

Area and Multiplication; Equivalent Fractions; Time & Volume

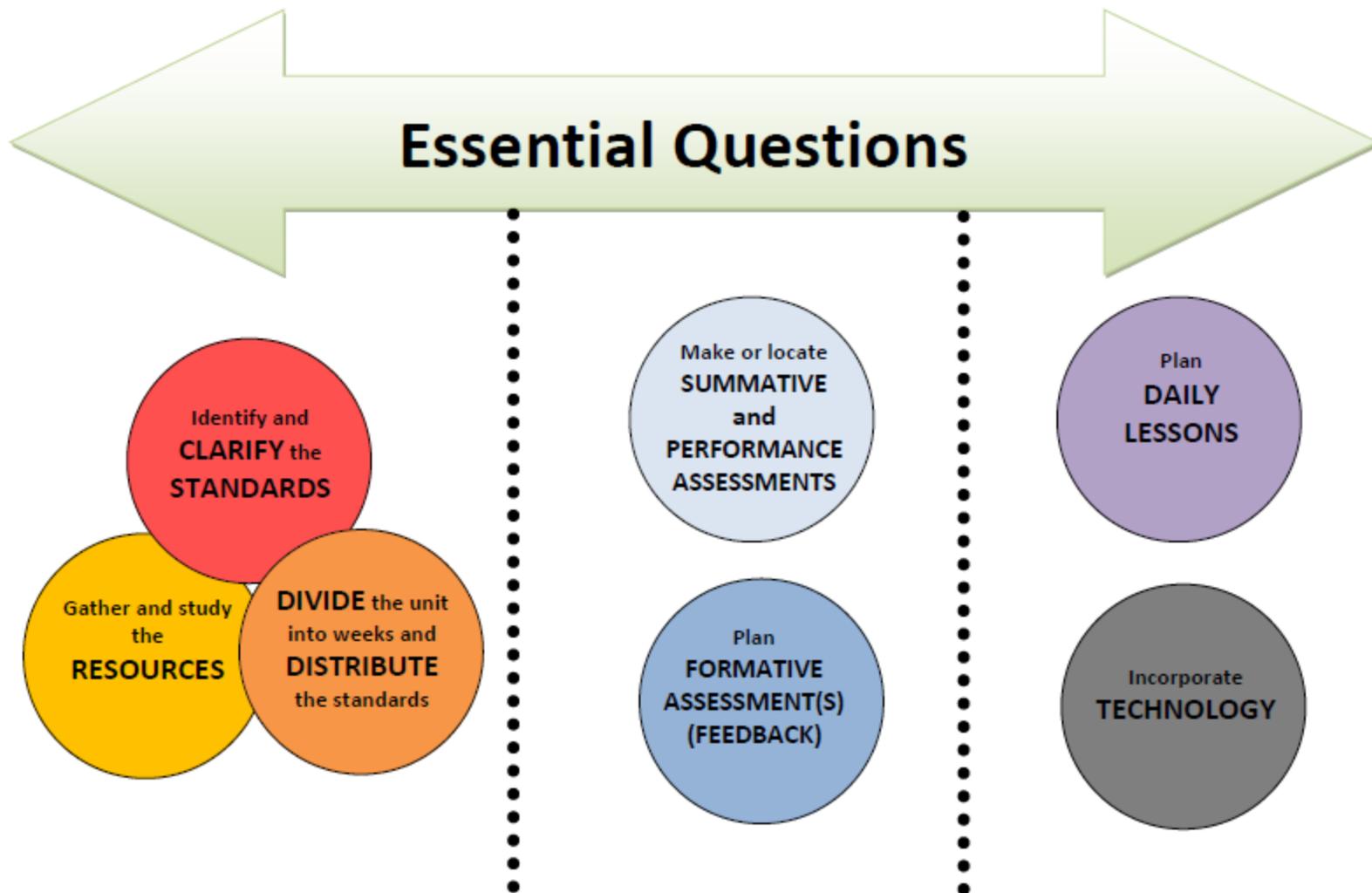
3rd Grade Unit 3



Unit Planning Team:

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Backward Unit Planning 1.0



Essential Questions



3rd Quarter (p. 1 of 2) Area and Multiplication; Equivalent Fractions; Time & Volume

Students will continue to use multiplication and division with all factors 0-10, working towards fluency by using the properties of operations. Students will reason about multiplication with area and arrays, applying the properties to these new models of multiplication. Students will recognize area as an attribute of plane figures and measure area by counting unit squares. In this unit, the focus for fractions is on equivalent fractions. Students will recognize and generate equivalent fractions both with the area model and as a point on the number line. Students will use addition and subtraction in contexts involving: time, volume, masses, and perimeters of polygons. Students will continue to use place value, properties of operations, and the relationship between addition and subtraction to add and subtract within 1000.

Essential Questions:

How can I efficiently solve problems using the properties of operations?

What is area and how do I measure it?

What are equivalent fractions?

Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = ? \div 3$, $6 \times 6 = ?$*

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5 Apply properties of operations as strategies to multiply and divide. [Students need not use formal terms for these properties.] *Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. [Commutative property of multiplication.] $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. [Associative property of multiplication.] Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. [Distributive property.]*

Multiply and divide within 100.

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operation. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. *[This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).]*

Number and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic. [A range of algorithms may be used.]

3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

How can I efficiently solve problems using the properties of operations?

3.OA.3, 3.OA.4, 3.OA.5, 3.OA.7, 3.OA.8, 3.NBT.2, 3.MD.8, 3.MD.1, 3.MD.2

What is area and how do I measure it?

3.MD.5, 3.MD.6, 3.MD.7

What are equivalent fractions?

3.NF.1, 3.NF.3

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Number and Operations in Base Ten

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3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

3.OA.3 Multiplication and Division Problems in context- continues all year. Make sure to pose problems that include equal groups, arrays, & measurement quantities. Notate with equations.

3.OA.4 Focus on the missing part. Could be done with number talks or in discussion- continues all year.

3.OA.5 Properties:

Commutative: $3 \times 2 = 2 \times 3$

Associative: $15 \times 2 = (3 \times 5) \times 2 = 3 \times (5 \times 2) = 3 \times 10 = 30$

Distributive: $7 \times 4 = (5 \times 4) + (2 \times 4)$

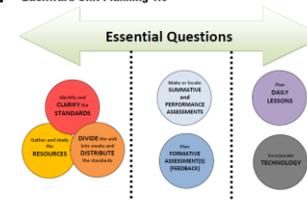
3.OA.7 Fluency this quarter should focus on using $\times 2$, $\times 3$, $\times 4$, $\times 5$, $\times 10$, etc. facts to derive the new facts with factors 6-9. Mastery by end of year.

