Measuring Parts of Units, Fraction Arithmetic with ½

Unit 8

Mathematical Understandings

- Fractions represent partial units.
- 1/b represents 1 copy of a unit partitioned into *b* congruent (equi-partitioned) parts.
- a/b represents *a* copies of a unit partitioned into *b* congruent parts.
- a/b can be interpreted as traveling from zero to the location a/b.
- a/b can also be interpreted as iterating a length of 1/b, a times.

Unit Overview

The lesson begins by establishing the need for a partial unit. A ft. strip is partitioned into two congruent parts (2-split), and each part is related to ½. Equal parts are explicitly contrasted with those that are not equal. The bottom number of the fraction is interpreted as the number of equal parts (2) and the top number as the number of copies of one of these equal parts. The property of *n* of 1/n = 1 is introduced informally by exploring the equivalence between 2 of ½ ft. and 1 ft. Only equi-partitioned parts have this property. Students create a 2-unit strip with each unit split in half. Students then "travel" *a* copies of ½, where *a* ranges from 1 to 5. Travel is related to addition and the split-copy to multiplication. Subtraction is related to the difference between paper lengths. The unit concludes by locating ½ ft. on every unit of the tape measure constructed during the previous unit, labeling the accumulation of ½ ft. units from the origin, establishing equivalence between 1ft. and $^2/_2$ ft., and using the tape measure to measure lengths of snakes that are not whole number units long. The formative assessment includes items ordering fractional unit lengths, locating ½ on a number line, and enacting the meaning of multiplication of 1 unit by a fraction, and of the meanings of addition and subtraction of fractional quantities.

Contents

Materials & Preparation

Read

- ☐ **Unit 8**
  Start by reading the unit to learn the content and become familiar with the activities.

- ☐ **Mathematical Background**
  Reread the Mathematical Background to anticipate the kinds of ideas and discussions you will likely see during instruction.

- ☐ **Measurement Construct Map**
  Read the construct map and look at the multimedia map to help you recognize the mathematical elements in student thinking, and to order these elements in terms of their level of sophistication (see 2 splits and compositions of 2 splits on ToML).

Gather

- ☐ Student math journals
- ☐ Teacher journal for note-taking
- ☐ Paper for personal unit strips
- ☐ Glue or tape
- ☐ Markers for labeling tape measures
- ☐ Chart paper, markers
- ☐ Several items to measure *(see page 15)*
- ☐ Footstrip rulers from previous units for demonstration

