Ribbon Paths: Walking and Representing Rectangles

UNIT

11

Mathematical Concepts

- An angle is a rotation — a turn.
- Turn angles are measured as parts of whole turns.
- A path can be composed of measured straight-line segments and measured turns.
- Blueprints (representations) of paths can be used to guide their construction.
- Representations of angle tell us at a glance the direction and amount of rotation.

Unit Overview

The goal is to write directions for, and then to construct, a rectangular walking path. A rectangular path includes both straight segments and turns. Students first consider what they know about rectangles. Then they decide how to iterate a yardstick to create sides of length measures of their choosing. For example, one side might be 3 yards long and the other 5 yards long. To measure turns, 1 rotation of the body is used as the standard: 1 Whole Turn. Students explore different measures of turn-angles: $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ Whole-Turn. Following these investigations, students use flags, ribbon (surveyor tape), and a yardstick to construct and mark rectangular paths. Students represent their constructions with a set of directions and with a diagram that shows turn angles and distances. During whole-group conversation, students consider how to modify their directions to create larger or smaller figures, or to create square paths.

Contents

Materials & Preparation

Read

- **Unit 11**
  Start by reading the unit to learn the content and become familiar with the activities.

- **Mathematical Background**
  Reread the mathematical background carefully to help you think about the important mathematical ideas within the unit. Reread the Student Thinking boxes to anticipate the kinds of ideas and discussions you will likely see during instruction.

- **Angle Measurement Construct Map**
  Read the Angle Measurement Construct Map to help you recognize the mathematical elements in student thinking, and to order these elements in terms of their level of sophistication.

Gather

- ☐ Surveyor tape of different colors
- ☐ Stakes with Flags to mark turning points
- ☐ Yardsticks
- ☐ Clipboards
- ☐ Copies of worksheet--Directions for Walking a Rectangle
- ☐ 8.5 × 11 in, blank paper
- ☐ Patty paper for the formative assessment

Prepare

Identify a large enough open space (playground, field, gym) in which students can work to measure distances in yards and use surveyor tape to represent the resulting path.

Mathematical Background

The big ideas of measurement emphasized in this unit include the distinction between units of length measure and units of angle measure. Paths are represented as compositions of straight line segments and turns. The measures of the path help students reason about properties of rectangles and squares and how one might consider a square as a kind of rectangle.

Straight

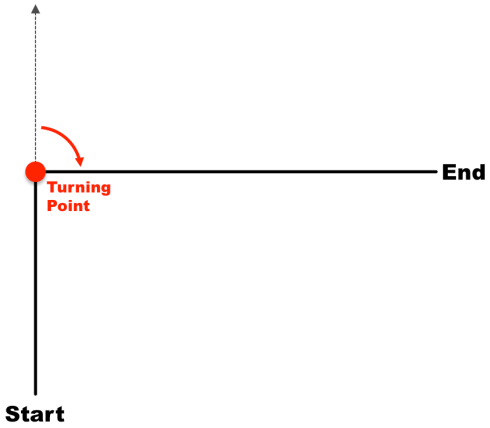
When is a line straight? Often, when we measure a distance between two points, we imagine a line. A traditional way of thinking about a straight line is as the shortest distance between two points. But it is more consistent with bodily experience to consider a line to be straight when it is formed by moving without any change in direction. For instance, walking at a constant heading while towing a piece of chalk ideally creates a straight line. Or walking toward a landmark while keeping it in one's line of sight creates a straight path.

Angle

An angle is a directed rotation from a heading.

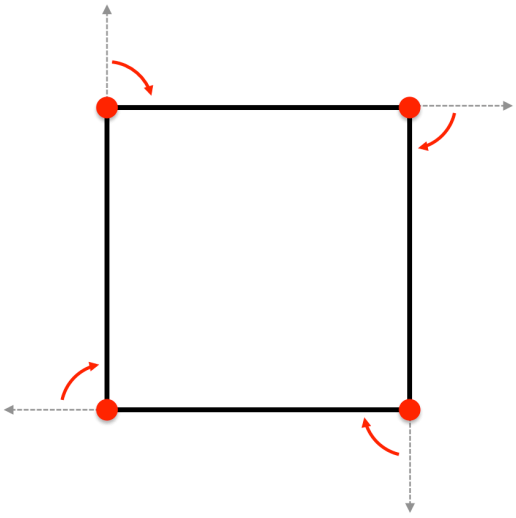
Representation

Angles-as-turns can be represented as illustrated below. From a starting point, travel to TP (the turning point). The dashed line shows the original heading (the direction of travel) and the solid line shows the heading that results from turning and then moving. The arc-arrow between the dashed and solid lines shows the direction of rotation.