3.OA.8 Multi-step problems using all four operations. Students need to represent their thinking with an equation. They will need to check for reasonableness of their answer. Introduction to the Order of Operations where multiplication and division are done before addition and subtraction.

i.e. $3 + 5 \times 10 = 3 + 50 = 53$; not $8 \times 10 = 80$

3.NBT.2 Continue working with addition and subtraction within 1000. Students should be developing strategies based on place value and the properties of operations.

Backward Unit Planning 1.0



Identify and
CLARIFY the
STANDARDS

Number and Operations - Fractions

[Grade 3 expectations in this domain are limited to fractions with denominators 2,3,4,6, and 8.]

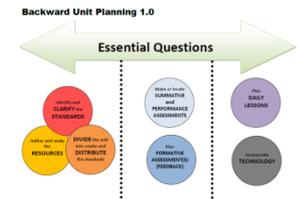
Develop understanding of fractions as numbers

3.NF.1	Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.
3.NF.3	<p>Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.</p> <p>a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.</p> <p>b. Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.</p> <p>c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. <i>Examples: Express 3 in the form $3=3/1$; recognize that $6/1=6$; locate $4/4$ and 1 at the same point of a number line diagram.</i></p> <p>d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparison with the symbols $<$, $>$, or $=$ and justify the conclusions, e.g., by using a visual fraction model.</p>

3.NF.1 Students will need to see a fraction (i.e. $\frac{1}{4}$) as 1 part when the whole is partitioned into (4) equal parts. Also that a/b (i.e. $\frac{3}{4}$) is $1/b$, a times ($\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$)

3.NF.3

- Tell if two fractions are equivalent by plotting them on a number line and comparing them.
- Tell if two fractions are equivalent by using a fraction area model and explain why they are the same
- Understand fractions that are the same as 1 such as $3/3$ or $4/4$ on number line or fraction model
- Compare fractions with the same numerator or denominator. When the denominators are the same, the higher the numerator the larger the fraction. (i.e. $2/6$ and $4/6$; $4/6$ is bigger because the shares are all the same size, but there are 4 instead of 2.) When the numerators are the same, the larger the denominator the smaller the share (i.e. $2/6$ and $2/3$; $2/3$ is bigger because $1/3$ size pieces are larger than $1/6$ size pieces and there are 2 of each). Comparison are valid only when referring to the same size whole.



Identify and CLARIFY the STANDARDS

Measurement and Data

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). *[Excludes compound units such as cm³ and finding the geometric volume of a container.]*
Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. *[Excludes multiplicative comparison problems (problems involving notions of "times as much")]*

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.
a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

3.MD.7 Relate area to the operations of multiplication and addition.
a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

3.MD.1 Tell time to the nearest min.

Solve problems with addition and subtraction of time (elapsed time).

3.MD.2 measurement units will be used as a context for problem-solving with addition, subtraction, multiplication, and division. Could be taught in Science- NS.1.3.5

3.MD.5 a) square units are used to measure area

b) Area follows the same rules as length measures. No gaps or overlaps!

3.MD.6 Count the squares to find area.

3.MD.7 a) Find area by tiling (filling in the space)

b) Develop the understanding that area is the number in each row x the number of rows.

c) Decompose areas into parts that can be solved separately and then put back together (i.e. a 12 by 6 rectangle could be solved by doing 10 x 6 then 2 x 6 and then adding the products), using the distributive property.

d) Break apart irregular areas (such as an L-shaped room), into rectangles, find the areas separately and then add them back together.

3.MD.8 Find perimeters in problem-solving. Find the unknown side. Generate rectangles with same perimeters and different areas or same areas and different perimeters.

Backward Unit Planning 1.0



Identify and
CLARIFY the
STANDARDS

Suggested Weekly Posing of Problems:

This is one way to think about clustering the standards. These ideas would be repeated each week.

Essential Question: How Can I Efficiently Solve Problems Using Properties of operations?

1 day a week- Work on addition and subtraction multi-digit fluency and 2-step problems with all four operations . Use measurement (masses, volumes, time, & perimeter) as a context for problem-solving (3.NBT.2 and 3.OA.8, 3.MD.1, 3.MD.2, 3.MD.8)

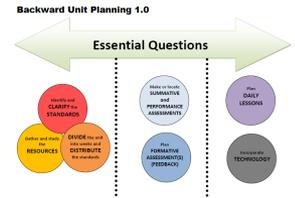
1 day a week- Work on multiplication and division problems and fluency. Focusing on properties of operations and the relationship between multiplication and division. (3.OA.3, 3.OA.4, 3.OA.5, 3.OA.7)

Essential Question: What is area and how do I measure it?

1-2 days a week- Work on area concepts. Pull in multiplication ideas. (3.MD.5, 3.MD.6, 3.MD.7)

Essential Question: What are equivalent fractions?

1-2 days a week- Work on fraction concepts, equal sharing and equivalency and comparing fractional amounts. (3.NF.1 & 3.NF.3)

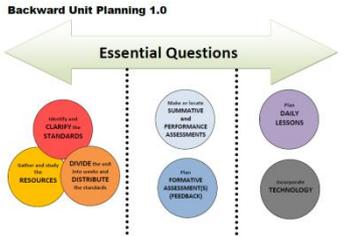


DIVIDE the unit into weeks and **DISTRIBUTE** the standards

COMMON CORE SHIFTS FOR MATHEMATICS

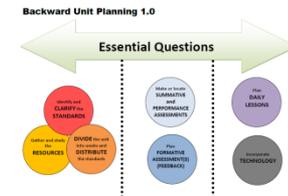
1. Focus strongly where the standards focus.
2. Coherence: think across grades, and link to major topics within grades.
3. Rigor: in major topics, pursue conceptual understanding, procedural skill and fluency, and application.

