling by 10s	
Step Counting / Adding / Subtracting	Step Counting / Repeated Adding Simple Doubling	
Strategies Based on Place Value and the Properties of Operations	Complex Doubling Partitioning one of the factors Partitioning both factors Compensating Doubling and Halving	

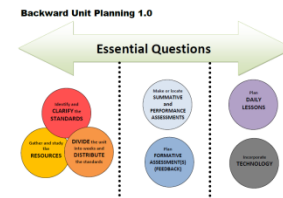
Identify and CLARIFY the STANDARDS



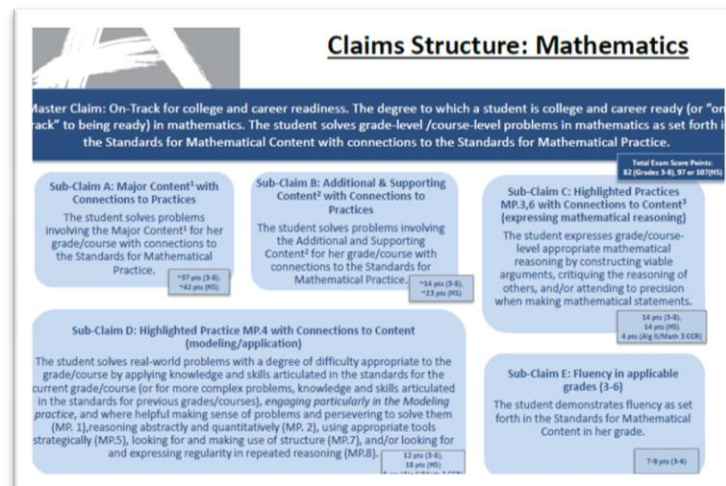
Fifth Grade: Whole Number Place Value and Operations; Volume (Unit 1: 3 Weeks)					Fifth Grade: Whole Number Place Value and Operations; Volume (Unit 1: 3 Weeks)								
Essential Questions: How are place value patterns repeated in numbers? How can place value help me multiply and divide? How do I solve real-world problems involving volume?					Essential Questions: How are place value patterns repeated in numbers? How can place value help me multiply and divide? How do I solve real-world problems involving volume?								
Common Core State Standards	Understand the place value system. 5.NBT.1 Depth opportunity Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	Understand the place value system. 5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	Perform operations with multi-digit whole numbers and with decimals to hundredths. 5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm. <i>Minimum expectation: Standard algorithm is not expected to be applied until 4th quarter.</i>	Perform operations with multi-digit whole numbers and with decimals to hundredths. 5.NBT.6 Depth opportunity Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	Common Core State Standards	Geometric measurement: understand concepts of volume and relate volume to multiplication and addition. 5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. (a) A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.	Geometric measurement: understand concepts of volume and relate volume to multiplication and addition. 5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. (b) A solid figure which can be packed without gaps or overlaps using unit cubes is said to have a volume of n cubic units.	Geometric measurement: understand concepts of volume and relate volume to multiplication and addition. 5.MD.4 Measure volumes by counting unit cubes, use cubic cm, cubic ft, and improvised units.	Geometric measurement: understand concepts of volume and relate volume to multiplication and addition. 5.MD.5 Depth opportunity Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. (a) Find the volume of a right rectangular prism with whole-number side lengths. Compute the volume by counting unit cubes of the prism or by using the formula $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of	5.OA.1 AND 2 Are imbedded in # talks			
						In-depth standards							
						equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as							
						the formula $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of							
Week 1:	X	X	X fluency		Week 1:							# talks	
Week 2:			X fluency	X	Week 2:							# talks	
Week 3:			X fluency		Week 3:	X	X	X	X	X	X	# talks	
Goal Statements					Goal Statements								
Notes	MP2,7 Whole numbers only...not decimal		fluency...push strategies	MP1,5 4 digit by 1 digit Use efficient strategies to push base 10	Notes	MP 7	MP 7	MP 7	Mathematical practices are match with standards on how they will be assessed in PARCC and how it lends itself to that standard				

Clustering of standards that are taught and assessed for coherence

Identify and CLARIFY the STANDARDS



of evidence	PBA	EOP	Evidence Statement Text	Clarifications	MP	Level 5: Distinguished Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Pa
PARCC Blueprints and Specs									
5.NBT.5-1	x		Multiply multi-digit algorithm.	Explicitly assess fluency. i) The given factors are such as to require an efficient/standard algorithm (e.g., 726-48). Factors in the task do not suggest any obvious ad hoc or mental strategy (as would be present for example in a case such as 725 - 40). ii) Tasks do not have a context. iii) For purposes of assessment, the possibilities are 2-digit x 3-digit.		Multiply with Whole Numbers 5.NBT.5-1 5.Int.1 5.NBT.Int.1	Solves unscaled word problems involving multiplication and multiplies three-digit by two-digit whole numbers using the standard algorithm.	unscaled word problems involving multiplication and multiplies three-digit by two-digit whole numbers using the standard algorithm.	Solves one problems in multiplicat
5.Int.1	x		Solve one-step word problems involving multiplying multi-digit whole numbers.	i) The given factors are such as to require an efficient/standard algorithm (e.g., 726 x 4871). Factors in the task do not suggest any obvious ad hoc or mental strategy (as would be present for example in a case such as 7250 x 400). ii) For purposes of assessment, the possibilities are 3-digit x 4-digit.	1, 7		Performs exact and approximate multiplications and divisions by mentally applying place value strategies when appropriate.	Performs exact and approximate multiplications and divisions by mentally applying place value strategies when appropriate.	
5.Int.2	x		Solve word problems involving three two-digit	i) The given factors are such as to require an efficient/standard algorithm (e.g., 76 x 48 x 39). Factors in the task do not suggest any obvious ad hoc or mental strategy (as would be present for example in a case such as 50 x 20 x 15).	1, 7				
Major, Supporting, and Additional Standards									
			of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	i) Tasks do not require students to illustrate or explain. ii) Tasks involve 3- or 4-digit dividends and one- or two-digit divisors.	1, 5	Quotients and Dividends 5.NBT.6 5.NBT.Int.1	Divides whole numbers up to four-digit dividends and two-digit divisors using strategies based on place value, the properties of operations and/or the relationship between multiplication and	Divides whole numbers up to four-digit dividends and two-digit divisors using strategies based on place value, the properties of operations and/or the relationship between multiplication and	Divides whole numbers up to four-digit dividends and one-digit divisors which are multiples of ten using strategies based on place value, the properties of operations and/or the relationship between
In-depth Standards									



A	B	C	D	E	F	G	H	I	J	K	L	M
Fifth Grade: Whole Number Place Value and Operations: Volume (Unit 1: 3 Weeks)					Fifth Grade: Whole Number Place Value and Operations: Volume (Unit 1: 3 Weeks)							
Essential Questions: How are place value patterns repeated in numbers? How can place value help me multiply and divide? How do I solve real-world problems involving volume?					Essential Questions: How are place value patterns repeated in numbers? How can place value help me multiply and divide? How do I solve real-world problems involving volume?							
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	Work It:	Work It:	Work It:	Work It:	Work It:	Work It:	Work It:	Work It:	Work It:	Work It:	Work It:	Work It:
	Goal Statements	Goal Statements	Goal Statements	Goal Statements	Goal Statements	Goal Statements	Goal Statements	Goal Statements	Goal Statements	Goal Statements	Goal Statements	Goal Statements
	Notes	Notes	Notes	Notes	Notes	Notes	Notes	Notes	Notes	Notes	Notes	Notes

