

Practice Task: The Sieve of Eratosthenes

STANDARDS FOR MATHEMATICAL CONTENT

MCC4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Many students may not understand why “one” is not a prime number. Be sure to allow for plenty of discussion about this. One is neither a prime nor a composite number. A prime number is one with exactly two positive, unique divisors, itself and one. One has only one positive, unique divisor. It cannot be written as a product of two factors, neither of which is the number itself, so one is also not composite. It falls in a class of numbers called units. These are the numbers whose reciprocals are also whole numbers. For more information go to:

http://mathforum.org/dr.math/faq/faq_prime_num.html

Students are asked to write an expression for given multiples in this task. Some may not be sure what the question is asking, providing a prime opportunity for discussion. Because all even numbers are multiples of two, the algebraic expression for multiples of two is $2n$. Similarly, the multiples of 5 can be expressed as $5n$.

ESSENTIAL QUESTIONS

- How do we know if a number is prime or composite?
- How can we determine whether a number is odd or even?
- How are factors and multiples defined?

MATERIALS

- “Sieve of Eratosthenes” Recording Sheet
- “Exploring the Sieve of Eratosthenes” Recording Sheet
- Colored pencils, markers, highlighters or crayons

GROUPING

individual or partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

This task allows students to create a list of prime numbers between 1 and 100.

Comments

Students should use a different color for each new prime number. Using different colors works better, but students might need to use a combination of crayons, highlighters, colored pencils, and markers. A different symbol could be used for each prime number (i.e., *, #, etc.), but using colors might be more effective. Have students draw only one line through numbers so when they cross out a number more than once, they can do so in a different direction. This will help students find numbers that are multiples of several numbers.

One way to introduce this task is to model the first one or two steps on the overhead. Discuss the patterns students see. Write these on the board or chart paper as students share them. Each student needs to complete his/her own Sieve of Eratosthenes. Let students share their observations. Record these on a chart or the board. The “Exploring the Sieve of Eratosthenes” student recording sheet will help students determine factors and multiples.

Task Directions

Use a 0-100 chart to complete the following:

1. Draw a line through the number 1, because it is not a prime number.
2. Circle the number 2 with yellow, because it is the smallest prime number. Draw a line through every multiple of 2 with yellow. How do you know which numbers to cross out? Write an algebraic expression for the numbers you crossed out with yellow.
3. Circle the number 3 with blue. This is the next prime number. Now, draw a line through every multiple of 3 with blue. What do you notice about the number 6? What do you think it means when a number is crossed out with two colors – in this case yellow and blue?
4. Circle the next open number, 5 with red. Draw a line through all multiples of 5 with red. Write an algebraic expression for the numbers you crossed out in red.
5. Circle the next open number with orange. Draw a line through all multiples of 7 with orange.
6. Continue doing this with different colors until all the numbers through 100 have either been circled or crossed out.
7. Write to explain what you noticed about the circled numbers.

FORMATIVE ASSESSMENT QUESTIONS

- What do all of the multiples of two have in common? Multiples of five?
- If you multiply by two will you always get an even answer (a multiple of two)? Why?
- If you multiply by five will you always get a multiple of five? Why?

DIFFERENTIATION

Extension

- Students can write a letter to an “absent” classmate about how to use the Sieve of Eratosthenes to find factors of numbers.

Intervention

- Have students use a calculator or the computer to generate the multiples of numbers.

Name _____ Date _____

Sieve of Eratosthenes

Eratosthenes (275-194 B.C., Greece) devised a 'sieve' to discover prime numbers. A sieve is like a strainer that you drain spaghetti through when it is done cooking. The water drains out, leaving your spaghetti behind. Eratosthenes's sieve drains out composite numbers and leaves prime numbers behind.



Use a hundred chart below to complete the following:

1. Put a square around the number 1, because it is neither prime number nor composite.
2. Circle the number 2 with yellow, because it is the smallest prime number. Draw a line through every multiple of 2 with yellow. How do you know which numbers to cross out? Write an algebraic expression for the numbers you crossed out with yellow.
3. Circle the number 3 with blue. This is the next prime number. Now, draw a line through every multiple of 3 with blue. What do you notice about the number 6? What do you think it means when a number is crossed out with two colors – in this case yellow and blue?
4. Circle the next open number, 5 with red. Draw a line through all multiples of 5 with red. Write an algebraic expression for the numbers you crossed out in red.
5. Circle the next open number with orange. Draw a line through all multiples of 7 with orange.
6. Continue doing this with different colors until all the numbers through 100 have either been circled or crossed out.
7. Write to explain what you noticed about the circled numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Name _____ Date _____

Exploring the Sieve of Eratosthenes Recording Sheet



Using your Sieve of Eratosthenes list the prime numbers up to 100.

_____, _____, _____, _____, _____, _____, _____, _____, _____, _____,
 _____, _____, _____, _____, _____, _____, _____, _____, _____, _____,
 _____, _____, _____, _____, _____

Complete the table below. Choose a number between 40 and 60 for the last two rows, and then complete the chart using the numbers chosen.

Find this number on your hundreds board:	Prime or Composite?	List the factors of the number
3		
9		
12		
15		
17		
21		
24		
29		

Look at your chart and record 3 observations from your work.