Week	Standards	Structure/Resources
1	3.OA.7 x and division fluency 3.MD.5 Measure area with square units 3.OA.3 Multiplication and division with arrays (connect to 3.OA.5 – properties of operations) 3.MD.8 Find perimeters 3.NBT.2 Addition and Subtraction Fluency	Mastering the Basic Math Facts Partitioning and Comparing Rectangles PDF (Rich Lehrer Area Unit 1) Problems to Pose to Promote Arrays (3.OA.3) Groceries, Stamps, and Measuring Strips Context for Learning Book Framing a Photo Fencing the Yard
2	3.OA.7 x and division fluency 3.MD.6 Measure area by counting unit squares 3.NF.1 Understand fractions (1/b) 3.OA.8 Two-step problems 3.MD.1 Tell and write time to the min. Solve word problems involving time.	Mastering the Basic Math Facts Comparing the Areas of Our Hands PDF (Rich Lehrer Area Unit 2) Equal Groups/Sharing Problems Elapsed Time Strategies Task - Holiday Travels Task - Kevin's Busy Day
3	3.OA.7 x and division fluency 3.MD.7 a) Find the area of rectangle by tiling 3.NF.3 Equivalent Fractions	Mastering the Basic Math Facts 3: Sweeping Area PDF (Rich Lehrer Lesson)- Use cardboard squares instead of squeegee and shaving cream. Will also need uncooked spaghetti) <i>Fraction Equivalence and Order ECM Book</i> Chapter 6 p.114-138 Problems to Pose p.139-143



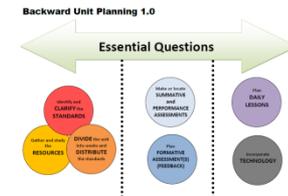
DIVIDE the unit into weeks and **DISTRIBUTE** the standards

Week	Standards	Structure/Resources
4	3.OA.7 x and division fluency 3.MD.7 b,c,d Find area of rectangular and irregular figures 3.OA.3 Multiply and Divide using measurement Quantities 3.MD.2 +, -, x, / word problems using measurement quantities 3.NF.3 Equivalent Fractions	Mastering the Basic Math Facts 4: Comparing Zoo Enclosures PDF Problem Solving situations with measurement quantities (use Chrome) Comparing Fractions (3.NF.3) Jon and Charlie's Run (3.NF.3) Comparing Fractions with a Different Whole (3.NF.3)
5	3.OA.7 x and division fluency 3.MD.5,6,7 Area Review 3.OA.3 Multiply and Divide (arrays, equal groups, measurement quantities) Tie in 3.OA.5 properties of operations 3.OA.4 Determine the unknown whole number in a x or division problem	Mastering the Basic Math Facts Building the Concept of Area Multiplication and Division Problem Types (3.OA.3, 3.OA.4, 3.OA.5) - Have students practice writing their explanation in words and/or critique the reasoning of others in words. Mrs. Tucker's Supplies (3.OA.8, 3.NBT.3) Cleaning with Mrs. Boyd (3.OA.8, 3.NBT.3)
6	3.OA.7 x and division fluency 3.MD.7b Find area in real world context 3.OA.8 Two step problems 3.OA.5 X and division properties of operations 3.MD.8 Perimeter and Area – rectangles with different perimeters and same areas, or vice versa	Mastering the Basic Math Facts Angel's Rectangle Painting the Door Area and Perimeter technology practice (use Chrome) Chocolate Bar Maker Rectangle Task (includes 3.MD.7)



DIVIDE the unit into weeks and **DISTRIBUTE** the standards

Week	Standards	Structure/Resources
7	3.OA.7 x and division fluency 3.NF.3 Equivalent Fractions	Mastering the Basic Math Facts Illustrative Mathematics Task Comparing Fractions Pizzas Made to Order Strategies for Comparing Fractions Trash Can Basketball Comparison problems on page 142 & 143 in ECM book are good for comparing fractions (3.NF.3d)
8	3.OA.7 x and division fluency 3.MD.7d Find areas of irregular figures by decomposing them into rectangles and adding the areas together 3.MD.8 Perimeter and Area – rectangles with different perimeters and same areas, or vice versa 3.OA.3 Multiply and Divide 3.OA.5 properties of operations 3.NBT.2 Addition and Subtraction Fluency	Mastering the Basic Math Facts Junior Architects: Finding Perimeter and Area Butterfly Gardens
9	3.OA.7 x and division fluency 3.MD.7 b,c,d Find area of rectangular figures by decomposing them into areas. 3.OA.3, 5, 4 Multiplication and Division, find the missing numbers and properties	Mastering the Basic Math Facts Give Assessment Mini-lessons for Early Multiplication and Division Pg.. 15 Patios A6 pg. 17 Gardens A8 Pg. 18 Floors A10 pg. 64-65 The open arrays, E11, E12, E13



DIVIDE the unit into weeks and **DISTRIBUTE** the standards

Lesson Resources

Extending Multiplication and Division
3.OA.3, 3.OA.7

Multiplication and Division Problem Types (3.OA.3, 3.OA.4, 3.OA.5)

Mastering the Basic Math Facts in Multiplication and Division: Strategies, Activities & Interventions to Move Students Beyond Memorization by Susan O'Connell & John SanGiovanni
Resource Guide for using *Mastering the Basic Math Facts in Multiplication and Division in Unit 2* (3.OA.1, 3.OA.2, 3.OA.3, 3.OA.4, 3.OA.5, 3.OA.6, 3.OA.7, 3.OA.9)

Additional Resources for 3.OA.7 and using *Mastering the Basic Math Facts: Multiplication and Division*. These resources are from Howard County Public Schools, MD, and one of the co-authors of the book, John SanGiovanni.

The Big Dinner

This unit begins with students' early multiplication strategies. As the unit progresses, the ratio table is introduced and students are supported to use the distributive property with large numbers. Strings of related problems guides learners toward computational fluency with whole number multiplication and build automaticity with multiplication facts by focusing on relationships.

Minilessons for Early Multiplication and Division
This book contains minilessons that you can choose from as you consider the needs of your students and can be used throughout the year. These are more guided and explicit and were designed to be used at the start of your math instruction - lasting 10 to 15 minutes. See the overview (p.5-9) for further details regarding the minilessons.

Lessons, Tasks, and Investigations The following lessons were written by the Georgia Department of Education and correspond with the standards in this unit. Some lessons may require additional days.

Multiplication and Division (3.OA.3, OA.4, OA.5, OA.6, OA.7)

- Family Scenarios
- Find the Unknown Number
- Multiplication Chart Mastery
- My Special Day
- Use What You Know
- What Comes First

Illustrative Mathematics Tasks: the following task comes from the Illustrative Mathematics site (headed by Bill McCallum, co-author of CCSSM). Illustrative Mathematics provides guidance to states, assessment consortia, testing companies, and content developers for illustrating the sense and direction of mathematical unit standards in a classroom.