Polygon

A polygon in the plane can be thought of as a path consisting only of straight segments joined by turns, where the person walking the path of the polygon begins and ends at the same place and is oriented in the same heading at the beginning and end of the walk. The path does not intersect itself. The turn angles of a polygon are called the *exterior angles*, and the sum of these turn angles is 1 whole turn. A square illustrates these properties. Imagine starting at the bottom left vertex facing due north or vertically.



Move straight for a distance, 1u, to produce a length of a side. Then instead of continuing to walk straight, represented by the dashed line, turn to the right $\frac{1}{4}$ of a whole turn. Walk straight again the same distance, 1u, to produce a length of another side of the square. The dashed line represents continuing straight ahead, but instead, turn to the right (clockwise) $\frac{1}{4}$ of a whole turn. Continue in this manner until the square is completed and you are facing in the original direction. The total number of turns is four and total amount of turn is 1 whole-turn (WT):

$\frac{1}{4}$ WT + $\frac{1}{4}$ WT + $\frac{1}{4}$ WT + $\frac{1}{4}$ WT = 1 WT

Measuring Distance

Whole Group

1. **You will be using this (show surveyor tape) to walk a path outside that makes a square or a rectangle.**

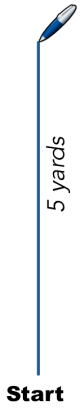
   What is a rectangle?

   What is a square?

Teacher Note
Elicit properties of rectangle. Highlight: pairs of congruent opposite sides, parallel opposite sides (meaning that the sides will not intersect even if extended), and 4 corners that look like the corners made when the walls of a room meet the floor or ceiling (students may say right angles).

2. **Let's think about how to measure the lengths of the sides of a rectangle.** We have been using a person's foot to measure length, but we know that when the distances get longer, it is easier to use units that are longer. So we will use this unit today—a yard. A yard is 3 feet long, and the foot is exactly this long. Hold up a conventional 1 foot ruler and demonstrate that three iterations of 1 foot is esactly as long as 1 yard. So, 1 yard is 3 times as long as 1 foot. And 1 foot is how many times as long as 1 yard? ($\frac{1}{3}$)

3. **Let's think about how to walk a rectangle.** If I start here (mark on whiteboard), and I use yards as my unit of length measure, I could represent traveling five yards from this starting point to here. When we walk outside, how can we stay straight like this line segment? (Keep a line of sight on a landmark)



4. **When we reach the end of the 5 yards,** if we kept going straight, we would do this (demonstrate with broken line the continued straight path).

5. **But we know that a rectangle's sides look like this (draw the second side).** So, what should we do at this point (gesture to the intersection of the two line segments).

Q: If we turned all the way around once, how much of a turn is that? (1 whole turn) Everyone get up and try that. Which direction did you turn? If you turned to the right, we can represent this as TR 1 whole-turn, or if to the left, as TL 1 whole-turn.

Q: How much and in what direction do we need to turn so that we can start walking the second side? (TR $\frac{1}{4}$ whole-turn or TL $\frac{3}{4}$ whole-turn). Let's use turning to the right, this is sometimes called clockwise. Why? (Refer to analog clock)

Q: When I write TR $\frac{1}{4}$ Whole-Turn, what does T mean? R? $\frac{1}{4}$ Whole-Turn?

Q: Let's get up and use bodies to turn. Let's all face so that when we look straight ahead we see _____ (landmark). I'll call out the turn and you turn:

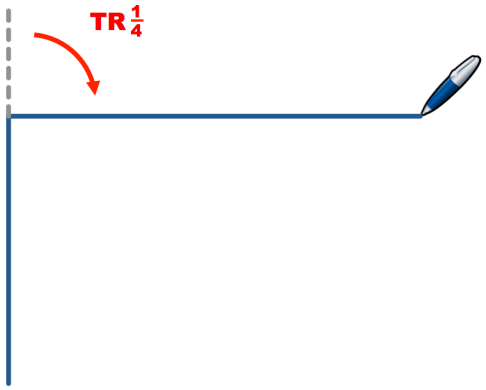
After each demonstration and disussion, have student reset their bodies to face the same landmark again:

Turn Right $\frac{1}{4}$ Whole-Turn
Turn Right $\frac{1}{2}$ Whole-Turn
Turn Right $\frac{3}{4}$ Whole-Turn
Turn Left $\frac{1}{4}$ Whole-Turn
Turn Left $\frac{1}{2}$ Whole-Turn
Turn Left $\frac{3}{4}$ Whole-Turn
Turn Right $\frac{1}{8}$ Whole-Turn

Q: How can I show turning at this point on my diagram?