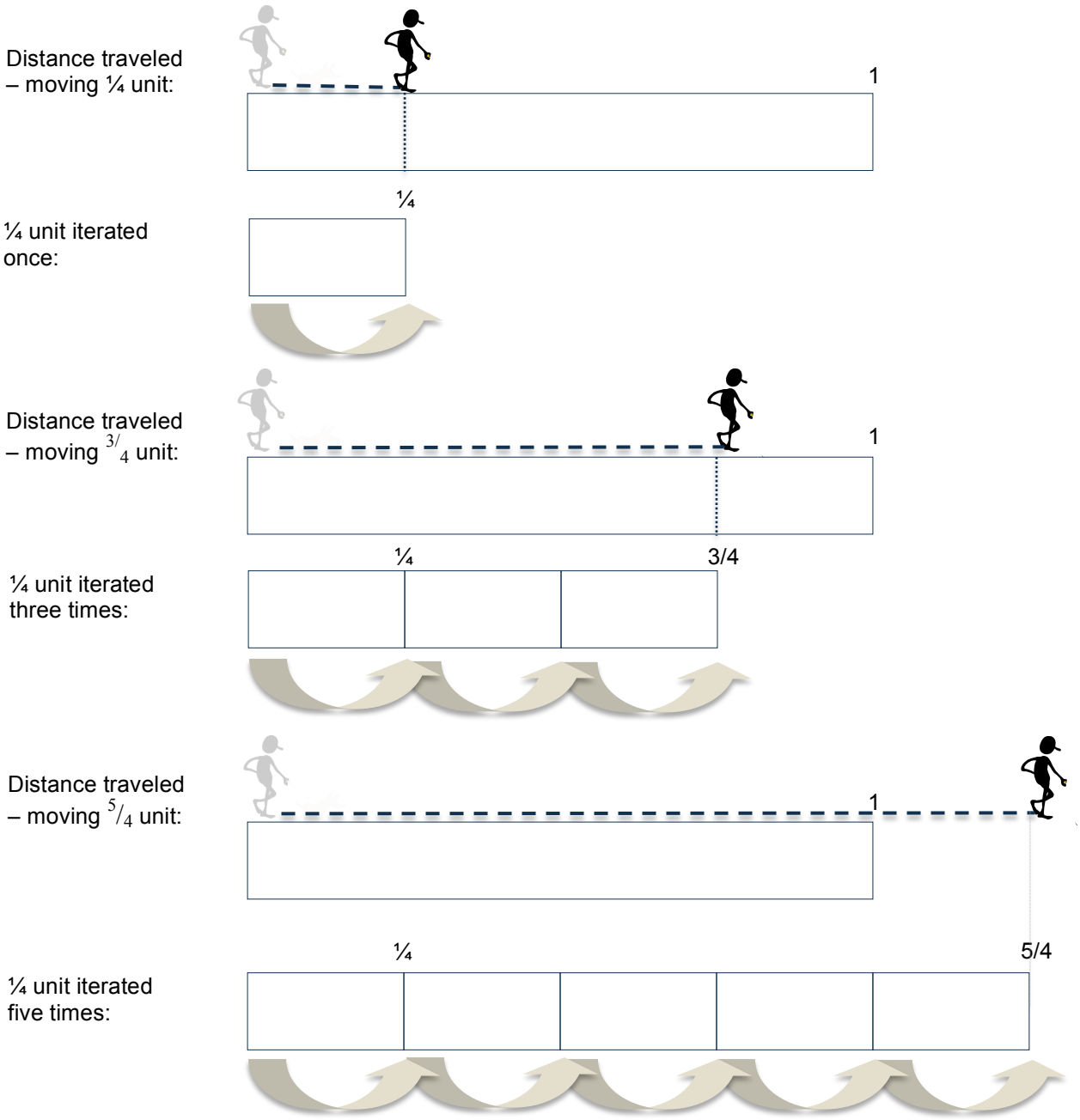
Prepare

- ☐ 5 ft-unit strips for each student (The ft-unit is the same length as the that used in the previous lessons.
- ☐ 10 to 15 unit strips for teacher demonstration.

Mathematical Background

Fraction Arithmetic (1/2) Unit 8

- Two metaphors are used to help students understand the meaning of a fractional quantity. One relies on unit iteration where, for example, ¾ unit means 3 iterations of ¼ unit. The second metaphor is distance traveled where, for example, $^{3}/_{4}$ means starting at 0 and moving to $^{3}/_{4}$ unit. Both of these metaphors, one static and the other dynamic, help students form images of fractions that will later help them locate fractions on the number-line, because the number-line is an idealized ruler.



Introducing 1/2

Whole Group

1. Last time, we decided that we wanted to be able to measure a distance, a length, that was in between units—for example, that was more than 1 unit but less than 2 units. **Show students a ft. strip tape with 1 and 2 marked and an object that is 1 ½ feet long.**

    a. Talk with your elbow partner and decide what we could do to measure this object's length. (Elicit suggestions.)

    b. Many of you said that we needed a half or about the middle of this unit (gesture to the second unit).

    c. Talk with your elbow partner, and then each of you use a ft. strip and try to make a half of a foot without cutting the paper.

        Q: How do you know that the unit is exactly half?
        Q: Here is a unit that has been split into 2 pieces (show a unit where the split is clearly not congruent). Is either of these parts ½ ft long?

2. **Demonstrate splitting a unit,** saying: I am going to take ½ of 1 ft. Match the ends and crease the midpoint, so that when unfolded the 2 congruent sections are clearly visible. To make certain that we really have split the unit into ½ ft., we should be able to fit two copies of the ½ ft. into the 1 ft. unit. Let's see if we can do that:

    a. Tape a ft. unit on the board.

    b. Tape a ½ ft. directly beneath it, aligned with the ft. unit.

    c. Demonstrate iteration of a copy of the ½ ft. unit.

    d. Conclude: The 1 ft. unit is 2 times (twice) as long as the ½ ft. unit. (Write this sentence on the board. Have students copy it in their math journals and make a drawing that

shows the iteration. This is important for promoting representation.)

Teacher note. If the split units are not congruent, then 2 iterations of either non-congruent part will not be congruent with the original 1 ft. unit. This is an informal proof that only equal partitions have the property that n of 1/n results in 1 unit (n x 1/n = 1). This property helps relate iteration to matching ends of the strip to form congruent partitions.

Individual

3. Have every child create a ½ unit by splitting a foot unit into equal partitions.

Whole Group

4. **We symbolize one half unit like this:** $\frac{1}{2}$

   Q: What do you think the bottom number tells us? (The number of equal parts) Sometimes, we call the bottom number the splitter.
   Q: Can the parts be of different sizes? (No! – Illustrate that if they are, they don't meet the iteration test.)
   Q: What do you think the top number tells us? (The number of copies of the equal parts.)