Gather and study the RESOURCES

Essential Questions



Unit 4

Personal Unit Tape Measure

This unit revisits concepts of measurement but in this lesson, the measurement units are not feet, and students are introduced to **fractions as partitions**. The goal for students is to create a tape measure composed of "personal" units, and to measure the length of various objects with this personal tape measure. A personal unit is a unit named after each student (e.g., 1 hand). Personal units are rectangular strips ranging from 1 by inches to 1 by 16 inches. (The long side of the rectangle is what is used to measure length, but in 2.10 students make students later visualize the results of folds.) The teacher assigns each student one pair of personal unit length (perhaps by folding). Classroom discussion focuses on splitting units, so that lengths that are not multiples of whole numbers can be measured. Fractions, $\frac{1}{2}$ are quantities representing a group of 2 congruent partitions of the unit. The explanation asks students to partition the personal unit into 2 congruent partitions by folding, and then drawing (marking) all of these partitions, starting at the zero. Hence, 1st unit represents traveling from the origin, $\frac{1}{2}$ to the end of the first of 2 equal partitions of the unit. Similarly, 2nd unit represents traveling from the origin to the end of the 2nd of 4 congruent partitions, and 3rd unit represents traveling from the origin to the end of 3rd of 6 congruent partitions, each an alternative representation of $\frac{1}{2}$. In contrast, 4th unit represents 4 units of 2 sections of $\frac{1}{4}$.

Materials

- A string (approximately 18 inches long)
- 2 Paper clips
- A paper fastener (clip that is 2 inches long)
- A pencil (at least 10 inches long)
- 1 Ruler (at least 10 inches long)
- 1 Yardstick (at least 36 inches long)
- 1 Meter (at least 100 centimeters long)

Unit 5

Thinking About Scale

Overview Students compare their personal tape measures to consider units of scale, including the origin of scale, and how to construct a scale so that markings on the scale are aligned with objects measured. After comparing their constructions, they compare their different measurements by using what an eye-based scaling which is better for determining to unit 1. (A measured length equals 4 units.)

Part One: Introducing the Unit

- 1. Whole group discussion about "Lesson Learned"
- 2. Partners and groups consider ways of labeling tape measures

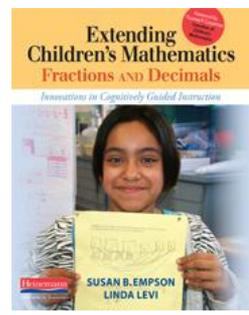
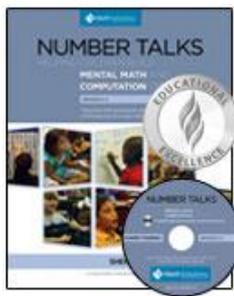
Unit 6

Revisiting Compositions of 2-Splits and Exploring Equivalences with Standard Units

Partial units are revisited with an eye toward composing repeated 2 splits (partitions). Compositions of partitions provide a way for students to begin think about the identity of fractions and their location on the number line. Fractions, $\frac{1}{2}$ are quantities representing a group of 2 congruent partitions. The explanation asks students to partition the personal unit into 2 congruent partitions by splitting (cutting) the unit, and then drawing (marking) all of these partitions. Hence, 1st unit represents traveling from the origin, $\frac{1}{2}$ to the end of the first of 2 equal partitions of the unit. In contrast, 4th unit represents traveling from the origin to the end of 4 units, because 4 splits that the partition is exactly the same length as the unit. From the perspective of unit division, 4 represents a single division of a unit into 4 equal parts, and 4 represents 4 sections (parts) of the unit. Equivalences is represented as the same division from the origin, to split the partition to 2nd unit (because 2 x 2 unit divided 4 unit and divided). The unit concludes with a number line representation of partitioning units and a discussion of different forms of $\frac{1}{2}$ and how to read the same distance traveled.

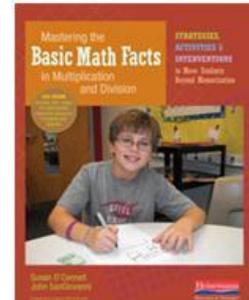
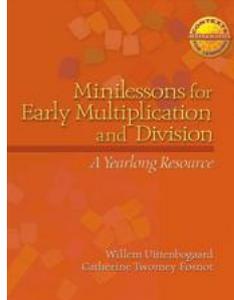
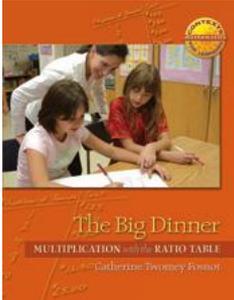
Introduction and Whole-Class Discussion

Introduce the task by asking students to summarize their ideas about splitting units from the last class—or choose several students to read their journal entries that summarize the values of their drawings. Then set the stage for the unit by asking students to consider how they might measure an object's length using a new unit ("1" that is really made from 12 small pieces) called Chinos (to choose whatever name you like, but measure must use the name). If the length of the object is between 1 to 100 and 1 to 100. What are the advantages to measurement if we use the same unit? The aim is to **evaluate the need to split the unit to provide better approximation**. The use of the same unit allows for easy comparison. This is a purpose of standardization. Discussion can easily be connected.

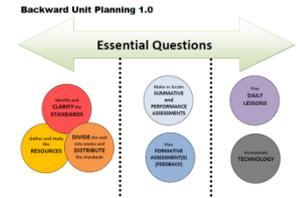


CGI Multiplication & Division Problem Types
*Adapted from Children's Mathematics Cognitively Guided Instruction by Carpenter, Stevens, Harter, and Empson

	Multiplication	Measurement Division	Partition Division
Grouping/Parting	Karen has 3 apple trees. There are 7 apples on each tree. How many apples are there all together?	Karen has some apple trees. There are 7 apples on each tree. All together there are 21 apples. How many apple trees does Karen have?	Karen has 3 apple trees. There are the same number of apples on each tree. All together there are 21 apples. How many apples are there on each tree?
Rate	Susan runs 4 miles an hour. How many miles does she run in 6 hours?	Susan runs 4 miles an hour. How many hours will it take her to run 24 miles?	Susan runs 24 miles. It took her 6 hours. If she runs the same speed the whole way, how far did she run in one hour?
Price	Cakes cost 7 dollars each. How much do 5 cakes cost?	Cakes cost 7 dollars each. How many cakes can you buy for \$35?	Jake bought 5 cakes. He spent a total of \$35. If each cake cost the same amount, how much did one cake cost?
Multiplicative Comparison	The box constructor is 8 times as long as the garden snake. The garden snake is 2 feet long. How long is the box constructor?	The box constructor is 16 feet long. The garden snake is 2 feet long. The box constructor is how many times longer than the garden snake?	The box constructor is 16 feet long. He is 8 times as long as the garden snake. How long is the garden snake?



