# Finding Common Partitions for Fractional Units

UNIT

14

## Mathematical Concepts

- A measure of length is obtained by iterating identical units.
- Addition means joining lengths.
- The measure of a sum of lengths is obtained by iterating identical units of measure.
- For partial units, identical units mean finding a common partition, so that $\frac{a}{b}$ unit + $\frac{c}{d}$ unit is $\frac{(ax+by)}{c}$ unit.
- For $\frac{a}{b}$ unit + $\frac{c}{d}$ unit, a common partition is found by composing splits of the same unit of measure, either by finding a common multiple or by composing b x d.
- Two fractional measures are equivalent when they are at the same distance from the origin of the scale of measure.

## Unit Overview

The lesson begins with a consideration of the problem of finding the measure of a sum when different splits of the same unit are joined. Students find that the measure of the sum can be expressed by a common partition obtained by composing the different splits (e.g., compose a 2-split and a 3-split to represent both as sixths) or by finding a common multiple (e.g., 2-split and 4-split have 4 as a common multiple).

Academic Language:
- Equivalent (or equal)
- Numerator as copier
- Denominator as splitter
- Common denominators

Contents

# Materials & Preparation





## Read

- ☐ **Unit 14**
  Start by reading the unit to learn the content and become familiar with the activities.
- ☐ **Mathematical Background**
  Reread the Mathematical Background to anticipate the kinds of ideas and discussions you will likely see during instruction.

## Gather

- ☐ BF strips
- ☐ Tape

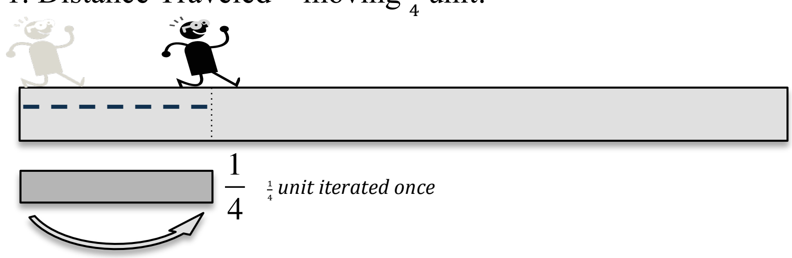
## Prepare

For each student: 8 BF strips

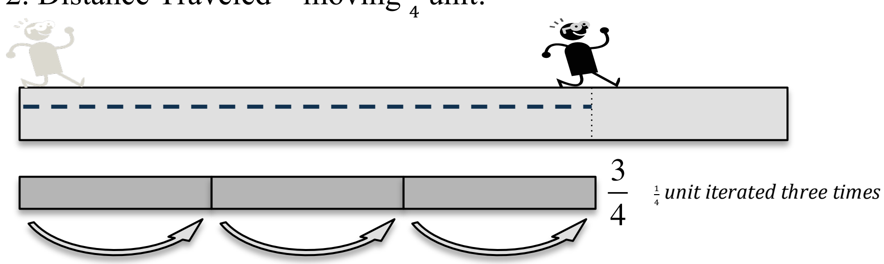
## Mathematical Background

See Measure Units 5-10 for expositions of the meaning of the arithmetic operations of addition, subtraction and multiplication for measured lengths. As in previous lessons, two metaphors are used to help students understand the meaning of a fractional quantity. One relies on unit iteration where, for example, $\frac{3}{4}$ means 3 iterations of $\frac{1}{4}$ unit. This sense of fraction relies on a copy of a partitioned unit, as in 3 copies (iterations) of $\frac{1}{4}$ unit results in $\frac{3}{4}$ unit. The second metaphor is distance traveled where, for example, $\frac{3}{4}$ means starting at 0 and moving $\frac{3}{4}$ unit. Both of these metaphors, one static and the other dynamic, help students form images of fractions that will later help them locate fractions on the number-line, because the number-line is an idealized ruler.

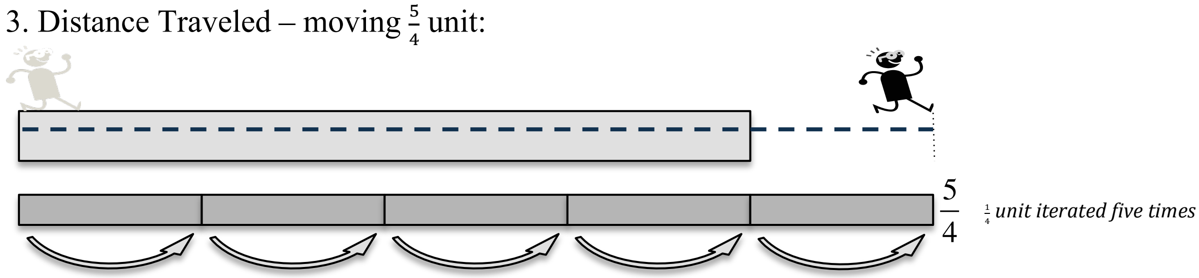
1. Distance Traveled – moving $\frac{1}{4}$ unit:




2. Distance Traveled – moving $\frac{3}{4}$ unit:




3. Distance Traveled – moving $\frac{5}{4}$ unit:



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Introducing the Problems of Establishing Common Partitions for Measuring with Partial Units
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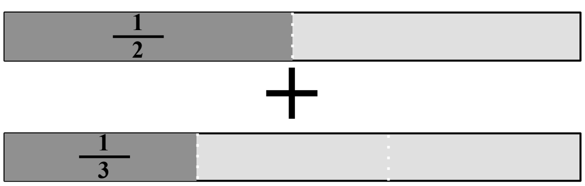
Adding lengths means joining them together, or when traveling, first traveling some straight distance *x* and then continuing to travel another straight distance *y*, for a total distance traveled of $x + y$. To measure the total distance traveled, we employ identical units of measure, so that 5 *pez* + 11 *pez* = 16 *pez*. The distance traveled then is 16 times as long as 1 *pez*. We can employ fractional measures of *pez* as well, so that 5 *pez* + 11 *pez* + $\frac{2}{3}$ *pez* = 16 $\frac{2}{3}$ *pez*. In a like manner, $\frac{5}{4}$ *pez* + $\frac{6}{4}$ *pez* = $\frac{11}{4}$ *pez*, meaning that the distance traveled is 11 times as long as $\frac{1}{4}$ *pez*, a distance of 2 $\frac{3}{4}$ *pez*.

When we use partial units, we assume that all the partitions are congruent, as in $\frac{3}{2}$ *pez* + $\frac{1}{2}$ *pez* = $\frac{4}{2}$ *pez*. This follows from our emphasis on identical units of measure.

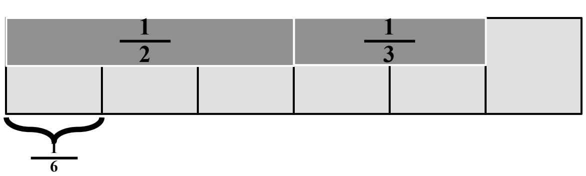
However, if we have a situation like the one depicted below, we need to establish a common partition of the unit in order to use the accumulation of these partitions as a measure of a sum.




To find a common partition, we compose the splitters a 3-split of a 2-split ($3 \times 2$) resulting in a 6 split.

To use the common splitter, we need to express each fractional unit in terms of the common split.

$\frac{1}{3}$ *pez* = $\frac{2}{6}$ *pez* Because each third is twice as long as each sixth, to travel the same distance, we need to use twice as many sixths as we did thirds. So, $\frac{1}{3}$ *pez* = $\frac{2}{6}$ *pez*. And, because each half is three times as long as each sixth, we need to use three times as many sixths as we did halves to travel the same distance, so $\frac{1}{2}$ *pez* = $\frac{3}{6}$ *pez*.




Once we have a common partition, then we can use the now identical units to measure the joined length. So $\frac{2}{6}$ *pez* + $\frac{3}{6}$ *pez* = $\frac{5}{6}$ *pez*.

## Introducing the Problems of Establishing Common Partitions for Measuring with Partial Units

### Whole Group

1. Here is a BF unit. **What does it mean to add 3 BF units to 5 BF units?** What I do? (join the measures). What is the result? (8 BF) Suppose I traveled 47 BF units and then 153 BF more, how far did I travel?
2. Here is a BF unit again. **This time I travel $\frac{5}{2}$ BF and then $\frac{19}{2}$ BF more. How far have I traveled?**
3. Here is the BF unit again. **This time, a bug walks for $\frac{1}{2}$ BF and then walks for another $\frac{1}{3}$ BF. How far has the bug traveled? Is it less or more than 1 BF?**

### Partner/ Small Group (each group has 3 or 4 BF strips)

4. **Using the BF strips to help you solve this problem: How can we measure $\frac{1}{2}$ BF and $\frac{1}{3}$ BF so that the splits of the unit are exactly the same—the split of the BF is the same for both fractions? And the number of copies of that common split will be the same distance from zero or the starting point for each partial unit. That means that $\frac{1}{2}$ BF and $\frac{copies}{common\ measure}$ are exactly the same distance from zero.** Try some things out and talk with your partner.

*Teacher Note*. As students work, decide whether or not some hints will be helpful. One hint might be to think of how they made their BF ruler. Are there any splits that could be used to make an equivalent measure? Another hint might be more specific: what happens if you split the half by three—how many equal partitions does that make ( $\frac{1}{3} \times \frac{1}{2}$ unit).

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## Whole Group: Guided Solution

5. **Thinking back to when you made the BigFoot ruler, could any of those partitions help—we made thirds, sixths, and twelfths?**

Q: If we take $\frac{1}{3}$ of $\frac{1}{2}$ unit, what is the result?