Individual

5. **Take two paper unit ft. strips and fold each into ½ ft**. **Unfold the units and tape them end-to-end with no spaces.** (Check to ensure that each child can do this, assist where needed or ask other children to assist.)

   a. Now, put your fingers like this at the very end of the strips (gesture grasping with thumb and finger). What do we call the beginning of the first strip—how far have we traveled on the strip? (0)

   b. Close your eyes. Now travel ½ ft. by sliding your fingers. Stop when you have traveled ½ ft. Open your eyes and see if you

stopped at the right place. Now continue to travel another ½ ft. Stop when you have traveled another ½ ft. How far have we traveled all together? ($^2/_2$ ft. or 1 ft.)

c. Open your eyes and look here (Write $^2/_2$ on the board.) What did we say the bottom number told us? (The number of equal parts that we split the unit into.) What did we say the top number told us? (The number of copies.) So, let's look at the first strip. Point to one of the equal parts. How many of these equal parts do we have? (2). So that is the meaning of $^2/_2$, and because we have now traveled to the end of the first unit, we say that $^2/_2$ ft. and 1 ft. mean the same thing.

Teacher note. An additional elaboration of $^2/_2$ would consist of taping a ft. strip on the board, and a ½ ft. strip above it or below it. Then use a third ½ ft strip to show that 2 copies—2 iterations of the ½ ft. strip, that is $^2/_2$ ft., are congruent with the first 1 ft. strip. This re-visits what was established earlier re the equi-partition into two parts and iteration.

d. Ok, go back to the beginning, close your eyes and try $^3/_2$. Now try $^4/_2$. Now try $^5/_2$ (fingers should go off the composite strips).

Teacher note. For some students, going beyond 1 unit is difficult, and so the literal enactment is very important. Enacting travel beyond 1 with a plastic figurine often helps students literally see the meaning of $^3/_2$.

Whole Group

6. **Here is a length. If this length is ½ of a whole unit, how could we draw the length of the whole unit?**

Teacher elicits student strategies and guides finding the unit by iterating the length representing ½ two times.

Teacher note. If necessary, remind students to think about how they could establish that each part was exactly the same length (Iterating the split-part twice to establish that it was as long as one unit.)

Addition

Whole Group

1. **What do we do when we add two lengths? (Join them.)** Talk with your elbow partner and decide what ½ + ½ means.

   Q: _____ come on up and show us with these strips (have 1 ft. and ½ ft. strips ready) what ½ + ½ means. What is the result? (½ + ½ = $^{2}/_{2}$ _)

Individual

2. **Use your ft. strips to find:**

   a. ½ ft. + ½ ft. + ½ ft. =

   b. $^{2}/_{2}$ ft. + ½ ft.=

   c. $^{3}/_{2}$ ft. + $^{5}/_{2}$ ft. =

Whole Group

3. **Students share solution strategies and teachers support the movement between symbolization and concrete enactment of solutions.** (Never just act. Never just symbolize. Always relate the symbolization to the action of joining strips and the action of joining to the symbolization.)

Subtraction

Whole Group

1. **What do we do when we subtract two lengths?** (Find the difference between them.) Talk with your elbow partner and decide what $^3/_2 - ½$ means.

   Q: _____ come on up and show us with these strips (have 1 ft. and ½ ft. strips ready) what $^3/_2$ ft. – ½ ft. means. What is the result? ($^3/_2$ - ½ = $^2/_2$)
   Q: What could you use ½ ft. and $^2/_2$ ft. to undo this result—to get back to the $^3/_2$ ft.? How could we write a number sentence that tells us to do this? ($^2/_2$ ft. + ½ ft = $^3/_2$ ft.)

Individual

2. **Use your ft. strips to find the result of (a-c). For each, write another sentence that "undoes" the result of the subtraction and obtains the first length.**

   a. $^3/_2$ ft. – $^2/_2$ ft.

   b. ½ ft. – ½ ft.

   c. $^4/_2$ ft. – $^3/_2$ ft.

Whole Group

3. **Students share solution strategies and teachers support the movement between symbolization and concrete enactment of solutions.** (Never just act. Never just symbolize. Always relate the symbolization to the action of finding the difference in length between the strips to the symbolization.) For each of (a)-(c), be sure to show how joining the difference between quantities to the subtracted quantity restores or is congruent with the first length in the subtraction number sentence.

