Culminating Task: Boxing Boxes

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

MCC5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

MCC5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

MCC5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

BACKGROUND KNOWLEDGE

"*Volume* typically refers to the amount of space that an object takes up" whereas "*capacity* is generally used to refer to the amount that container will hold," Van de Walle (2006) (p. 265). To distinguish further between the two terms, consider how the two are typically measured. Volume

is measured using linear measures (ft, cm, in, m, etc) while capacity is measured using liquid measures (L, mL, qt, pt, g, etc). However, Van de Walle reminds educators, "having made these distinctions [between volume and capacity], they are not ones to worry about. The term *volume* can also be used to refer to the *capacity* of a container" (p. 266).

Van de Walle, J. A. & Lovin, L. H. (2006). *Teaching students-centered mathematics: Grades 3-5*. Boston: Pearson Education, Inc.

ESSENTIAL QUESTIONS

- Can different size containers have the same volume?
- How can we measure volume?

MATERIALS

- "Boxing Boxes" student recording sheet
- Snap cubes and/or 1" grid paper (several sheets per student), scissors, and clear tape
- "Boxing Boxes, Part II" student recording sheet (optional)

GROUPING

Individual/Partner Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students explore volume while packing shipping boxes with various-sized merchandise boxes.

Comments

This task can be introduced by asking small groups of students to create the different sized merchandise boxes using grid paper or snap cubes. If using grid paper, students will need to sketch the nets for the boxes described on 1" grid paper and then cut the nets out and fold them to create the rectangular prisms. If using snap cubes, students can create the required rectangular prisms with snap cubes using the dimensions required. Students can then use these models while working on the task.

Allow students to create their own chart for the "Boxing Boxes" task that makes sense to them. Then allow students to share their chart with students in their small group and choose two or three students who created different charts to share their work with the class.

Notice that the capacity of the standard shipping box is 12 ft^3. Therefore, the sum of the volumes of the merchandise boxes packed must equal 12 ft^3 for each packing plan (see table below).

Volume of Merchandise Boxes (Number of Boxes × Volume of each Box in ft 3)					
Packing Plans	**Merchandise Box W**	**Merchandise Box X**	**Merchandise Box Y**	**Merchandise Box Z**	**Total Volume**
1				**12 × 1 ft^3**	**12 ft^3**
2	**2 × 6 ft^3**				**12 ft^3**
3	**1 × 6 ft^3**	**1 × 4 ft^3**		**2 × 1 ft^3**	**12 ft^3**
4	**1 × 6 ft^3**			**6 × 1 ft^3**	**12 ft^3**
5		**3 × 4 ft^3**			**12 ft^3**
6		**2 × 4 ft^3**		**4 × 1 ft^3**	**12 ft^3**
7			**1 × 8 ft^3**	**4 × 1 ft^3**	**12 ft^3**
8		**1 × 4 ft^3**		**8 × 1 ft^3**	**12 ft^3**

Merchandise Packing Guide

Number of Merchandise Boxes in Each Standard Packing Box				
Packing Plans	**Merchandise Box W**	**Merchandise Box X**	**Merchandise Box Y**	**Merchandise Box Z**
1				12
2	2			
3	1	1		2
4	1			6
5		3		
6		2		4
7			1	4
8		1		1

The volume of the merchandise boxes are as follows:

Merchandise Box W: 1 ft x 3 ft x 2 ft = 6 ft^3
Merchandise Box X: 1 ft x 2 ft x 2 ft = 4 ft^3
Merchandise Box Y: 2 ft x 2 ft x 2 ft = 8 ft^3
Merchandise Box Z: 1 ft x 1 ft x 1 ft = 1 ft^3

The capacity of the standard shipping box is 2 ft × 3 ft × 2 ft = 12 ft^3

Additionally, students will need to write a letter to their boss explaining how to use the chart they created.

Task Directions

Students will follow the directions below from the "Boxing Boxes" student recording sheet.

You have been hired by Boxes Unlimited to help determine the best way to package merchandise for shipping.

Boxes Unlimited has a standard shipping box which will hold merchandise measuring 2 ft by 3 ft by 2 ft.

Boxes Unlimited needs to pack merchandise they receive into the standard shipping box. The merchandise arrives in four different box sizes.

Merchandise Box W is 1 ft. x 3 ft. x 2 ft.
Merchandise Box X is 1 ft. x 2 ft. x 2 ft.
Merchandise Box Y is 2 ft. x 2 ft. x 2 ft.
Merchandise Box Z is 1 ft. x 1 ft. x 1 ft.

Your task is to create a chart for employees to use as a reference when they are packing boxes for shipment. Be sure to include the volume of each merchandise box and the capacity of the standard shipping box on your chart. Convert the capacity of the standard shipping box from cubic feet to cubic yards.

Write a report to your boss explaining how to read your chart.

FORMATIVE ASSESSMENT QUESTIONS

- Have you found all of the possible ways to fill the standard shipping box? How do you know?
- What is the total capacity of the standard shipping box? Will the merchandise completely fill the standard shipping box? How do you know?
- How are you organizing your packing chart? Why did you choose this type of organizational chart?
- Can you explain how your chart could be used by the employees who pack boxes?

DIFFERENTIATION

Extension

- Ask students to consider a large shipping box with dimensions of 3 ft × 3 ft × 3 ft. What are the ways that this packing box could be filled with the given merchandise boxes? Students could work the task with this large shipping box rather than the regular shipping box. Next, students who worked with the large shipping box could be paired with students who worked on the standard shipping box. Partners could then be asked to determine which size box would be a better choice and justify their thinking.

Intervention

- Encourage students to use snap-cubes to create models of the merchandise boxes.

- Students who would benefit from a chart in which to record their work should be provided one. A sample is given below. See "Boxing Boxes, Part II" student recording sheet.

Name_____ Date _____

Boxing Boxes

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Write a report to your boss explaining how to read your chart.

Name_____ Date _____

Boxing Boxes
Part II

The volume of the merchandise boxes are as follows:

Merchandise Box W: _____

Merchandise Box X: _____

Merchandise Box Y: _____

Merchandise Box Z: _____

The capacity of the standard shipping box is _____.