Constructing Task - Comparing MP3s

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.NF.1 Use equivalent fractions as a strategy to add and subtract fractions.
Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

MCC5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

MCC5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

MCC5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

MCC5.NF.5 Interpret multiplication as scaling (resizing), by:

a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

MCC5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS

- How can we model an area with fractional pieces?
- How can modeling an area help us with multiplying fractions?
- What does it mean to decompose fractions or mixed numbers?
- How can decomposing fractions or mixed numbers help us multiply fractions?
- How can decomposing fractions or mixed numbers help us model fraction multiplication?

MATERIALS

- Comparing MP3s Task
- Pencil, ruler
- Grid paper
- Accessible manipulatives

GROUPING

Pair/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Comments:

This task was developed to give students a real world application of multiplying fractions that would be engaging. It also allows students to take the concept of arrays, which they should have worked with extensively in their study of multiplication, and apply this familiar concept to another area of multiplication – fractions.

Students should be allowed to draw representations of their thinking. This allows them to "talk through" their process which in turn enables students the opportunity to attend to precision as they explain and reason mathematically.

BACKGROUND KNOWLEDGE

Students engaging in this task should be familiar with arrays and part-whole thinking as applied to multiplication. See Teaching Student Centered Mathematics, Vol. 2, Slicing Arrays Activity, pg. 66.

Teacher Notes:

Before the lesson, work with students on a computation discussion involving whole number multiplication. Use the following problems, one at a time, giving think time and time to share after each one. Discuss strategies and give students opportunities to make connections to other areas in mathematics.

10 x 16
5 x 16
16 x 1
16 x ½
16 x 1 ½
16 x 2 ½
12 x 2 ½

Introduce the problem and be sure everyone is clear with the context. You may wish to use the pictures included at the end of this task to help develop this context.
Facilitate a preliminary discussion with the class, before students get to working on the problem. Allow students to share their initial thoughts, then ask them to work in pairs to investigate the following:
One of the most popular mp3 players is Apple's iPod. However, companies such as Samsung and Microsoft are trying to take some business away from Apple with their own products – some of which can cost much less. As of right now, Apple's iPod is holding on to its share of the mp3 market, but some of the other mp3 players offer larger screens that might lure consumers from the Apple.
Apple wants our help to make a mock-up. They think if they can make the length of the new iPod Touch 1 ¼ times longer than it is now and the width 1 ½ times longer than it is now, the iPod Touch will still be small enough to fit in a pocket or purse, but will have a screen size that is easier to use and work with and be large enough to compete with the largest touch screen mp3 player out there.

How much larger is each dimension of the prototype iPod than the original?
How much larger is the area of the prototype iPod than the original?
Samsung's model (Galaxy Player 5.0 with wi-fi) has dimensions of 3 1/10 x 5 3/5. Will Apple's new prototype be larger than Samsung's?

Possible struggles students may have can be turned into wonderful inquiries! As students investigate the screen areas, you may notice them:

Cutting each screen into familiar pieces first, such as wholes, then the fractional pieces. This strategy may lead them to an open array model.

Using multiplication of fractions algorithm. This strategy may promote discussion, so please allow students the freedom to make sense of this in the closing part of the lesson.

FORMATIVE ASSESSMENT QUESTIONS

- How can you tell that your answer is correct?
- How far away from a whole is your fractional area? How do you know?
- Did you develop a strategy to find your answers?
- Did you identify any patterns or rules? Explain!

After enough time has been devoted to the task, ask pairs of students to make posters to prepare for the closing of the lesson. Posters should be clear enough for others in the class to understand their thinking, but should not just be the figuring that was initially done copied over again. The posters should be clear and concise presentations of any important ideas and strategies students wish to present.
Some ideas to encourage discussion about in the presentations of student work:
Estimation, using just the whole number dimensions can help determine whether areas are in the ballpark.
How mixed numbers are decomposed can make a difference.
Students who use any algorithm should also make sense of the algorithm used with some kind of model.

DIFFERENTIATION

- **Extension**
  Students should work on contextual problems such as those found in Teaching Student Centered Mathematics, by John Van de Walle, pgs 167-172. Possible student representations are also presented in these pages.
- **Intervention**
  Students requiring intervention should also use contextual problems such as those found in Teaching Student Centered Mathematics, by John Van de Walle, pgs 167-172. Students should be talking their way through the problems with teacher support and questioning.

Technology
http://www.bbc.co.uk/schools/ks2bitesize/maths/number/ games and activities for students to use for practice working with fractions and decimals, as well as whole numbers.

http://www.counton.org/games/map-fractions/falling/ in this game, students find fractions of whole numbers to collect leaves falling from a tree. The player with the most leaves at the end of the game is the winner.