Essential Questions



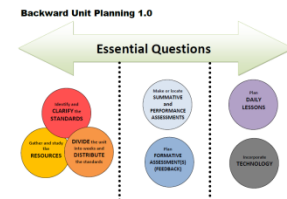
Identify and
CLARIFY the
STANDARDS

Statement of Evidence	PEA	EOY	Evidence Statement Text	Clarifications	MP	Level 5: Distinguished Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Partial
5.NBT.5-1	x		Multiply multi-digit numbers using the standard algorithm.	i) Tasks do not explicitly assess fluency. ii) The given factors are such as to require an efficient/standard algorithm (e.g., 726-48). Factors in the task do not suggest any obvious ad hoc or mental strategy (as would be present for example in a case such as 725 - 40). iii) Tasks do not have a context. iv) For purposes of assessment, the possibilities are 2-digit x 3-digit.	1, 7	Multiply with Whole Numbers 5.NBT.5-1 5.Int.1 5.NBT.Int.1	Solves multi-step unscaffolded word problems involving multiplication and multiplies three-digit by two-digit whole numbers using the standard algorithm.	Solves two-step unscaffolded word problems involving multiplication and multiplies three-digit by two-digit whole numbers using the standard algorithm.	Solves one-step problems involving multiplication.
5.Int.1	x		Solve one-step word problems involving multiplying multi-digit whole numbers.	i) The given factors are such as to require an efficient/standard algorithm (e.g., 726 x 4871). Factors in the task do not suggest any obvious ad hoc or mental strategy (as would be present for example in a case such as 7250 x 400). ii) For purposes of assessment, the possibilities are 3-digit x 4-digit.	1, 7		Performs exact and approximate multiplications and divisions by mentally applying place value strategies when appropriate.		
5.Int.2	x		Solve word problems involving three two-digit numbers.	i) The given factors are such as to require an efficient/standard algorithm (e.g., 76 x 48 x 39). Factors in the task do not suggest any obvious ad hoc or mental strategy (as would be present for example in a case such as 50 x 2 x 15).	1, 7				
5.NBT.6	x		Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	i) Tasks do not require students to illustrate or explain. ii) Tasks involve 3- or 4-digit dividends and one- or two-digit divisors.	1, 5	Quotients and Dividends 5.NBT.6 5.NBT.Int.1	Divides whole numbers up to four-digit dividends and two-digit divisors using strategies based on place value, the properties of operations and/or the relationship between multiplication and division.	Divides whole numbers up to four-digit dividends and two-digit divisors using strategies based on place value, the properties of operations and/or the relationship between multiplication and division.	Divides whole numbers up to three-digit and one-digit dividends which are multiples of ten using strategies based on place value, the properties of operations and/or the relationship between multiplication and division.

Mathematical Practices Look-fors

MP.7: Look for and make use of structure. (how numbers and shapes are organized)		
The Math Task:	The Students:	The Teacher:
<ul style="list-style-type: none"> Is an interesting problem Has more than one solution path which may be generalizable Requires cognitive effort Connects to real world Relates to grade level CCSS Builds student understanding of grade level standard Leads students to look back and reflect on answer Explicitly asks for justification or explanation 	<ul style="list-style-type: none"> Recognizes that quantities can be represented in different ways Uses properties of operations to make sense of problems Recognizes how numbers and shapes are organized Looks for patterns and structures in the number system Uses models to prove equations Recognizes how symbols help represent relationships and can be applied to new situations 	<ul style="list-style-type: none"> Shows students time to discuss connections Brings students back to the rule or properties being used Helps students look for patterns and structures in the number system Helps make connections between the structure used, equation, student thinking, and content standard Helps make connections to real world
MP.8: Look for and express regularity in repeated reasoning. (number pattern)		
The Math Task:	The Students:	The Teacher:
<ul style="list-style-type: none"> Is an interesting problem Has more than one solution path which may be generalizable Requires cognitive effort Connects to real world Relates to grade level CCSS Builds student understanding of grade level standard Leads students to look back and reflect on answer Explicitly asks for justification or explanation 	<ul style="list-style-type: none"> Notifies number patterns are repeated Applies more efficient computation strategies using number patterns Looks both for general methods and for shortcuts 	<ul style="list-style-type: none"> Encourages students to connect task to prior concepts taught Helps make connections between the structure used, equation, student thinking, and content standard

New Learning Unit 1



Week	Standards	Structure/Resource
1	<p>5.NBT.1 A digit in one place represents 10 times as much...</p> <p>5.NBT.2 Explain patterns in number of zeros when multiplying by powers of 10</p> <p>5.NBT.5 Fluent multiply multi digit numbers pushing strategies</p> <p>5.OA.1 Use parentheses, brackets, or braces</p> <p>5.OA.2 Write simple expressions</p>	<p>Push strategies using base 10 (multiplication)</p> <p>ECM problems (# in each group is a multiple of 10)</p> <p>Number Talks and problem discussion</p> <p>Look online at unit resources and choose those related to the standards being taught each week.</p>
2	<p>5.NBT.1 A digit in one place represents 10 times as much...</p> <p>5.NBT.2 Explain patterns in number of zeros when multiplying by powers of 10</p> <p>5.NBT.6 Find whole number quotients 4 digit by 1 digit</p> <p>5.OA.1 Use parentheses, brackets, or brace</p> <p>5.OA.2 Write simple expressions</p>	<p>Push strategies using Base 10 (measurement division)</p> <p>ECM measurement division problems (number in each group is a multiple of 10)</p> <p>Number Talks and problem discussion</p> <p>Look online at unit resources and choose those related to the standards being taught each week.</p>
3	<p>5.MD.3 Cubic Units</p> <p>5.MD.4 Counting Cubic Units</p> <p>5.MD.5 Connecting packing to the formula</p> <p>5.OA.1 Tie to 5.MD.5 Through use of properties of operations</p> <p>5.OA.2 Tie to 5.MD.5 through use of properties of operations</p>	<p>Look online at unit resources and choose those related to the standards being taught each week.</p> <p>https://grade5commoncoremath.wikispaces.hcpss.org/Grade+5+Home</p> <p>http://www.engageny.org/resource/grade-5-mathematics</p>

DIVIDE the unit
into weeks and
DISTRIBUTE
the standards

Exemplars Resources - Unit 1

Technology Resources

Games and Activities

Literature Connections

Number Talks for Unit 1

Assessments

Lesson Resources



Extending Children's Mathematics: Fractions and Decimals
by Susan B. Empson and Linda Levi

Student Recording Sheets for problems

The following documents are student recording sheets for problems presented in Chapter 7. "Multiplication and Measurement Division problems with larger numbers can engage students in thinking about base-ten number concepts beyond two-digit numbers." p.152 (5.NBT.1, 5.NBT.2)

Mr. Jones (p. 152)

Crystal Clear Water Company (p. 152)

Photospot Mega-Servers (p. 152)

Habitat for Humanity (p. 152)

National Debt (p. 153)



The Box Factory

This Contexts for Learning Mathematics unit focuses on deepening and extending students' understanding of multiplication and the extension of student's understanding of two-dimensional rectangular arrays to three dimensional arrays within rectangular prisms. Students work with these big ideas: properties of multiplication; dimensions of length and width can be used to produce a sq unit of area for rectangles; dimensions of length, width, and height can be used to produce a cubic unit of measurement; surface area of rectangular solids increase as the measures of the 3-Dimensions diverge. 10 lessons (5.MD.3, 5.MD.4, 5.MD.5)

Lessons, Tasks, and Investigations The following lessons were written by the Georgia Department of Education and correspond with the standards in this unit. Some lessons may require additional days.

Powers of Ten and Exponents

Patterns R Us Part 1

Power-ful Exponents

Multiplication and Division

Multiplication Three in a Row

Division Four in a Row

Patterns R Us Part 2

Area and Volume

Differentiating Area and Volume

How Many Ways

Exploring with Ropes



Home District K-5 Curriculum 6-8 Curriculum 9-12 Curriculum Calendar Assessment PD Employment Links

Whole Number Place Value & Operations; Volume

Planning Options

Lesson Resources

Technology Resources

Games and Activities

Literature Connections

Number Talks for Unit 1

Assessments

Understanding Multiplication & Division of Fractions

Intranet » K-5 Curriculum » 5th Grade Curriculum » Math » Curricular and Instructional Resources » Whole Number Place Value & Operations; Volume

Whole Number Place Value & Operations; Volume

3 weeks



Students will build on their work from Fourth grade using various strategies based on place value to multiply and divide multi-digit whole numbers. Students will only be scored on four digits by one digit in first quarter. They will continue to use these different strategies (i.e. area model, base ten model, array, etc.) throughout the year to solidify their understanding until the standard algorithm is applied in the fourth quarter. Students will experience finding volume of rectangular prisms and understand concepts related to volume. Notation for finding volume will develop from these experiences.

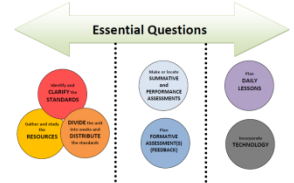
Essential Questions

How are place value patterns repeated in numbers?

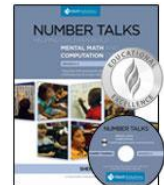
How can place value help me multiply and divide?

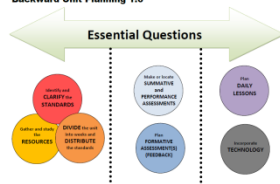
How do I solve real world problems involving volume?

Backward Unit Planning 1.0



Gather and study the
RESOURCES



5th Math, Unit1 (3 weeks)

Whole Number Place Value & Operations; Volume

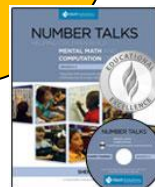
Unit 1: Instructional Strategies and Background Knowledge for 5th Grade Math CCSS

Students will build on their work from Fourth grade using various strategies based on place value to multiply and divide multi-digit whole numbers. Students will only be scored on four digits by one digit in first quarter. They will continue to use these different strategies (i.e. area model, base ten model, array, etc.) throughout the year to solidify their understanding until the standard algorithm is applied in the fourth quarter. Students will experience finding volume of rectangular prisms and understand concepts related to volume. Notation for finding volume will develop from these experiences.

