Constructing Task: Thoughts About Triangles

Adapted from a lesson in Navigating Through Geometry in Grades 3-5 by NCTM

STANDARDS FOR MATHEMATICAL CONTENT

MCC. 4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

MCC.4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students should have the following background knowledge.

- Be able to use a straight edge or ruler to draw a straight line.
- Know how to use a ruler, and how to identify right angles (90 degrees), obtuse angles, and acute angles (using the corner of an index card or another object with a known angle of 90 degrees).
- Understand that the side across from an angle on a triangle can be described as an opposite side
- Know parallel means that lines will never intersect or cross over each other no matter how long they are extended.
- Understand that perpendicular means lines or segments intersect or cross forming a right angle. (Some students may use a known 90 degree angle to show an angle is a right angle.)
- Know that a property is an attribute of a shape that is always going to be true. It describes the shape.
- Be able to use a ruler to measure sides to verify they are the same length.

Some properties of triangles that should be discussed are included below. As students draw conclusions about the relationships between different figures, be sure they are able to explain

their thinking and defend their conclusions. Much of the information below may come out as a result of students' explorations. This is information to look for and highlight as they explore the triangles to pull out, not a list of understandings that you must teach them beforehand.

- A shape is a triangle when it has exactly 3 sides and is a polygon. (To be a polygon the figure must be a closed plane figure with at least three straight sides and having no curved lines.)
- A right triangle is a triangle with one angle that measures 90 degrees. A right triangle can be either scalene or isosceles, but never equilateral.
- An obtuse triangle has one angle that measures greater than 90 degrees. There can only be one obtuse angle in any triangle.
- An acute triangle has three angles that measure less than 90 degrees.
- An equilateral triangle has three equal angles and three sides of equal length.
- An isosceles triangle has two equal angles and two sides of equal length.
- A scalene triangle has three sides that are not equal and no angles that are equal.

ESSENTIAL QUESTIONS

- What are triangles?
- How can you create different types of triangles?
- How are triangles alike and different?
- What are the properties of triangles?
- How can triangles be classified by the measure of their angles?

MATERIALS

For Each Group:

- Geoboard with one rubber band for each student
- A copy of "Geodot Paper for Geoboard"
- Paper
- Pencils

GROUPING

Partner/Small Group Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Comments

Make sure that students complete this activity in partners or small groups to encourage mathematical discussion while they make their triangles and test conjectures. You may wish to have students explore some on their own and then come together to discuss their findings. Students can then explain and defend their conclusions as a group.

The purpose of this task is for students to become familiar with the properties of triangles.

Working in pairs, students will create the following triangles: right triangles, obtuse triangles, acute triangles, isosceles triangles, scalene triangles, and equilateral triangles. They will identify the attributes of each triangle, then compare and contrast the attributes of different triangles. Though the standards only specifically state that students are to identify right triangles as a category for classification, the exploration of the attributes of all triangles is vital to students differentiating between right triangles and all other triangles.

Task Directions

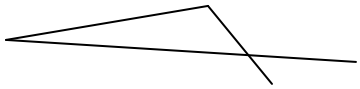
This task is a collection of investigations into triangles through the use of guiding questions. For each question students should (1) make a conjecture, (2) explore, using their geoboards, and (3) discuss their findings as a group. The class should come to a general consensus during their discussion. As students and the class come to a consensus about triangles, keep an anchor chart or running list of "true" ideas about triangles.

Make sure to guide discussion during explorations and discussion time through the use of questioning rather than intervening by answering their questions. For example, if students incorrectly identify a polygon as a right triangle, rather than telling them it's not a right triangle, ask them to explain how they know it is a right triangle and then discuss together the definition of a right triangle.

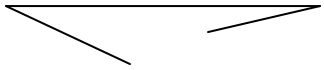
These questions lend themselves nicely to student reflection in math journals. The journal entries can be used as evidence of learning for the students. There is a sample journal entry question at the end of each exploration.

Question #1: Is it possible to make a three-sided polygon that is not a triangle?

- Have students make their conjectures and record the conjectures as a group.
- Have students explore answering and explaining their answer using their geoboards explorations.
- If students make a three-sided figure like the one below, ask students if their figure is closed with no lines crossing.



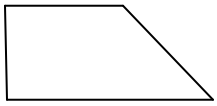
- If students make a figure like the one below, refer students back to the origin of the word triangle (three angles).



- At closing discussion, make a class list of all the properties of triangles, including triangles having three angles, three sides, and being classified as a polygon.
- Journal Reflection Question: What have you learned about triangles from this investigation?

Question #2: Is it possible for a triangle to have two right angles?

- Have students make their conjectures and record the conjectures as a group.
- Have students explore answering and explaining their answer using their geoboards explorations.
- Students may use the corner of an index card or another known right angle to tests for right angles.
- If students create a figure like the one shown below that has 2 right angles, ask students if their figure has all the properties of a triangle.