Q: How many sixths in $\frac{1}{2}$ BF? Try it and see if you are right. We say that $\frac{1}{2}$ BF = $\frac{3}{6}$ BF. They are the same distance from the starting point.

Q: $\frac{1}{2}$ BF is _________times as long as $\frac{1}{6}$ BF. Test your answer. (Show that 3 iterations of $\frac{1}{6}$ BF is the same distance traveled as $\frac{1}{2}$ BF)

Q: How many sixths in $\frac{1}{3}$ BF? Try it and see if you are right.

Q: $\frac{1}{3}$ BF is ______ times as long as $\frac{1}{6}$ BF/ Test your answer (Show that 2 iterations of $\frac{1}{6}$ BF is the same distance traveled as $\frac{1}{3}$ BF)

Q: So we can say that $\frac{1}{3}$ BF = $\frac{2}{6}$ BF.

Q: $\frac{3}{6}$ BF + $\frac{2}{6}$ BF = ______.

Q: $\frac{2}{6}$ BF + $\frac{3}{6}$ BF = ______.

## Partners / Small Group (use the strips as needed)

6. **Use a common split (partition) of 12ths to solve $\frac{1}{2}$ BF + $\frac{1}{2}$ BF.**

## Whole Group: Guided Solution

Q: $\frac{1}{2}$ BF = $\frac{\_\_}{12}$ BF.

Q: $\frac{1}{2}$ BF is ____ times as long as $\frac{1}{12}$ BF. (Iterate $\frac{1}{12}$ BF 6 times to show that this must be true.)

Q: $\frac{1}{3}$ BF = $\frac{\_\_}{12}$ BF.

Q: $\frac{1}{3}$ BF is _____times as long as $\frac{1}{12}$ BF. (Iterate $\frac{1}{12}$ 4 times to show that this is true.)

Q: $\frac{6}{12}$ BF + $\frac{4}{12}$ BF = $\frac{10}{12}$ BF

Q: Does $\frac{10}{12}$ BF = $\frac{5}{6}$ BF?

## Whole Group: Generalizing a Method

7. **When we have 2 different splits and we want a common split so that we can measure them with the same partial unit, then we can multiply the two splitters, just as we have done before: $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ . After we find a common split, we need to re-express the distance traveled by each fractional unit as the same distance traveled in this common unit. So, for example, $\frac{1}{3}$ unit = $\frac{2}{6}$ BF and $\frac{1}{2}$ BF = $\frac{3}{6}$ BF.**

8. **Another way to generate a common unit is to skip count by each denominator until a common number of splits is found. For $\frac{1}{3}$ and $\frac{1}{2}$ , 3, 6, 9, 12 and 2, 4, 6, 8, 10, 12. 6 and 12 are both common, so either can be the common split of the unit. Or, for $\frac{1}{4}$ u + $\frac{1}{6}$ u, skip counting by 4 produces 4, 8, 12 and skip counting by 6 produces 6, 12 with a common split of 12. Hence, $\frac{3}{12}$ u + $\frac{2}{12}$ u = $\frac{5}{12}$ u.**

## Partner / Small Group

9. **Try both of these methods for the following (use the BF strips to check your results but first try to just use the fractional units).**

   (a) $\frac{1}{3}$ BF + $\frac{1}{4}$ BF = ____________

   (b) $\frac{2}{3}$ BF + $\frac{1}{4}$ BF = ____________

   (c) $\frac{1}{8}$ BF + $\frac{1}{4}$ BF = ____________

   (d) $\frac{5}{8}$ BF + $\frac{2}{4}$ BF = ____________

## Whole Group

10. **Review solution strategies and make sure students use the strips to check their results.**

# Formative Assessment





For each problem, show your work:

1. $\frac{5}{8}$ unit + $\frac{11}{8}$ unit = __________
2. $\frac{1}{3}$ unit + $\frac{1}{2}$ unit = __________
3. $\frac{1}{8}$ unit + $\frac{1}{4}$ unit + $\frac{1}{2}$ unit = __________
4. $\frac{5}{6}$ unit + $\frac{1}{2}$ unit = __________
5. $\frac{2}{3}$ unit + $\frac{3}{4}$ unit = __________

# Formative Assessment Record

Finding Common Partitions Unit 14

Name:___________________________________ Date:____________

| Item | Student Response | Strategies |
|---|---|---|
| $\frac{5}{8}+\frac{11}{8}$ | | |
| $\frac{1}{3}+\frac{1}{2}$ | | |
| $\frac{1}{8}+\frac{1}{4}+\frac{1}{2}$ | | |
| $\frac{5}{6}+\frac{1}{2}$ | | |
| $\frac{2}{3}+\frac{3}{4}$ | | |