*How are place value patterns repeated in numbers? How can place value help me multiply and divide?
How do I solve real-world problems involving volume?*

Week 1		Monday	Tuesday	Wednesday	Thursday	Friday
# Talks that push fluency		During # talks you will want to use the area model, rectangular arrays and base 10 models, # talks. You will also want to focus on the power of 10. You will want students to use the notation of finding volume				
		5.NBT.5 Use area model rectangular arrays, and base 10 models 5.OA.1 5.OA.2 Resource Guide for using Number Talks in Unit 1 Supplemental Number Talks for Unit 1 Number Talks Resources for 3rd-5th Grades				
Lesson	Standards/MP	5.NBT1 (2,7), 5.NBT2 (2,7), 5.NBT5 (F), 5.OA1 (7), 5.OA2 (7)				
	Goal and Task: *4-digit by 1-digit multiplication problems with multiples of 10 in each group. *Build order of operation through discussion and Mindson Math	I use my understanding of place value understanding to solve problems. I see patterns when I multiplying by multiples of 10.				
		(4 x 100) (25 X 100) (258 x 100)	(8 x 1000) (26 X 1000) (478 X 1000)	(36,000 x 10) (30,450 x 10) (14,560 X 100)	Foldable for exponents 100 x 100 1000 x 1000 10 ³ x 10 ⁵ 10 ² x 10 ⁴ 3 x 10 ³ ○ 4 x 10 ² 4 x 10 ⁴ ○ 7 x 10 ⁵	9 x 10 ³ 720 x 10 ⁴ 4025 x 10 ³ 10 ⁴ = 40000
Fluency/ Assessment		5.NBT.5 Make the Largest Product Make the Smallest Product 5.NBT.2 Multiplying a Whole Number by a Power of 10 Dividing a Whole Number by a Power of 10 Assessment of learning				

Gather and study the
RESOURCES



Assessment Tasks

Turn
and
TALK

External Measures

State Tests

District Tests

Classroom Assessment Tasks

FORMATIVE

Provides instructive feedback about progress intended for improvement

- observation
- assignments & quizzes
- student self- assessment

SUMMATIVE

Provides summary of achievement at a point in time

- performance tasks
- tests for comprehension and thinking
- end of unit tests

Evaluating Student Performance

Grading &
Record Keeping
for Improvement

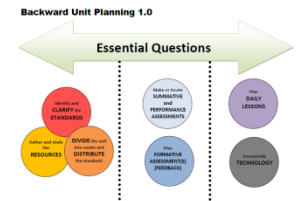
Grade Books

Reporting

Report Cards
Progress
Reports

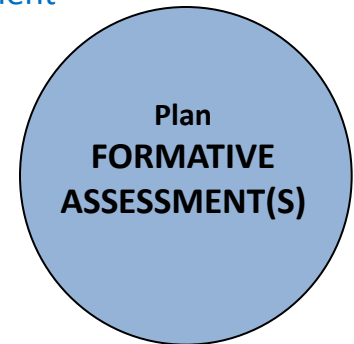
Formative Assessment is used during the learning process. It's purpose is to inform the teacher and the students how well the learning is going.

- Anne R. Reeves



Examples:

Fact Fluency Interviews/Assessment



Math Operations and Algebraic Thinking 1st Grade

Report Card Descriptor	Relates addition and subtraction
Standard To Be Measured	1.OA.3. Apply properties of operations as strategies to add and subtract. [Students need not use formal terms for these properties.] Examples: If $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known. (Commutative property of addition.) To add $2 + 6 = 4$, the second two numbers can be added to make a ten, so $2 + 6 = 4 + 2 = 10 + 2$. (Associative property of addition.) 1.OA.4. Understand subtraction as an unknown-addend problem. For example, subtract $10 - 8$ by finding the number that makes 10 when added to 8. Add and subtract within 20.
Quarterly Expectation	1st Quarter 2nd Quarter 3rd Quarter 4th Quarter
Student	

Revised 5/24/13

Problem: _____ Date: _____

Continuum of Solution Strategies for Multi-Digit Addition

Addition Strategy Level	Specific Strategy Within Each Level	Students who are using that strategy
Non-Valid Strategy		
Direct Modeling	Direct Modeling by 1's	
	Direct Modeling by 10's and 1's	
Counting	Counting On	
Strategies Based on Place Value and the Properties of Operations	Combining like Place Value Units	
	Incrementing	
	Compensating	

Problem: _____ Date: _____

Continuum of Solution Strategies for Multi-Digit Division

Division Strategy Level	Specific Strategy Within Each Level	Students who are using that strategy
Non-Valid Strategy		
Direct Modeling	Direct Modeling by 1's	
	Direct Modeling by 10's	
Skip Counting / Adding / Subtracting	Skip Counting / Repeated Adding or Subtracting	
	Simple Doubling	
	Complex Doubling	
Relational Thinking (Property Based Strategies)	Building up/down (Finding dividends*)	
	Compensating*	
	Ratios	

*Strategies based on Base-Ten will be more powerful.

Name: _____ Date: _____

Addition Facts Assessment - teacher note-taking sheet
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Fact	Answer	Strategy
5 + 5		
7 + 3		
8 + 3		
2 + 7		
6 + 4		
5 + 6		
4 + 8		
2 + 8		
6 + 6		
7 + 7		
7 + 5		
7 + 6		
9 + 7		
5 + 7		
6 + 8		
8 + 9		
6 + 9		
8 + 7		

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Name of Student: _____ ©Levi and Jaslow, 2012

Addition Fact Assessment Summary Sheet

Date	Strategy Used Most Often (DM, counting, RT, recall)	Most sophisticated strategy	Evidence of use of Commutative Property of + (Look at: $2 + 7$; $2 + 8$ and others.)	Evidence of use of associative property of + (Write out relationship used.)	Can compare numbers?	Number of Facts at efficient rate for grade level

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p. 18

Exit Tickets

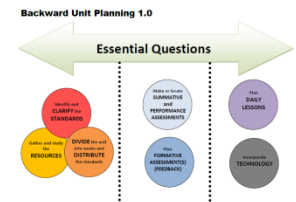
Student Interviews

Open Ended Tasks

Summative Assessment are

Examples:

Plan Summative Assessment(S)



Fifth Grade: Whole Number Place Value and Operations; Volume (Unit 1: 3 weeks)

1. The following equations involve different quantities and use different operations, yet produce the same result. Show and explain why this is true.

$$413 \times 10^2 = 4130 \qquad 4,130,000 \div 10^2 = 4130$$

Fifth Grade: Whole Number Place Value and Operations; Volume

2. Solve the equation below

a. $432 \div 7 = \underline{\hspace{2cm}}$

3.

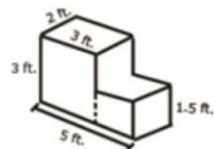
a. A rectangular container that has a length of 30 cm, a width of 20 cm, and a height of 10 cm. Calculate the volume and label in cubic units.

b. The rectangular container from Part A is filled with water to a depth of 5 cm. If an additional 6.5 liters of water is poured into the container, some of the water will overflow the container. Use words, pictures, and numbers to explain how much water will overflow the container. (Remember $1 \text{ cm}^3 = 1 \text{ mL}$)

Explain how estimation will help check the reasonableness of your answer.

a. $432 \div 7 = \underline{\hspace{2cm}}$

4. Calculate the volume and label in cubic units. Write and explain your work.



A Progression Toward Mastery				
Assessment Task Item and Standards Assessed	STEP 1 Little evidence of reasoning without a correct answer. (1 Point)	STEP 2 Evidence of some reasoning without a correct answer. (2 Points)	STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)	STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)
1 5.NBT.1 5.NBT.2	The student is unable to provide a correct response.	The student attempts but is not able to accurately show or explain reasoning fully.	The student correctly shows but does not show full reasoning, or explains reasoning fully. Work may not match explanation.	The student correctly: • Shows movement of digits. • Explains movement of units to the left for multiplication and movement of units to the right for division.
2 5.NBT.1 5.NBT.2 5.NBT.6	The student is unable to solve and estimate either the dividend or the divisor to a one-digit fact.	The student solves and estimates the dividend and divisor, but not to a one-digit fact.	The student correctly solves and estimates to a one-digit fact for either Part (a) or Part (b).	The student correctly solves and estimates both Part (a) and Part (b) to a one-digit fact. estimate a. $420 \div 70 = 6$ b. $1200 \div 600 = 2$ Solve a and b.
3 A & B 5.MD.3 5.MD.5	The student is unable to find the volume of the water that has overflowed and is unable to explain the reasoning used.	The student finds the volume of the water that has overflowed, but is unable to explain the reasoning used.	The student makes a calculation error in finding the volume of the water that has overflowed, but is able to clearly explain the reasoning used.	The student finds the volume of the water that has overflowed to be 1.1 L and uses words, numbers, and pictures to clearly explain the reasoning used.
4 5.MD.4 5.MD.5	The student is unable to find the volume and write the equation.	The student can find the volume but not write the equation.	The student makes a calculation error in finding the volume, but is able to write the equation.	The student finds the volume and writes the correct equation.