Options for Assessment – available online for Unit 3



Essential Question:

How can I efficiently solve problems using the properties of operations?



Choose the equation that shows 48 crayons being divided into groups of 6.

- A. $6 \times 48 = 8$
- B. $48 \div 6 = 8$
- C. $48 \times 6 = 8$
- D. $6 \div 48 = 8$

Which of the equations could be used to solve the problem above? Mark all that apply.

- $6 \times 8 = c$
- $6 \times 6 \times 6 \times 6 \times 6 \times 6 = c$
- $6 + 6 + 6 + 6 + 6 + 6 + 6 = c$
- $48 - 6 = c$
- $8 \times 6 = c$

3.OA.5

Which **two** statements can be represented by the expression 4×8 ?

- A. A teacher puts 8 chairs at each of the 4 tables.
- B. Tom buys 4 red markers and 8 black markers.
- C. Marie shares her 8 marbles equally among 4 friends.
- D. There are 4 rows of flowers and there are 8 flowers in each row.
- E. There are 8 ducks in the pond. Then, 4 more ducks join them.

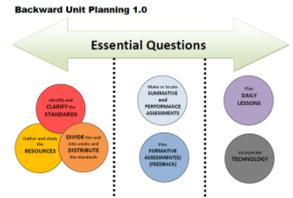
Make or locate
SUMMATIVE
and
PERFORMANCE
ASSESSMENTS



Options for Assessment – available online for Unit 3

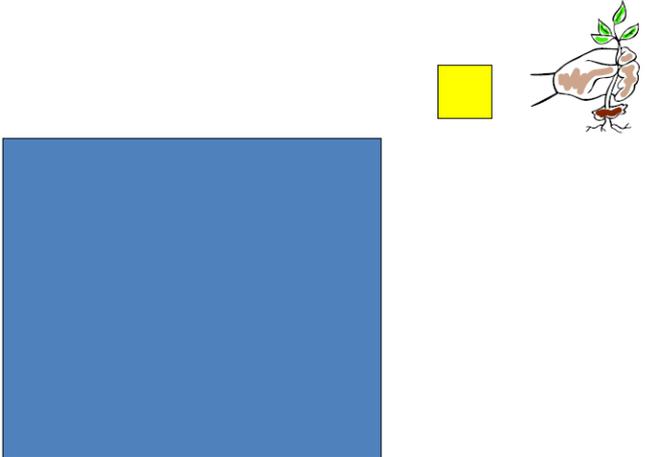
Essential Question:

What is area and how do I measure it?

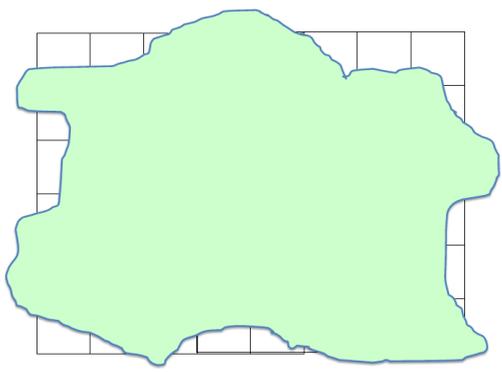


Name: _____

One plant can be planted in each square. How many plants can be planted in the space below. Explain your thinking.



This was a map of a garden Cody was going to plant. He was going to plant one plant in each of the squares. Someone spilled ink on his map. How many plants can Cody plant? Show or explain your thinking.



**Make or locate
SUMMATIVE
and
PERFORMANCE
ASSESSMENTS**

Apples come in crates. The crate has 6 rows going across and 5 apples are in each row. How many apples are in each crate?

At the grocery store they sell boxes with 24 peaches in each box. There were 3 peaches in each row. How many rows of peaches are there?

Kayla is making decorating a bulletin board. The board is 5 feet by 7 feet. How many square feet of paper will she need to cover the bulletin board?

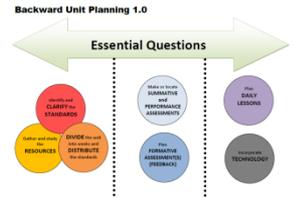
Mia was making a garden. She wanted it to be 48 square feet. If she made length of the garden 6 feet, how many feet is the width?



Options for Assessment – available online for Unit 3

Essential Question:

What is area and how do I measure it?



An art teacher will tile a section of the wall with printed tiles made by students in three art classes.

- Class A made 10 tiles.
- Class B made 14 tiles.
- Class C made 16 tiles.

Part A
What is the total number of tiles that are to be used?
 tiles

Part B
The grid shows how much wall space the art teacher can use. Use the grid to create a rectangular array showing how the art teacher might arrange the tiles on the wall. Select the boxes to shade them. Each tile should be shown by one shaded box.

Part C
Andy created a rectangular array showing how he would place 56 small tiles on the wall. He placed 7 tiles in each row. He wrote a multiplication equation using R to stand for the number of rows he used.
Write an equation using R that Andy could have written.

Grade 3	The art teacher's rectangular array
Type	Type III - 3 points
Evidence Statement	3.D.1: Solve multi-step contextual word problems with degree of difficulty appropriate to Grade 3, requiring application of knowledge and skills articulated in the Evidence Statements on the PBA (excludes Reasoning Evidence Statements). Clarification: 1) Tasks may have scaffolding if necessary in order to yield a degree of difficulty appropriate to Grade 3.
Most Relevant Standards for Mathematical Content	This standard is major content in the grade based on the PARCC Model Content Frameworks.
Most Relevant Standards for Mathematical Practice	Students must create a model of the situation using equations as well as the technology-enabled response space (MP.4). Note that there are multiple possible solutions, an increasingly important component of modeling across the grades. In order to create these models, students will need to reason abstractly and quantitatively with that context (MP.2).
Item Description and Assessment Qualities	This application task requires students to use their understanding of the four operations to solve a real-world problem. This is a two-step problem because students must add to determine the total number of tiles, and then determine how those tiles can be displayed in a rectangular array within a 10×10 grid. Students then write an equation with a letter to represent an unknown quantity to represent a similar situation using different numbers. This item allows for a variety of rectangular arrays and emphasizes that there are multiple representations by requiring students to write an equation. The rectangular array can be placed on the grid in a wide variety of ways but each correct representation will represent $6 \times 8 = 48$ or $8 \times 6 = 48$. Similarly the student should recognize that if 56 tiles are used with 7 tiles placed on each of R rows, then an equation such as $7 \times R = 56$ is another representation of the situation. The requirement of the use of a multiplication equation reinforces the relationship between multiplication and division. The response boxes are technology-enhanced so they can be electronically scored. Unlike traditional multiple choice, it is difficult to guess the correct answer or use a choice elimination strategy.
Scoring Information	Scoring Rubric Task is worth 3 points. Task can be scored as 0, 1, 2, or 3. Scoring consists of 2 points for modeling and 1 point for computation.