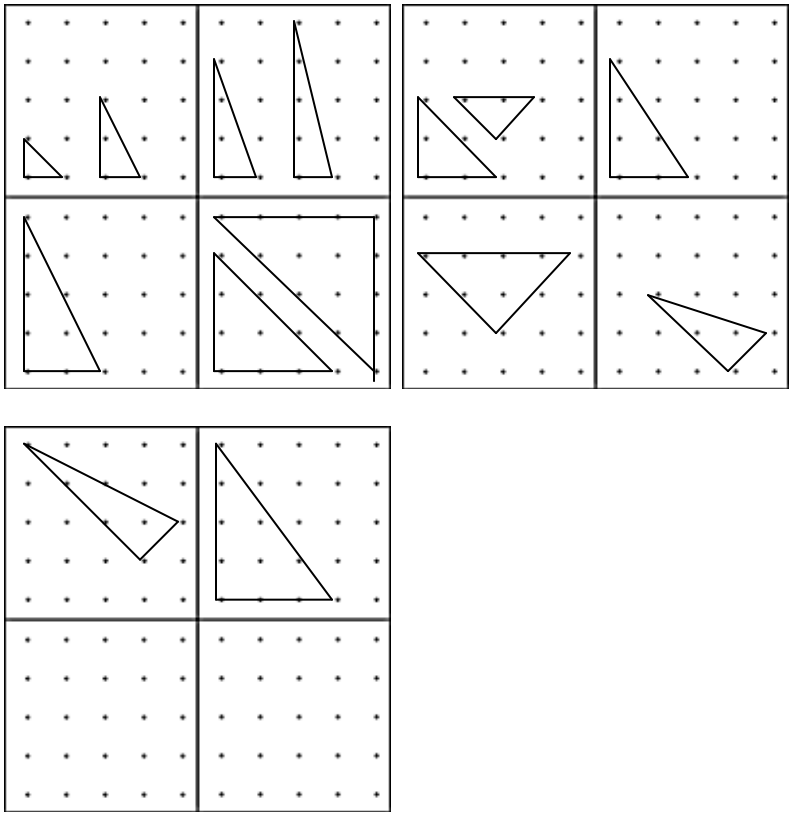


- At closing discussion, guide students to determine that there is a category of triangles referred to as right triangles because these have one right angle.
- Journal Reflection Question: If you could make a triangle that was as large as you wanted, would you be able to make one that has two right angles? Explain your thinking.

Question #3: How many different right triangles can be made on the geoboards?

- Have students make their conjectures and record the conjectures as a group.
- In the introduction of this exploration, discuss what different means. For the purposes of this exploration, if a triangle can be flipped or turned and matched up, it is not "different."
- For this exploration, it would be helpful for students to record all their triangles on dot paper so that they can compare their right triangles.
- Use guided questions to keep students on track during the exploration.
  - Have you found all of the right triangles that can be made? How do you know?
  - What is your strategy to make sure you have them all?
- If your students have difficulty coming up with a strategy for ensuring they find them all, model your approach. For example, "I started with a right triangle with a base of one and a height of one. Then I changed the height by one…"
- Teachers should attempt this task before students do in order to devise your own strategy for making sure all solutions are found and to experience what the students will experience and see during the exploration.
- Journal Reflection Question: Write everything you know that is true about all right triangles.

The 14 right triangles that can made on a 5 by 5 pin geoboard are shown below.



Question #4: How many different types of triangles can you find?

- Have students make their conjectures and record the conjectures as a group.
- Show the students examples of a right triangle to review the definition of a right triangle. Show non-examples of a right triangle to stimulate discussion about differing length of sides and angle size. Encourage students to use a known right angle and rulers (if needed) to differentiate between angle size and lengths of sides. (Students have not necessarily learned to measure angels to the degree yet, so having them simply classify the angels as acute, right, or obtuse using a known right angle is sufficient for this exploration.)
- Have students record their triangles on dot paper.
- NOTE: It is not possible to make an equilateral triangle on a geoboard. Some students may claim that some are, but if you measure the sides they will find them to have differing lengths.
- Have students share the triangles with each other in a group. Have students cut out the triangles and sort them into piles that are the same and label them with their defining characteristic. In order the help guide students to grouping, beyond just having the exact same measurements, feel free to set restrictions on the sorting rules such as there must be at least 3 piles and at least 3 triangles in each pile.
- Students should create posters with triangles displayed by category and should present and explain their groupings to the class. After the presentations, have a class

discussion and introduce the terms *acute, obtuse, scalene,* and *isosceles*. DO NOT introduce these terms until after the presentations. These geometric terms will come about naturally from the student classifications.

- Journal Reflection Question: Write in your own words the definitions for the new geometric terms we have found (*acute, obtuse, scalene,* and *isosceles*).

Summary

- After all explorations, have students complete the following journal entries with as many different answers as possible:
  - All triangles have….
  - Some triangles have…

FORMATIVE ASSESSMENT QUESTIONS

- What make a triangle a triangle?
- How do you know which triangles are right triangles?
- How can you classify or group triangles?
- Were students able to easily create the different types of triangles?
- Were students able to identify similarities and differences between two triangles?
- Were students able to identify right angles, obtuse angles, and acute angles within the triangles?

DIFFERENTIATION

Extension

- Using straws of different length or a computer geometry program such as The Geometer's Sketchpad, students can consider and explore the following questions:
  - Can a triangle be made with segments measuring five, six, and seven units? Can more than one triangle be made? Why or Why not?
  - If you are given any three lengths, can you always make a triangle? Why or why not?
  - Using several different sets of three lengths, try to make triangles. Can you make up a rule about the lengths of the sides of the triangles?

Intervention

- Have students create the triangles using straws of different lengths rather than geoboards so they can more easily compare side lengths.

Thoughts About Triangles

Dot Paper