Exit Tickets

Student Interviews

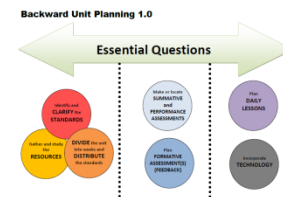
Open Ended Tasks

**Unit 2 – Understanding Multiplication & Division of Fractions** (p. 1 of 2)

Students will understand the relationship between fractions and division and use multiplicative relationships to solve problems. They will begin by solving *equal sharing* problems with answers that are mixed numbers and problems with answers that are fractions less than 1. Students will represent their solutions with equations and diagrams. They will then solve *multiple groups* problems where the number of groups is a whole number and problems where the number of groups is less than 1 or a mixed number. Emphasis should be on student strategies that use multiplicative relationships to solve problems. Students will then solve open number sentences that focus on the relationship between multiplication and division, for example: $8 \times \frac{3}{4} = ?$ $? \times \frac{3}{4} = 15$ $? \times 1 \frac{1}{4} = 25$

6 W

Essential Questions

**Essential Questions:**

How can I use my understanding of multiplication and division to solve real world problems involving multiplication and division of fractions?

How can I use information from the line plot to solve problems involving operations with fractions?

Number and Operations - Fractions**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**

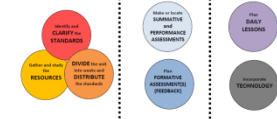
5.NF.3	Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. <i>For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?</i>
5.NF.4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
5.NF.5	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
5.NF.6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
5.NF.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. b. Interpret division of a whole number by a unit fraction, and compute such quotients. <i>For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.</i> c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. <i>For example, how much chocolate will each person get if 3 people share $1/2$ lb of chocolate equally? How many $1/3$-cup servings are in 2 cups of raisins?</i>

- 1. How can I use my understanding of multiplication and division to solve real world problems involving multiplication and division of fractions?**
- 2. How can I use information from the line plot to solve problems involving operations with fractions?**



Unit 2 – Understanding Multiplication & Division of Fractions (p.1 of 2) 6 Weeks

Students will understand the relationship between fractions and division and use multiplicative relationships to solve problems. They will begin by solving *equal sharing* problems with answers that are mixed numbers and then solve problems with answers that are fractions less than 1. Students will represent their solutions with equations, with an emphasis on linking addition and multiplication and reflecting a multiplicative understanding of fractions. After *equal sharing* problems, students will solve *multiple groups* problems where the number of groups is a whole number and the number in each group is a fraction less than 1 or a mixed number. Emphasis should be on student strategies that use multiplicative relationships to solve these problems. Students will then solve open number sentences that focus on the relationship between fractions and whole numbers, for example: $8 \times \frac{1}{4} = ?$ $? \times \frac{1}{4} = 15$ $? \times 1 \frac{1}{4} = 25$



Identify and CLARIFY the STANDARDS

Essential Questions:

How can I use my understanding of multiplication and division to solve real world problems involving multiplication and division of fractions?

How can I use information from the line plot to solve problems involving operations with fractions?

Number and Operations - Fractions

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	
5.NF.3	Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. <i>For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people each have $1/5$ of a pizza, how much pizza is there in all? If 3 people each have $1/5$ of a pizza, how much pizza is there in all? If 3 people each have $1/5$ of a pizza, how much pizza is there in all?</i>
5.NF.4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
5.NF.5	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
5.NF.6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
5.NF.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a whole number by a unit fraction, and compute such quotients. <i>For example, interpret $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.</i> b. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. <i>For example, how much chocolate will each person get if 3 people share $1/2$ lb of chocolate equally? How many $1/3$-cup servings are in 2 cups of raisins?</i>

Pose equal sharing problems.

Equal Sharing problem types:

- # of objects is greater than the # of shares that result in a mixed number
- # of shares is greater than the # of objects that result in a proper fraction

Finding area using fractions

Scaling-this is during the entire unit...part of discussions when checking for reasonableness of answers

Whole Number x Fraction
Whole Number ÷ Fraction
May involve area

Multiple Groups problem types:

- Whole number x unit fraction (Multiplication... $8 \times \frac{1}{4} = \underline{\hspace{1cm}}$)
- Whole number x unit fraction (Measurement Division... $\underline{\hspace{1cm}} \times \frac{1}{4} = 2$)
- Whole number x non-unit fraction (Multiplication... $8 \times \frac{3}{4} = \underline{\hspace{1cm}}$)
- Whole number x non-unit fraction (Measurement Division... $\underline{\hspace{1cm}} \times \frac{3}{4} = 6$)



Unit 2 – Understanding Multiplication & Division of Fractions (p. 2 of 2)

Measurement and Data	
Represent and interpret data.	
5.MD.2	Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Use operations on fractions for this grade to solve problems involving information presented in line plots. <i>For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.</i>
Number and Operations in Base Ten	
Perform operations with multi-digit whole numbers and with decimals to hundredths.	
5.NBT.5	Fluently multiply multi-digit whole numbers using the standard algorithm. <i>Minimum expectation: Standard algorithm is not expected to be applied until 4th quarter.</i>
5.NBT.6	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Operations and Algebraic Thinking	
Write and interpret numerical expressions.	
<i>These standards will not be taught in isolated lessons, but will be addressed during Number Talks and Problem Solving discussion and notation.</i>	
5.OA.1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
5.OA.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. <i>For example, express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.</i>

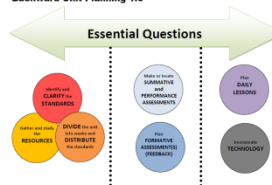
Equal Sharing problem types:

- # of objects is greater than the # of shares that result in a mixed number
- # of shares is greater than the # of objects that result in a proper fraction

Multiple Groups problem types:

- Whole number \times unit fraction (Multiplication... $8 \times \frac{1}{4} = \underline{\hspace{1cm}}$)
- Whole number \times unit fraction (Measurement Division... $\underline{\hspace{1cm}} \times \frac{1}{4} = 2$)
- Whole number \times non-unit fraction (Multiplication... $8 \times \frac{3}{4} = \underline{\hspace{1cm}}$)
- Whole number \times non-unit fraction (Measurement Division... $\underline{\hspace{1cm}} \times \frac{3}{4} = 6$)

Backward Unit Planning 1.0



Identify and
CLARIFY the
STANDARDS

Denominators are limited to $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$

Posing of math problems/ approach during number talks and equal sharing problems

5.NBT.6 in this quarter is 4 digit \times 1 digit

This can be brought out during # talks and posing of problems. In this unit you would pose equal sharing and multiple group problems.

Build order of operations through discussions (connecting strategies to notation)

Fifth Grade: Understanding Multiplication and Division of Fractions (Unit 2: 6 Weeks)							
Essential Questions: How can I use my understanding of multiplication and division to solve real world problems involving multiplication and division of fractions?							
Common Core State Standards	Apply and extend previous understandings of multiplication and division to multiply and divide fractions. 5.NF.3 (1) Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). (2) Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 2 whole are shared equally among 4 people, each person has a share of size $1/2$. $4/3$ people want to share a 50-pound sack of rice equally by weight. How many pounds of rice should each person get? Between what two whole numbers does your answer lie?	Apply and extend previous understandings of multiplication and division to multiply and divide fractions. 5.NF.4 (Depth opportunity) Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. (a) Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths; and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles; and represent fraction products as rectangular areas.	Apply and extend previous understandings of multiplication and division to multiply and divide fractions. 5.NF.5 Interpret multiplication as scaling (resizing), by: (a) Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.	Apply and extend previous understandings of multiplication and division to multiply and divide fractions. 5.NF.5 Interpret multiplication as scaling (resizing), by: (b) Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case), explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.	Apply and extend previous understandings of multiplication and division to multiply and divide fractions. 5.NF.6 (3) Solve real world problems involving multiplication of fractions and (2) mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	Apply and extend previous understandings of multiplication and division to multiply and divide fractions. 5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (b) Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.	Apply and extend previous understandings of multiplication and division to multiply and divide fractions. 5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (c) Solve real world problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual models and equations to represent the problem. For example, how many $3/4$ cups of raisins are in 2 cups of raisins?
Week 4:	X Equal Sharing problems		X	X			
Week 5:	X Equal Sharing problems		X	X			
Week 6:		X	X	X	X		X
Week 7:		X	X	X	X	X	X
Week 8:	X Equal Sharing problems	X	X	X	X	X	X
Week 9:			X	X			
Goal Statements							
Notes	MP 2 - 5.NF.3 (1) Interpret MP 1, 4, 5 - 5.NF.3 (2) Solve	MP 2, 5 Area	MP 7, 8 Part of discussion when checking for reasonableness of answer		MP 1, 4, 5 - 5.NF.5-1 whole number X fraction whole number divided by fraction	MP 5, 7 whole number X fraction whole number divided by fraction May involve area	MP 2, 5, 7