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Part A

- 1 computation point is earned for determining that the total number of tiles is 48 tiles.

Part B

- 1 modeling point is earned for stating the number of rows needed AND creating a rectangular array using the provided model that is 6 boxes wide and 8 boxes tall (or 8 boxes wide 6 boxes tall). For example, here are two of the many possible solutions:

Part C

- 1 modeling point is earned for writing a correct equation using R .
 $7 \times R = 56$ or $R \times 7 = 56$ or $56 = R \times 7$ or $56 = 7 \times R$

Note: An incorrect computation in Part A may be carried through Part B to receive the modeling credit of 1 point for Part B.

Find this interactive Assessment on the [PARCC site!](https://www.parc.org)



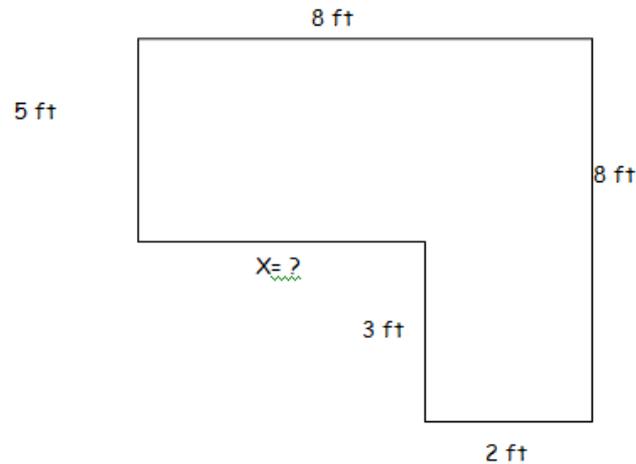
Options for Assessment – available online for Unit 3

Essential Question:

What is area and how do I measure it?



Chris built a dog pen for his new puppy. He made a drawing of the pen shown below. Find the total area of feet that his puppy has to play in. Explain your thinking with words, drawings, or equations.



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Options for Assessment – available online for Unit 3



Essential Question:

What are equivalent fractions?

Make or locate
**SUMMATIVE
and
PERFORMANCE
ASSESSMENTS**

Grade 3 – The Field - Part A

Part A
A farmer plants $\frac{3}{4}$ of the field with soybeans.
Drag the soybean to the field as many times as needed to show the fraction of the field that is planted with soybeans.

Part B
Type a fraction different than $\frac{3}{4}$ in the boxes that also represents the fractional part of the farmer's field that is planted with soybeans.

$$\frac{\boxed{3}}{\boxed{4}} = \frac{\boxed{}}{\boxed{}}$$

Farmer's Fields

Reset

Explain why the two fractions above are equal.

Grade 3	The Field – Part A
Type	Type I, Claim A & C
Most relevant Standard(s) for Mathematical Content	3.NF.1. Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.
Most relevant Standard(s) for Mathematical Practice	MP.2 enters in a simple way (Reason abstractly and quantitatively), as students in Grade 3 must link initially abstract symbols such as $\frac{3}{4}$ to the quantities they represent. MP.7 also enters (Look for and make use of structure), since the task involves spatially structuring the grid, and since it requires close attention to the numerator and denominator of the given fraction.
Item description and assessment qualities	This is an example of a fairly traditional fraction task in a technological setting. Note that the student is asked to show $\frac{3}{4}$ on a field that is divided into 8 equal parts. A student who drags only 3 soybeans onto the grid might be attending to the numerator of the given fraction but not the denominator. To find the answer, a student might mentally structure the grid so that it divides the field into 4 equal parts (e.g., the four rows in the array shown). Unlike traditional multiple choice, it is difficult to guess the correct answer or use a choice elimination strategy. Unlike traditional multiple choice, there is more than one correct solution. Unlike paper and pencil tests, students can create a visual representation even though the task is scored automatically.
Scoring	There are 28 possible ways to drag 6 soybeans onto a grid of 8 squares; all such responses are correct. Many students might fill the top three rows in the grid or the bottom three rows, as these two arrangements make it most obvious that $\frac{3}{4}$ of the field is planted.



Find this interactive Assessment on the [PARCC site!](https://www.parc.org)

Find this assessment [here](#) on the Illustrative Mathematics Site!



Essential Question:

What are equivalent fractions?

3.NF.3 Equivalent Fractions

Alec and Felix are brothers who go to different schools. The school day is just as long at Felix' school as at Alec's school. At Felix' school, there are 6 class periods of the same length each day. Alec's day is broken into 3 class periods of equal length.

One day, it snowed a lot so both of their schools started late. Felix only had four classes and Alec only had two. Alec claims his school day was shorter than Felix' was because he had only two classes on that day. Is he right?

Make or locate
**SUMMATIVE
and
PERFORMANCE
ASSESSMENTS**

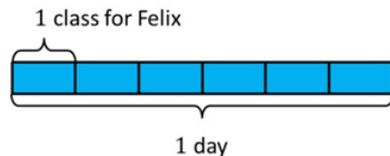
Commentary

The purpose of this task is for students to investigate a claim about a comparison of two fractions in a context. Many fraction problems are set in food contexts or a situation where a physical thing is being shared. It is important for students to work on more abstract quantities like time as well. This task addresses MP3, Construct viable arguments and critique the reasoning of others.

Solutions

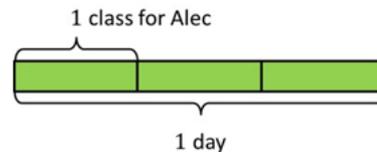
Solution: 1

Felix has six equal class periods each day.