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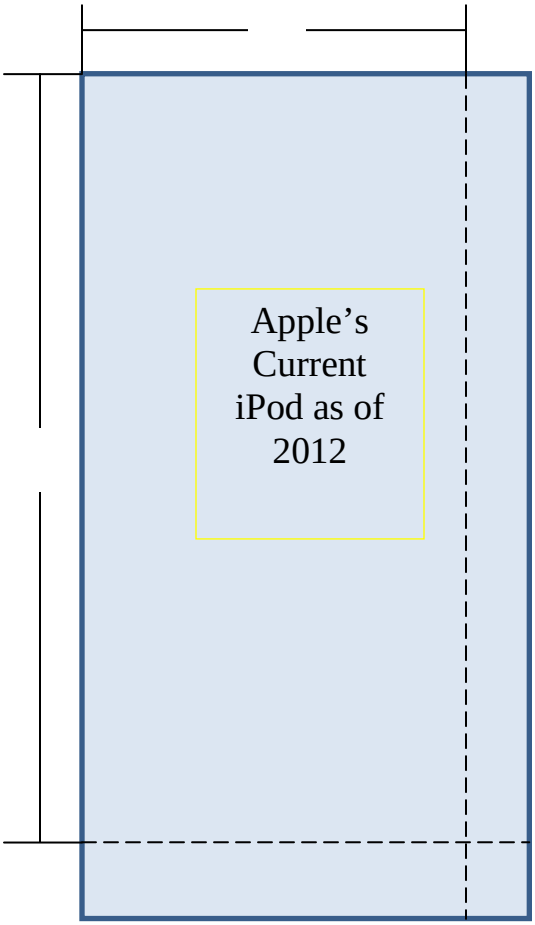
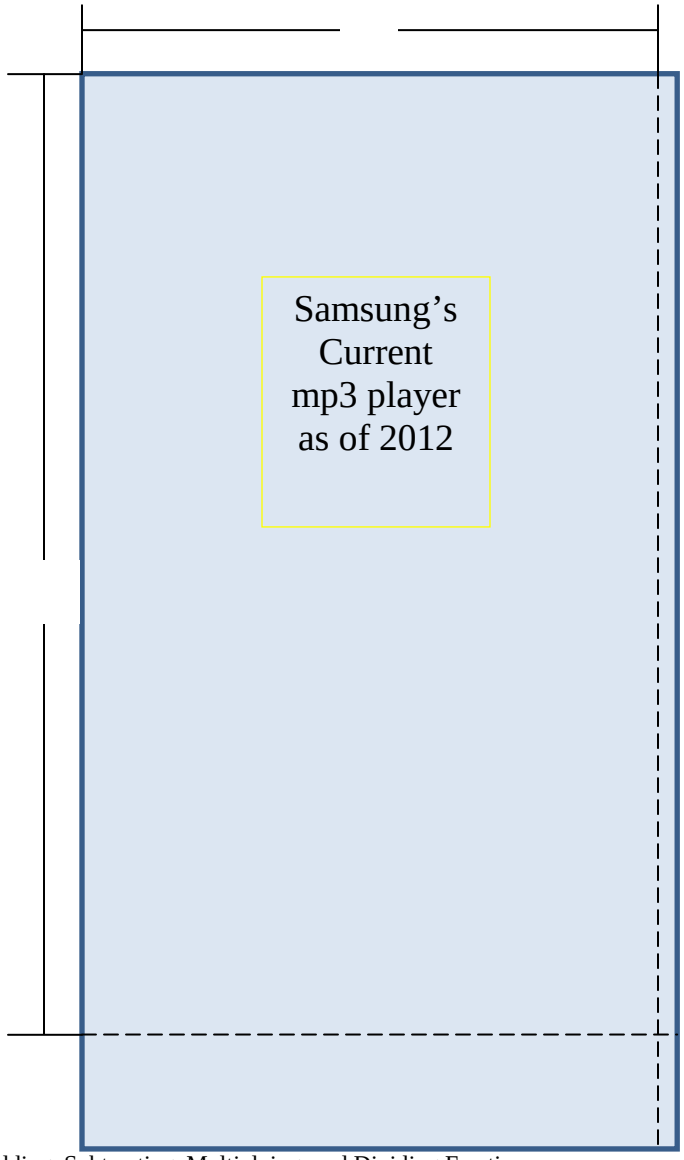
Apple wants your help to make a mock-up. They think if they can make the length of the new iPod Touch 1 ¼ times longer than it is now and the width 1 ½ times longer than it is now, the iPod Touch will still be small enough to fit in a pocket or purse, but will have a screen size that is easier to use and work with and be large enough to compete with the largest touch screen mp3 player out there.

Current iPod dimensions:
2 1/3 x 4 2/5



Current Samsung Galaxy 5.0 dimensions:
3 1/10 x 5 3/5



What will the dimensions (length and width) of the new iPod Touch mock-up be, if the length and width are increased according to Apple's specifications?

Use grid paper, a pencil, and a ruler, or a drawing or word processing program to create your mock-up. Use the dimensions you found in step one to create the rectangle dimensions.

Find the area of each mp3 player including the new iPod Touch mock-up and make a decision based on your mathematics as to whether the new iPod Touch mock-up will be able to compete with Samsung's model. Be sure to include the areas of the screens, the differences in their dimensions, which one is larger and whether you think current iPod Touch users would like this change in size in your decision support statement.