Essential Questions

Identify and
CLARIFY the
STANDARDS

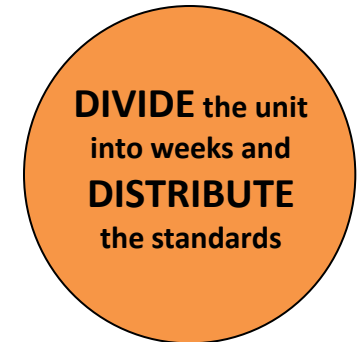
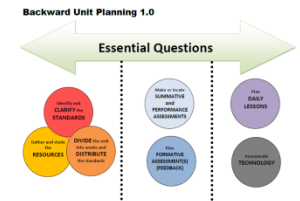
Statement of Evidence	PEA	EOY	Evidence Statement Text	Clarifications	MP	Level 5: Distinguished Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Partial
5.NBT.5-1	x		Multiply multi-digit numbers using the standard algorithm.	i) Tasks do not explicitly assess fluency. ii) The given factors are such as to require an efficient/standard algorithm (e.g., 726-48). Factors in the task do not suggest any obvious ad hoc or mental strategy (as would be present for example in a case such as 725 - 40). iii) Tasks do not have a context. iv) For purposes of assessment, the possibilities are 2-digit x 3-digit.	Multiply with Whole Numbers 5.NBT.5-1 5.Int.1 5.NBT.Int.1	Solves multi-step un scaffolded word problems involving multiplication and multiplies three-digit by two-digit whole numbers using the standard algorithm.	Solves two-step un scaffolded word problems involving multiplication and multiplies three-digit by two-digit whole numbers using the standard algorithm.	Solves two-step scaffolded word problems involving multiplication of a three-digit by a one-digit whole number.	Solves one-step problems involving multiplication.
5.Int.1	x		Solve one-step word problems involving multiplying multi-digit whole numbers.	i) The given factors are such as to require an efficient/standard algorithm (e.g., 726×4871). Factors in the task do not suggest any obvious ad hoc or mental strategy (as would be present for example in a case such as 7250×400). ii) For purposes of assessment, the possibilities are 3-digit x 4-digit.	1, 7	Performs exact and approximate multiplications and divisions by mentally applying place value strategies when appropriate.	Performs exact and approximate multiplications and divisions by mentally applying place value strategies when appropriate.		
5.Int.2	x		Solve word problems involving three two-digit numbers.	i) The given factors are such as to require an efficient/standard algorithm (e.g., $79 \times 48 \times 39$). Factors in the task do not suggest any obvious ad hoc or mental strategy (as would be present for example in a case such as $50 \times 20 \times 15$).	1, 7				
5.NBT.6	x		Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	i) Tasks do not require students to illustrate or explain. ii) Tasks involve 3- or 4-digit dividends and one- or two-digit divisors.	Quotients and Dividends 5.NBT.6 5.NBT.Int.1	Divides whole numbers up to four-digit dividends and two-digit divisors using strategies based on place value, the properties of operations and/or the relationship between multiplication and division.	Divides whole numbers up to four-digit dividends and two-digit divisors using strategies based on place value, the properties of operations and/or the relationship between multiplication and division.	Divides whole numbers up to four-digit dividends and one-digit divisors which are multiples of ten using strategies based on place value, the properties of operations and/or the relationship between multiplication and division.	Divides whole numbers up to three-digit and one-digit divisors which are multiples of ten using strategies based on place value, the properties of operations and/or the relationship between multiplication and division.

Mathematical Practices Look-fors

MP.7: Look for and make use of structure. (how numbers and shapes are organized)		
The Math Task:	The Students:	The Teacher:
<ul style="list-style-type: none"> Is an interesting problem Has more than one solution path which may be generalizable Requires cognitive effort Connects to real world Relates to grade level CCSS Builds student understanding of grade level standard Leads students to look back and reflect on answer Explicitly asks for justification or explanation 	<ul style="list-style-type: none"> Recognizes that quantities can be represented in different ways Uses properties of operations Recognizes how numbers and shapes are organized Looks for patterns and structures in the number system Identifies strategy for basic facts and reflects on answer Recognizes how symbols help represent relationships and can be applied to new situations 	<ul style="list-style-type: none"> Spends students time to discuss connections Brings students back to the rule or properties being used Helps students look for patterns and structures in the number system Helps make connections between the structure used, equation, student thinking, and content standard Helps make connections to real world
MP.8: Look for and express regularity in repeated reasoning. (number patterns)		
The Math Task:	The Students:	The Teacher:
<ul style="list-style-type: none"> Is an interesting problem Has more than one solution path which may be generalizable Requires cognitive effort Connects to real world Relates to grade level CCSS Builds student understanding of grade level standard Leads students to look back and reflect on answer Explicitly asks for justification or explanation 	<ul style="list-style-type: none"> Notifies number patterns are repeated Applies more efficient computation strategies using number patterns Looks both for general methods and for shortcuts 	<ul style="list-style-type: none"> Encourages students to connect task to prior concepts taught Helps make connections between pattern, equation, student thinking, and content standard

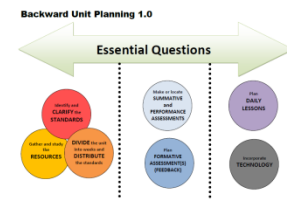
New Learning Unit 2

Week	Standards	Structure/Resource
1	5.NF.3 5.NF.5A During discussion 5.NF.5B During discussion 5.NBT.6 (4 digit X 1 digit) 5.OA.1 Use parentheses, brackets, or braces 5.OA.2 Write simple expressions (push strategies using base 10 division)	Pose equal sharing problems Whole number divided by whole number = whole number Whole number divided by whole number = mixed number Whole number divided by whole number = fraction
2	5.NF.3 5.NF.5A During discussion 5.NF.5B During discussion 5.NBT.6 (4 digit X 1 digit) 5.OA.1 Use parentheses, brackets, or braces 5.OA.2 Write simple expressions (push strategies using base 10 division)	Pose equal sharing problems Whole number divided by whole number = whole number Whole number divided by whole number = mixed number Whole number divided by whole number = fraction
3	5.NF.4 5.NF.6 5.NF.7 5.NF.5A During discussion 5.NF.5B During discussion 5.NBT.5 5.NBT.6 (4 digit X 1 digit) 5.OA.1 Use parentheses, brackets, or braces 5.OA.2 Write simple expressions (push strategies using base 10 division)	Multiple group problems Whole number x fraction Fluency 4 digit divided by 1 digit (number talks)



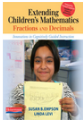
**Layer in
Continuous
Standards**

Week	Standards	Structure/Resource
4	5.NF.4 5.NF.6 (4 digit X 1 digit) 5.NF.7 5.NF.5A During discussion 5.NF.5B During discussion 5.NBT.5 5.NBT.6 (4 digit X 1 digit) 5.OA.1 Use parentheses, brackets, or braces 5.OA.2 Write simple expressions (push strategies using base 10 division)	Multiple group problems Whole number x fraction Fluency 4 digit divided by 1 digit (number talks)
5	5.NF.3 5.NF.4 5.NF.6 (4 digit X 1 digit) 5.NF.7 5.NF.5.A During discussion 5.NF.5.B During discussion 5.NBT.6 (4 digit X 1 digit) 5.OA.1 Use parentheses, brackets, or braces 5.OA.2 Write simple expressions (push strategies using base 10 division)	Look online at unit resources and choose those related to the standards being taught each week. https://grade5commoncoremath.wikispaces.hcpss.org/Grade+1+Home http://www.engageny.org/resource/grade-5-mathematics
6	5.MD.2 5.NF.5.A 5.NF.5.B	Denominators $\frac{1}{8}$, $\frac{1}{2}$, $\frac{1}{4}$ Look online at unit resources and choose those related to the standards being taught each week. https://grade5commoncoremath.wikispaces.hcpss.org/Grade+1+Home http://www.engageny.org/resource/grade-5-mathematics