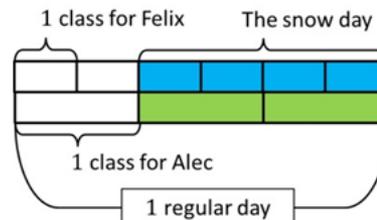
So each class period lasts for $\frac{1}{6}$ of the day.

Alec has three equal class periods each day.



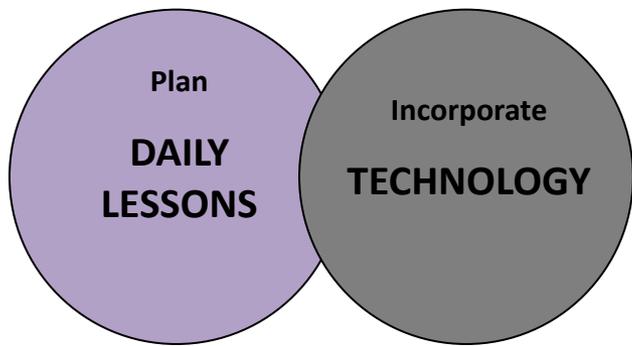
So each class period lasts for $\frac{1}{3}$ of the day.

Felix only had 4 class periods, so he went to school for $\frac{4}{6}$ of a full day. Alec only had 2 class periods, so he went to school for $\frac{2}{3}$ of a full day.



But a full day is equal for the two brothers, so two of Felix' class periods are the same length as one of Alec's. The brothers actually went to school for the same amount of time on the snow day.

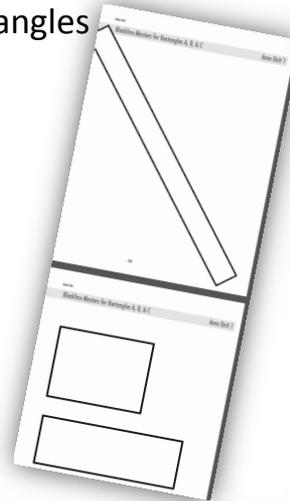




Week 1

Possible Lesson

Rich Lehrer Area Unit 1-
Partitioning and Comparing Rectangles
In this lesson, students will reason about 3 rectangles
And determine which has the greatest area.



Partitioning and Comparing Rectangles

Mathematical Concepts

- We call the space enclosed by a 2-dimensional figure an area.
- A 2-dimensional figure A can be partitioned (dissected) into two or more pieces. If the pieces of figure A cover all of figure B without any leftovers or overlaps, we say that the areas of the figures are congruent.
- A subdivision can be privileged to constitute a unit of measure. For example, a square or rectangular or triangular partition can serve as a unit of area measure.
- The measure of the area of a figure is the ratio of the area of the figure to the area of a unit. Practically, this is established by counting the number of units that cover the figure.
- Areas of different figures can be compared without re-arranging pieces by counting units, if the units are identical and tile the plane (as in units consisting of squares, rectangles, or triangles).

Unit Overview

This unit encourages students to spatially structure, and re-structure, 2-dimensional spaces as they compare the space covered (area) of three different-looking rectangles. Without using rulers or other metrics, students partition the rectangles and attempt to establish relations among the rectangles by re-arrangement of the partitions. Students typically propose privileging one of the partitions, most often a rectangle, but occasionally a square, and use the count of that unit to compare the space covered by the three rectangles. The unit ends with student investigation of re-arrangements of 12 unit squares to produce shapes with the same area but often with different perimeters and of different appearances. The formative assessment is aimed at firmly differentiating units of length measure (perimeter) from units of area measure.

UNIT

1

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Mathematical Background	4
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Additive Congruence	8
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1

Rich Lehrer Area Lessons

The following units of study were developed by Rich Lehrer, PhD, Professor of Education at Vanderbilt University and a team of other educators. His research focuses on children's mathematical and scientific reasoning in the context of schooling, with a special emphasis on tools and notations for developing thought.

All of the following resources are found at: disme.org (use "guest" to login and change the password to *geometry*)

[Area Construct](#) - describes how students come to understand the foundations of area

1: Partitioning and Comparing Rectangles [PDF](#) [WebPage](#) [Formative Assessment](#)

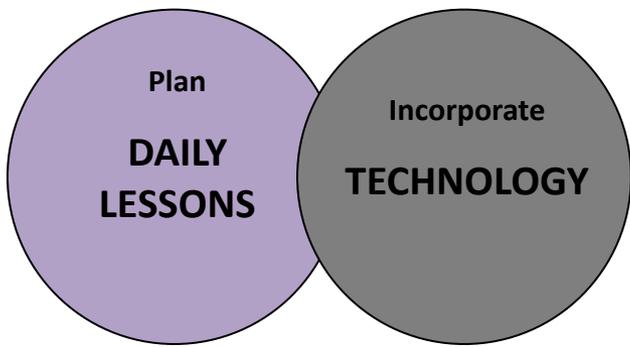
2: Comparing the Areas of Our Hands [PDF](#) [WebPage](#) [Formative Assessment](#)

3: Sweeping Area [PDF](#) [WebPage](#) [Formative Assessment](#)

4: Comparing Zoo Enclosures [PDF](#) [WebPage](#) [Formative Assessment](#)

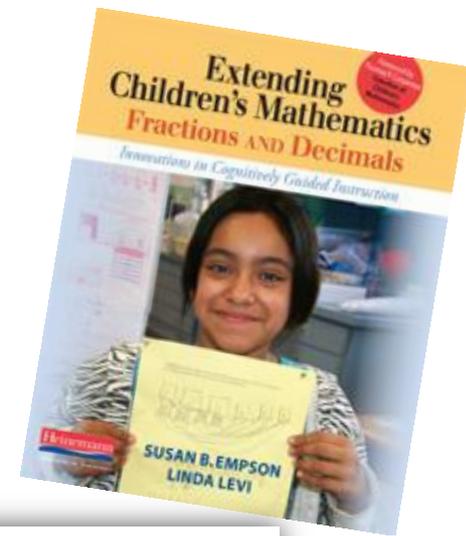
5: Justifying and Using Formulas to Find Area Measure... [PDF](#) [WebPage](#) [Formative Assessment](#)

1-5 Combined in one [PDF document](#)



Week 3

Possible Lesson



Launch

-APK –What’s your favorite thing to do at recess?

Picture of kids playing soccer – What are they doing?