Multiplication

Whole Group

1. **What is 3 of 2 ft.? How long is that? Who can show that?** (Look for literal enactment of 3 of a composite unit of 2 ft.) **What about 3 of 1 ft. How long is that? Who can show that? How about 3 of ½ ft.? Who can show that?** (Look for or demonstrate 3 copies of ½ ft, which is the same as $^3/_2$ ft.)

    Q: Talk with your elbow partner. What does $^3/_2$ of 1 ft. mean? The bottom number is the splitter and the top number is the number of copies. What about $^2/_2$ of 1 ft.?

2. **The teacher elicits student responses and demonstrates the meaning of the operation by splitting 1 ft. unit into 2 congruent parts, and then iterating the fractional unit length 3 times.** Because we can fit ½ ft. into $^3/_2$ ft. 3 times, we can say that $^3/_2$ ft. is 3 times as long as ½ ft.

Individual

3. **Use a ft. unit. Write $^3/_2$ x 1 ft. and show how to find the result.** (Teacher roves and checks student understanding.)

Ruler Equivalence

Individual

1. **Use your ½ unit to mark your ruler to show the ½ split for every unit.** Label your ruler so that someone else could easily find $^{3}/_{2}$ ft. or $^{5}/_{2}$ ft.

2. **Use your ruler to measure the snakes with rattles.** How long are those snakes? (The snakes with rattles have lengths of $^{7}/_{2}$ ft. and $^{5}/_{2}$ ft.)

Whole Group

3. **What labels on your ruler were different numbers but meant the same thing—that is, they meant the same exact length or distance traveled from the starting point?** ($^{2}/_{2} = 1$, $^{4}/_{2} = 2$, etc.)

Formative Assessment — Fraction Arithmetic (1/2) Unit 8

Name _____

1. **For each problem, write the result and show how you found it with the paper strip units.**

   (a) $\frac{1}{2}$ of 1 unit

   (b) $\frac{1}{2}$u + $\frac{1}{2}$u + $\frac{1}{2}$u

(c) $\frac{3}{2}$ u **+** $\frac{2}{2}$ u

(d) $\frac{5}{2}$ **of 1 unit**

Formative Assessment

Fraction Arithmetic (1/2) Unit 8

(e) $\frac{6}{2}$ u – $\frac{1}{2}$ u

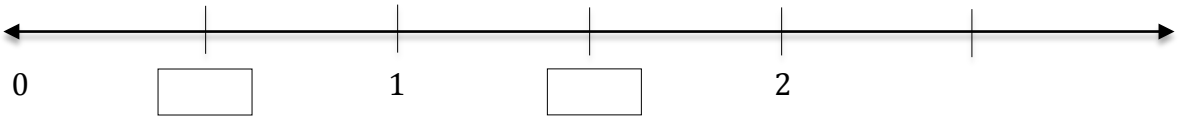
2. Put these into order from smallest length to longest length:

$\frac{3}{2}$ ft. $\frac{1}{2}$ ft. $\frac{2}{2}$ ft. $\frac{5}{2}$ ft.

Formative Assessment

Fraction Arithmetic (1/2) Unit 8

3. What numbers go in the boxes on this numberline?



4. True or False?

(a) $\frac{2}{2} = 1$

(b) $\frac{5}{2} = 2$

Formative Assessment Record Fraction Arithmetic (1/2) Unit 8

Student _____ Date _____

Indicate the levels of mastery demonstrated by circling those for which there is clear evidence:

Construct/Concept	Evidence for	Notes
ToML 4E **½ of 1**	Student folds strip into 2 congruent parts for Item 1a.	
Addition as Joining of Lengths Fractions>1 **½ + ½ + ½** $^3/_2 + {}^2/_2$	Student joins 3 2-split units, says result is $^3/_2$. (Item 1b) Student joins 5 2-split units, says result is $^5/_2$. (Item 1c)	
Multiplication of a Measured Length by a Fraction: N copies of 2-split $^5/_2$ **of 1 unit**	Student shows 5 iterations of ½. (Item 1d)	
Subtraction as Difference $^6/_2 -$ **½**	Student shows difference between 6 ½ unit strips or 3 units and ½ unit strip is $^5/_2$ or 2 ½ units. (Item 1e)	
Order fractional measures	Student orders fractions from least to greatest. (Item 2)	
Numberline Representation	Student locates ½ correctly and labels the 1 ½ either as $^3/_2$ or as 1 ½. (Item 3)	
Equivalence	Identifies $^2/_2$ unit and 1 unit as equivalent Identifies $^5/_2$ unit and 2 units as not equivalent (Item 4)	

Academic Language: Indicate academic words the student is familiar with by recording them here.