DIVIDE the unit
into weeks and
DISTRIBUTE
the standards

Lesson Resources



Extending Children's Mathematics: Fractions and Decimals

by Susan B. Empson and Linda Levi

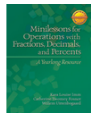
This book is referenced throughout the unit and school year to guide your instruction with fractions. The authors share how a student's understanding of whole-number operations can be extended to fractions and decimals. Full of examples and student work, it provides rich understanding and insight to problem solving with fractions. Page xxiv provides correlation between this book and the CCSS. [Guide for Using Extending Children's Mathematics](#)

Instructional Progression of Fraction and Decimal Operations



Field Trips and Fundraisers: Introducing Fractions

This Contexts for Learning Mathematics unit introduces fractions and fair sharing through a series of investigations set in context of the fair sharing of submarine sandwiches on a field trip. Students work with these big ideas: fractions are relations - the size or amount of the whole matters; fractions may represent division with a quotient less than one; with unit fractions, the greater the denominator, the smaller the piece is; pieces don't have to be congruent to be equivalent; and for equivalence, the ratio must be kept constant. 10 days of lessons (5.NF.3, 5.NF.4)



Minilessons for Operations with Fractions, Decimals, & Percents

This book contains minilessons that you can choose from as you consider the needs of your students and can be used through out the school year. These are more guided and explicit and were designed to be used at the start of your math instruction. [Further details regarding the minilessons](#)

Contextual Problem Solving Situations In the task with fractions through contextual situations that will help students in a "suggested" instructional sequence that begins with strong intuitive understanding. Most students can draw a picture or use a number line to help them solve these problems. These are meaningful to students.

Home

District

K-5 Curriculum

6-8 Curriculum

9-12 Curriculum

Calendar

Assessment

PD

Employment

Links

Whole Number Place Value & Operations; Volume

Understanding Multiplication & Division of Fractions

Lesson Resources

Games and Activities

Assessments

Number Talks for Unit 2

Planning Options

Intranet » K-5 Curriculum » 5th Grade Curriculum » Math » Curricular and Instructional Resources » Understanding Multiplication & Division of Fractions

Understanding Multiplication & Division of Fractions

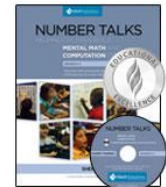
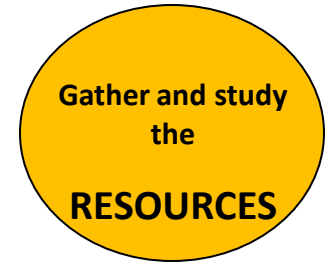
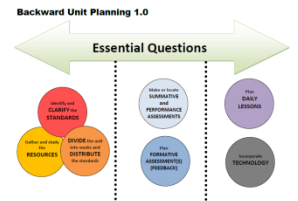
6 weeks

Students will understand the relationship between fractions and division and use multiplicative relationships to solve problems. They will begin by solving equal sharing problems with answers that are mixed numbers and then solve problems with answers that are fractions less than 1. Students will represent their solutions with equations, with an emphasis on linking addition and multiplication and reflecting a multiplicative understanding of fractions. After equal sharing problems, students will solve multiple groups problems where the number of groups is a whole number and the number in each group is a fraction less than 1 or a mixed number. Emphasis should be on student strategies that use multiplicative relationships to solve these problems. Students will then solve open number sentences that focus on the relationship between fractions and whole numbers, for example: $8 \times \frac{3}{4} = ?$ $? \times \frac{3}{4} = 15$ $? \times 1 \frac{1}{4} = 25$

Essential Questions

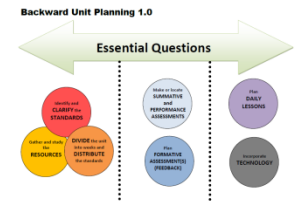
How can I use my understanding of multiplication and division to solve real world problems involving multiplication and division of fractions?

How can I use information from the line plot to solve problems involving operations with fractions?



Formative Assessment is used during the learning process. It's purpose is to inform the teacher and the students how well the learning is going.

- Anne R. Reeves



Examples:

Fact Fluency Interviews/Assessment

Math: Operations and Algebraic Thinking 1st Grade

Report Card Descriptor	Relates addition and subtraction
Standard To Be Measured	1.OA.3. Apply properties of operations as strategies to add and subtract. [Students need not use formal terms for these properties.] Examples: If $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known. (Commutative property of addition.) To add $2 + 6 + 4$, the second two numbers can be added to make a ten, so $2 + 6 + 4 = 2 + 10 = 12$. (Associative property of addition.) 1.OA.4. Understand subtraction as an unknown-addend problem. For example, subtract $10 - 8$ by finding the number that makes 10 when added to 8. Add and subtract within 20.
Quarterly Expectation	1st Quarter 2nd Quarter 3rd Quarter 4th Quarter
Student	

Revised 5/24/13

Problem: _____ Date: _____

Continuum of Solution Strategies for Multi-Digit Addition

Addition Strategy Level	Specific Strategy Within Each Level	Students who are using that strategy
Non-Valid Strategy		
Direct Modeling	Direct Modeling by 1's	
	Direct Modeling by 10's and 1's	
Counting	Counting On	
Strategies Based on Place Value and the Properties of Operations	Combining like Place Value Units	
	Incrementing	
	Compensating	

Problem: _____ Date: _____

Continuum of Solution Strategies for Multi-Digit Division

Division Strategy Level	Specific Strategy Within Each Level	Students who are using that strategy
Non-Valid Strategy		
Direct Modeling	Direct Modeling by 1's	
	Direct Modeling by 10's	
Skip Counting / Adding / Subtracting	Skip Counting / Repeated Adding or Subtracting	
	Simple Doubling	
	Complex Doubling	
Relational Thinking (Property Based Strategies)	Building up/down (Finding dividends*)	
	Compensating*	
	Ratios	

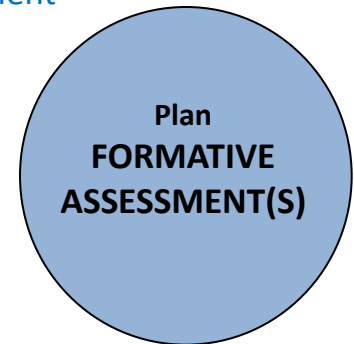
*Strategies based on Base-Ten will be more powerful.

Name: _____ Date: _____

Addition Facts Assessment – teacher note-taking sheet
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Fact	Answer	Strategy
5 + 5		
7 + 3		
8 + 3		
2 + 7		
6 + 4		
5 + 6		
4 + 8		
2 + 8		
6 + 6		
7 + 7		
7 + 5		
7 + 6		
9 + 7		
5 + 7		
6 + 8		
8 + 9		
6 + 9		
8 + 7		

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Name of Student: _____ ©Levi and Jaislow, 2012

Addition Fact Assessment Summary Sheet

Date	Strategy Used Most Often (DM, counting, RT, recall)	Most sophisticated strategy	Evidence of use of Commutative Property of + (Look at: 2 + 7; 2 + 8 and others.)	Evidence of use of associative property of + (Write out relationship used.)	Can compare numbers?	Number of Facts at efficient rate for grade level

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p. 18

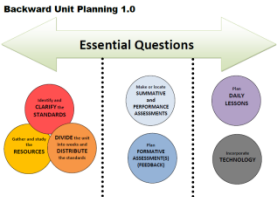
Exit Tickets

Student Interviews

Open Ended Tasks

Summative Assessment *are*

Examples:



Fifth Grade: Understanding Multiplication and Division of Fractions (Unit 2: 6 weeks)

1. Jill is collecting honey from 9 different beehives, and recorded the amount collected, in gallons, from each hive in the line plot shown:

a. She wants to write the value of each point marked on the number line above in terms of the largest possible whole number of gallons, quarts, and pints. to fill in the blanks with the correct conversions. (The first one is done for you.)

a. 0 gal 3 qt 0 pt

b. gal qt pt

c. gal qt pt

d. gal qt pt

2. Shiloh wants to make 5 pitchers of tea. Each recipe calls for $\frac{1}{4}$ cup of sugar. How many cups of sugar will she have more or less than 1 whole cup of sugar? Explain your reasoning.

3. A bakery bakes 728 trays of cookies in a day. How many trays of cookies can they bake in 43 days?

Explain how you found your solution.

Fifth Grade: Understanding Multiplication and Division of Fractions (Unit 2: 6 weeks)

4. Mrs. Allen needs 60 square tiles to cover the family room floor. The tiles come in boxes of 8.

How many boxes does Mrs. Allen need?