-Pose the problem

Students

Independently Work

-Students work to solve the problem

-Teacher listens, notices and confers

-Teacher selects strategies to share

Discussion

-Compare and analyze strategies, mathematical understanding, notation, misconceptions, etc.

Who Gets More?

Name: _____
Date: _____

A. A play group is having a snack. There are different tables set up around the room.

- At the red table, 3 kids are sharing an apple. At the blue table, 4 kids are sharing an apple. Who gets more apple, the kids at the red table or the kids at the blue table? Explain your thinking.
- At the green table, 4 kids are sharing two apples. At the yellow table, 6 kids are sharing 2 apples. Who gets more apple, the kids at the green table, or the kids at the red table? Explain your thinking.



3.NF.3

District Purchased Resource –

Extending Children’s Mathematics
Fractions and Decimals

Week 5

Possible Lesson

3rd Grade

Building the Concept of Area

Units 3 & 4

Lesson 4:

- Give each student a loop of string (not stretchy) that measures 24 inches long and some [one-inch grid paper](#).
- Have students come up with as many different sized rectangles as they can with a perimeter of 24 inches on their grid paper. Have students record each rectangle and label the areas of each on the grid paper.
- Have students share their ideas. Create a class anchor chart of their ideas, recording their findings in a chart. Ask students what they noticed about each rectangle they found.
- **BIG IDEA:** Area and perimeter are different. Shapes can have the same perimeter, but different areas.

Lesson 5

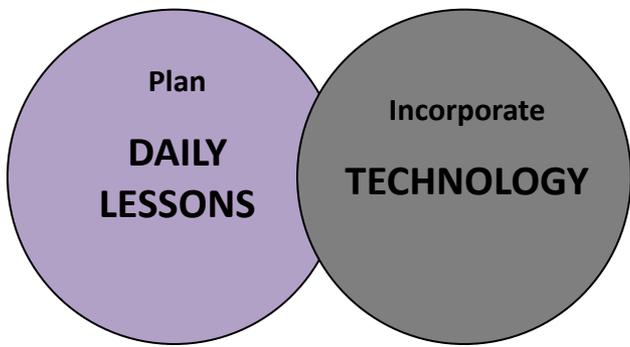
- Give each student 24 color tiles.
- Ask students to find as many different rectangles as they can with an area of 24 (using all 24 tiles to make filled-in rectangles – not just outlines of rectangles)
- Have students use some [one-inch grid paper](#) to record each rectangle with an area of 24 units and have them also find the perimeters of each.
- Have students share their ideas. Create a class anchor chart of their ideas, recording their findings in a chart. Ask students to discuss why it is that they all had the same area, but different perimeters.
- **BIG IDEA:** Area and perimeter are different. Shapes can have the same area, but different perimeters.

Plan
**DAILY
LESSONS**

Incorporate
TECHNOLOGY

Find
It
Here

Building the Concept of Area This series of mini-lessons builds the concept of area. (3.MD.5, 3.MD.6, 3.MD.8)



Week 7

Possible Lesson

Launch

-APK –What’s your favorite thing to do at recess?

Picture of kids playing soccer – What are they doing?

-Pose the problem

Students

Independently Work

-Students work to solve the problem

-Teacher listens, notices and confers

-Teacher selects strategies to share

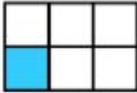
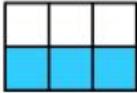
Discussion

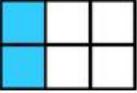
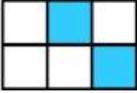
-Compare and analyze strategies, mathematical understanding, notation, misconceptions, etc.

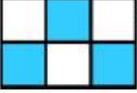
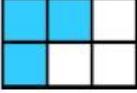
a. A small square is a square unit. What is the area of this rectangle? Explain.



b. What fraction of the area of each rectangle is shaded blue? Name the fraction in as many ways as you can. Explain your answers.

A.  B. 

C.  D. 

E.  F. 

G.  H. 

c. Shade $\frac{1}{2}$ of the area of rectangle in a way that is different from the rectangles above.

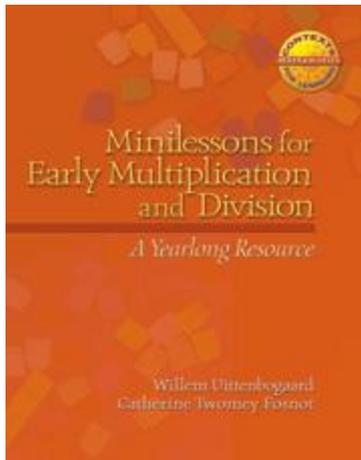
Find this lesson here!

Week 7

Possible Lesson

Week 9

Number Talk



See E11 for details (page 64).

The array has 48 squares and 6 rows.
The array has 48 squares and 8 rows.
The array has 48 squares and 4 rows.
The array has 48 squares and 12 rows.
The array has 48 squares and 2 rows.
The array has 48 squares and 24 rows.
The array has 24 squares and 4 rows.

How many columns?

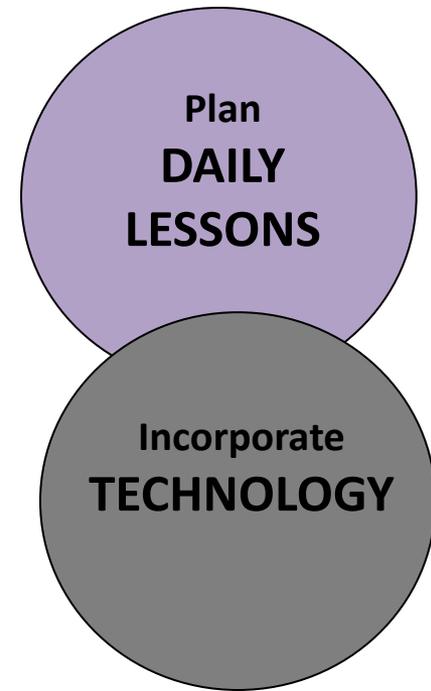
The Open Array · E13

Relating Multiplication to Division, Doubling, Halving

See E11 for details (page 64).

The array has 36 squares and 3 rows.
The array has 36 squares and 12 rows.
The array has 36 squares and 4 rows.
The array has 36 squares and 9 rows.
The array has 36 squares and 2 rows.
The array has 36 squares and 18 rows.
The array has 18 squares and 9 rows.

How many columns?



1. Pose the problems to students.
 2. Have them solve mentally.
 3. Add notation to their thinking!
- 😊

NEW Teacher Created Resources pages!!!



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