Teacher Note

Use the notation of the dashed line to represent the original heading and an arc with arrowheads suggesting direction of rotation. Notate as TR $\frac{1}{4}$ whole-turn. *Be sure to make certain that this representation is understood by students*, perhaps by inviting one or more to enact with their fingers what is happening in the diagram.



Partners

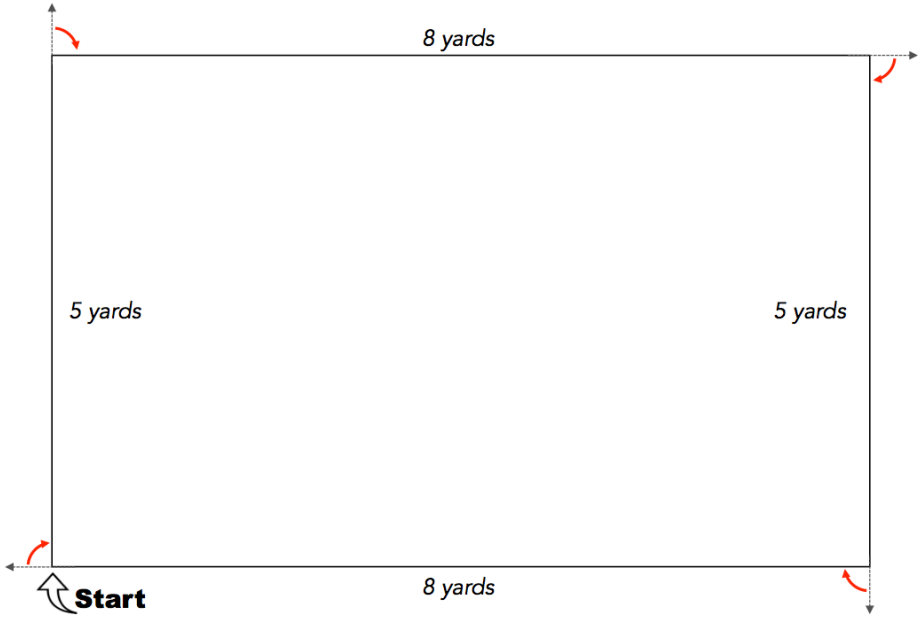
Each team of partners will get 4 flags (show), a yardstick, and some surveyor tape. We wll use the flags and ribbons to make a rectangle walking path on the lawn. Everyone look at the "Directions for Walking a Rectangle Path." Work with your partner to complete these directions as you make your ribbon path. When you are done, on the next page, draw a diagram of your rectangle that shows how far you walked for each side and how much you turned.

Teacher Note 1

Ideally, students will write something like:

Look at the wall of the school.
Walk straight 5 yards. Turn Right $\frac{1}{4}$ whole-turn.
Walk straight 8 yards. Turn Right $\frac{1}{4}$ whole-turn.
Walk straight 5 yards. Turn Right $\frac{1}{4}$ whole-turn.
Walk straight 8 yards. Turn Right $\frac{1}{4}$ whole turn.

Ideally, students will draw something like:



Teacher Note 2

As partners work, assist thinking by asking or suggesting the following:

a. How do you know you are walking straight? Have you chosen something to keep in sight as you walk.

b. With ribbon that is a different color, use that ribbon to represent what would happen at a vertex if one continued to walk straight. Then sweep the differently colored ribbon $\frac{1}{4}$ of a whole turn so that it is superimposed on the side of the rectangle. Help children see that this sweeping motion is the same as the rotation of their bodies $\frac{1}{4}$ turn.

c. Have students re-enact the walking path, raising their right hands to turn right or their left hands to turn left.

d. For students with a firm grasp on the construction, challenge them to make a bigger (larger area) rectangle walking path.

e. For students with a firm grasp on the construction, challenge them to make a bigger path that includes a fractional unit, such as $5\frac{1}{2}$ yards.

Instruction

Whole Group

1. **After working outside to make ribbon paths,** students take a photo. For whole class conversation, choose three to four different paths with accompanying directions and diagrams. For at least one of the paths, represent it with a diagram on the whiteboard. Have different students enact walking the sides and turning. You might let students use a token to represent a person walking.

Teacher Note 1

Take photos of students' constructions to preserve for future versions of this lesson and to have a record that students can see of their path craft.

Teacher Note 2

During the whole class conversation, highlight the difference between measures of length and angle (turn).