5.

$$50 \div 2 + 8 - 3 = 2$$

Explain, using what you know about order of operations, how the problem above was solved to get the answer of 2. Hint: you need to add parenthesis to solve this problem correctly.

6. Alex and Chet both collect cards. Write an algebraic equation to show that Alex has twice as many cards as Chet. Let c represent the number of cards Chet has.

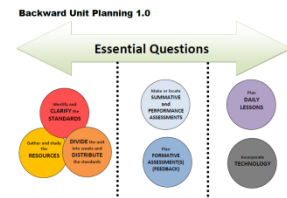
Robin can bike 4 miles in one hour. Write an algebraic expression to show how many miles she can bike in h hours?

A Progression Toward Mastery				
Assessment Task Item and Standards Assessed	STEP 1 Little evidence of reasoning without a correct answer. (1 Point)	STEP 2 Evidence of some reasoning without a correct answer. (2 Points)	STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)	STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)
1 S.NF.3 S.NF.4 S.NF.6 S.NF.7 S.MD.1 S.MD.2	The student has two or fewer correct answers.	The student has three correct answers.	The student has five correct answers.	The student correctly answers all seven items: a. 1 gal, 2 qts 2 gal, 1 pt 2 gal, 2 qt, 1 pt b. 13 gal, 1 pt c. 2 1/9 gal d. 1 7/12 gal e. 1/12 gal f. 6 c g. 12 bottles
2 S.NF.4	The student is unable to find a solution and is unable to explain the reasoning used.	The student finds the solution, but is unable to clearly explain the reasoning used.	The student makes a calculation error in finding a solution, but is able to clearly explain the reasoning used.	The student is able to clearly explain that the product is greater than 1. For example, $5 \times \frac{1}{4} = \frac{5}{4} = 1\frac{1}{4}$ or greater than 1.
3 S.NBT.5	The student is able to solve one part of the problem and is unable to clearly explain the reasoning used, or the student is unable to solve any part of the problem and is unable to clearly explain the reasoning used.	The student is able to solve one part of the problem and is able to clearly explain the reasoning used.	The student is able to solve both parts of the problem but is unable to clearly explain the reasoning used.	The student is able to efficiently solve both parts of the problem and is able to clearly explain the reasoning used.
4 S.NBT.6	The student is unable to solve the problem and is	The student is able to solve the problem, but is unable to	The student is unable to solve the problem but is able to clearly	The student is able to efficiently solve the problem and is able to

Exit Tickets

Student Interviews

Open Ended Tasks



Essential Questions

Unit 1

1. *How are place value patterns repeated in numbers?*
2. *How can place value help me multiply and divide?*
3. *How do I solve real-world problems involving volume?*

Unit 2

1. *How can I use my understanding of multiplication and division to solve real world problems involving multiplication and division of fractions?*
2. *How can I use information from the line plot to solve problems involving operations with fractions?*

Backward Unit Planning 1.0

Essential Questions

Identify and
CLARIFY the
STANDARDS

Gather and
the
RESOURCES

DIVIDE the unit
into weeks and
DISTRIBUTE
the standards

Make or locate
SUMMATIVE
and
PERFORMANCE
ASSESSMENTS

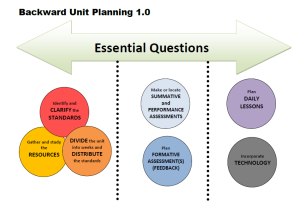
Plan
FORMATIVE
ASSESSMENT(S)
(**FEEDBACK**)

Plan
DAILY
LESSONS

Incorporate
TECHNOLOGY

Summative Assessment *is designed and administered to “sum up” learning that has taken place during a lesson, a unit, or a course.*

- Anne R. Reeves



Considerations:

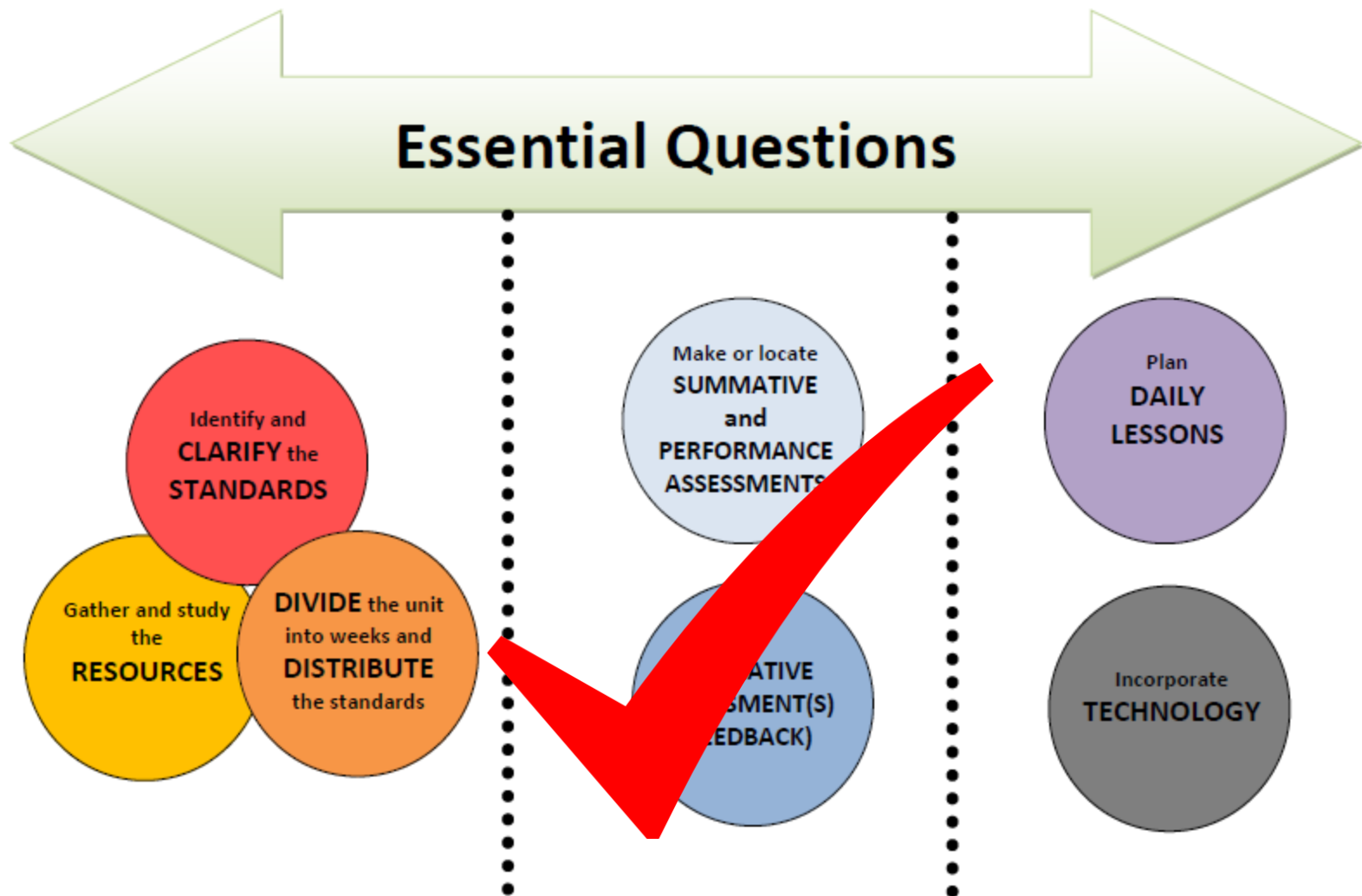
1. **Assess CCSS**
2. **Address Essential Questions**
3. **Incorporate Mathematical**



4.

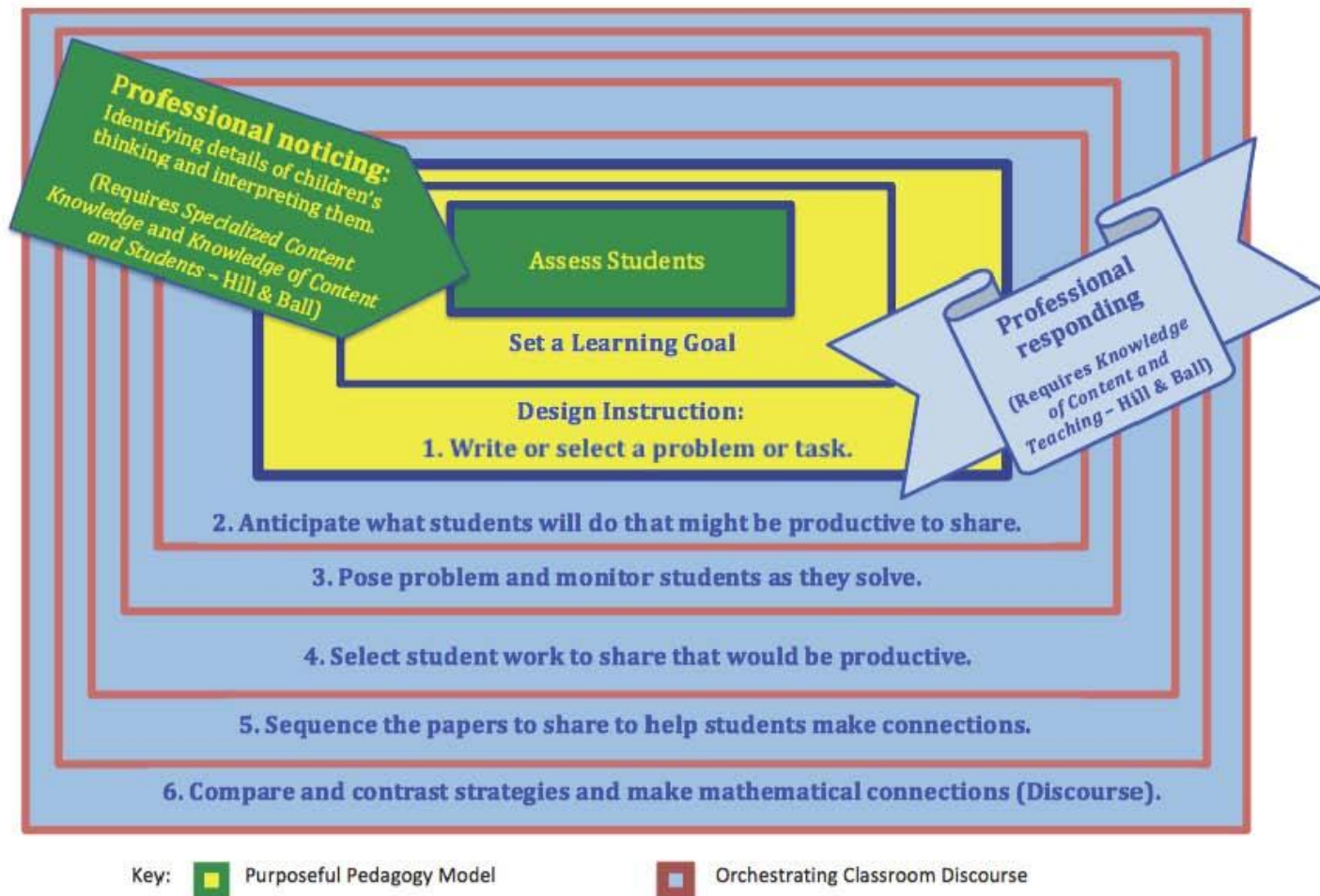


Backward Unit Planning 1.0



Purposeful Pedagogy in Math

Arkansas CCSSM Professional Development Purposeful Pedagogy and Discourse Instructional Model



GANAG is a daily lesson structure that allows teachers to plan for student use of research based instructional strategies.

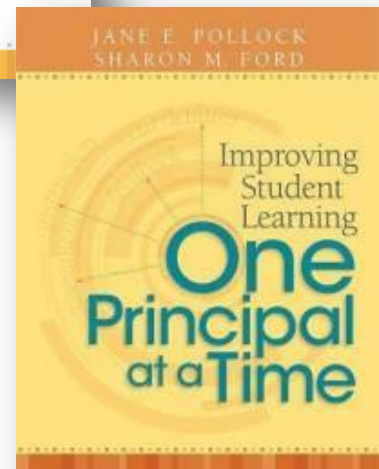
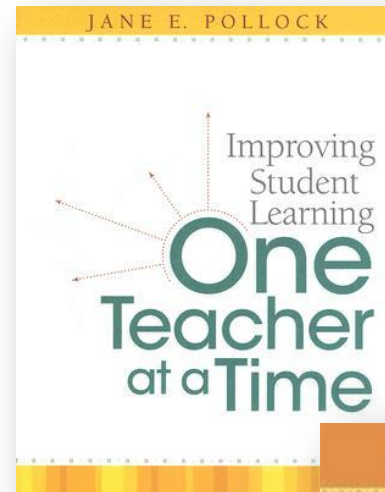
G= goal

A= access prior knowledge

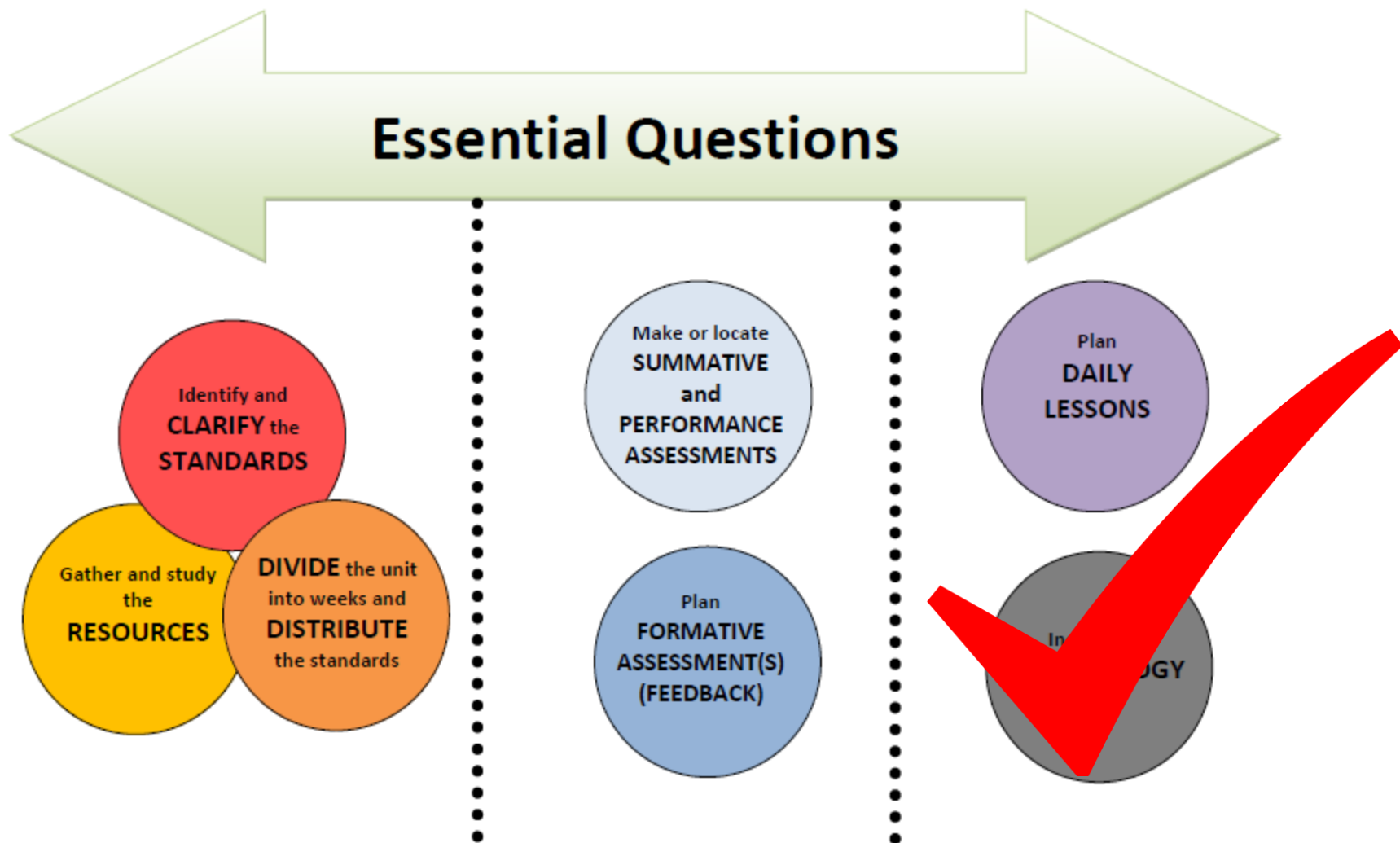
N= new information

A= application

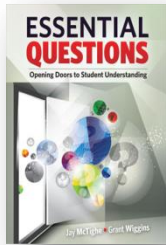
G= generalize the goal



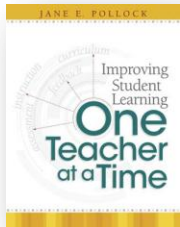
Backward Unit Planning 1.0



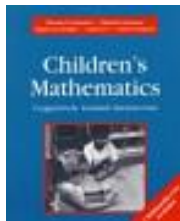
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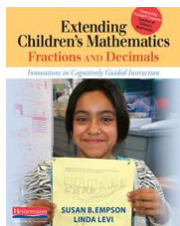
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