Many students think that the direction of the turn will change at each vertex. Help them enact the turn at each vertex with their bodies and with the token figure to see that, for example, 4 clockwise $\frac{1}{4}$ turns will bring a walker back to the initial heading.

2. **Select one rectangle path and challenge students to change the directions to make a larger rectangle.** Also, challenge students to find the total number of yards walked (the perimeter) in the original and in the enlarged rectangle.

Teacher Note 3

Some students may suggest increasing the magnitude of the turn or its direction. You may wish to have students enact this suggestion so they can understand why changing the turns will not have the desired effect.

Formative Assessment Record

Formative Assessment

Give the formative assessment. Then select responses that help students revisit the difference between measures of length and measures of turning angle. Revisit the properties of a square and of a rectangle, with an eye toward helping students understand the rationale for modifying only the distances traveled in the path directions, not the turning angles. For the second item, it is often useful to physically enact a turn at each vertex, so that students can see that the measure of the turn is not affected by the length of the line segments.

Formative Assessment Record — Ribbon Paths Unit 11

Name: _____ Date:_____

1. After selecting a starting point, complete these directions to make a rectangle:

   Walk straight 6 yards. Turn Right $\frac{1}{4}$ whole-turn.

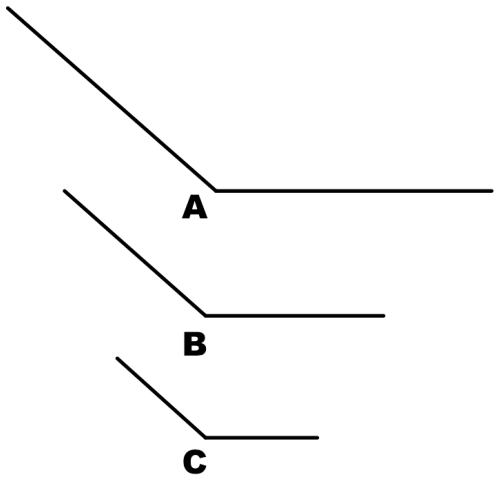
   Walk straight 10 yards. Turn Right $\frac{1}{4}$ whole-turn.

   Walk straight ____ yards. Turn _____.

   Walk straight ____ yards. Turn _____.

   How would you change your directions to change the path into a square?

2. Here are three paths and some patty paper to use if you like:



Circle all the statements that are true:

1. **A** has the largest turn angle.
2. **C** has the smallest turn angle.
3. **A**, **B**, and **C** have the same turn angle.
4. **B**'s turn angle is more than **C**'s.

For each statement that you said was true, tell why you think it is true:

1. _____
2. _____
3. _____
4. _____

Formative Assessment Record — Ribbon Paths Unit 11

Student _____ Date _____

Indicate the levels of mastery demonstrated in students' work outdoors and in their responses to the formative assessment items by circling those for which there is clear evidence:

Level	Description	Notes
Figure -as-path	+ Completes rectangle directions correctly. Modifies directions to make a square successfully, indicating that s/he differentiates and coordinates length and turn. 0 Completes rectangle directions correctly but fails to modify for square. - Cannot successfully complete either formative assessment item.	
ToMAL4E	Partition and compose partitions by factors of 2, and use the partitions as a unit when an object cannot be measured in whole units. Performance: Outdoor path lengths include 1/2 yards. Student represents fractional path lengths in diagram.	
ToML4D	Symbolize measure as whole number unit distance traveled. Performance: Diagram of outdoor path notates yard lengths of the sides.	
ToML4B	Use and justify standard (including conventional) unit. Performance: Student uses yardstick to iterate length.	
ToMA^{0}3A	Recognize and compare amount of turns. Performance: Student successfully incorporates magnitude and direction of turns into paths, so is correct for both items. May use patty paper to confirm that angles are the same for item 2.	
ToMA^{0}2B	Compare angles as turns. Performance: Student successfully incorporates turns into paths. Student ignores relative length of segments for item 2 and focuses turning one segment onto another, noticing that the amount of turn is about the same.	
ToMA^{0}1C	Compare openings for paths in item 2, does not mention turns. Notices they are all the same or establishes their equivalence with patty paper.	
ToMA^{0}1A	Confounds lengths of segments for item 2 with angle measure.	

Academic Language: Indicate academic words the student is familiar with by recording them here.

Worksheet — Ribbon Paths Unit 11

Name: _____ Date:_____

Directions for a Rectangle/Square Ribbon Path

START FACE_____

WALK STRAIGHT _____TURN_____

WALK STRAIGHT _____TURN_____

WALK STRAIGHT _____TURN_____

WALK STRAIGHT _____TURN_____

STOP

Diagram of Path:

Start