Partitioning and Comparing Rectangles

UNIT

1

Mathematical Concepts

- We call the space enclosed by a 2-dimensional figure an area.
- A 2-dimensional figure A can be partitioned (dissected) into two or more pieces. If the pieces of figure A cover all of figure B without any leftovers or overlaps, we say that the areas of the figures are congruent.
- A subdivision can be privileged to constitute a unit of measure. For example, a square or rectangular or triangular partition can serve as a unit of area measure.
- The measure of the area of a figure is the ratio of the area of the figure to the area of a unit. Practically, this is established by counting the number of units that cover the figure.
- Areas of different figures can be compared without re-arranging pieces by counting units, if the units are identical and tile the plane (as in units consisting of squares, rectangles, or triangles).

Unit Overview

This unit encourages students to spatially structure, and re-structure, 2-dimensional spaces as they compare the space covered (area) of three different-looking rectangles. Without using rulers or other metrics, students partition the rectangles and attempt to establish relations among the rectangles by re-arrangement of the partitions. Students typically propose privileging one of the partitions, most often a rectangle, but occasionally a square, and use the count of that unit to compare the space covered by the three rectangles. The unit ends with student investigation of re-arrangements of 12 unit squares to produce shapes with the same area but often with different perimeters and of different appearances. The formative assessment is aimed at firmly differentiating units of length measure (perimeter) from units of area measure.

Contents

Materials and Preparation

Area Unit 1

Read

- **Unit 1**
  Start by reading the unit to learn the content and become familiar with the activities.

Gather

- Sets of three rectangles constructed from unmarked chart paper with (horizontal x vertical) dimensions of: 12×1, 4×3, and 2×6. A template for each rectangle is found in the Appendix. Label the rectangles A, B, and C (See Figure 1). Rectangles must have no folds or tape lines. Do not laminate or use paper with lines, grids, or other markings.
- If you are working with a group or a whole class, make larger rectangles with the dimensions scaled as:
  - A = 48 $in. \times 4$ $in.$
  - B = 16 $in. \times 12$ $in.$
  - C = 8 $in. \times 24$ $in.$

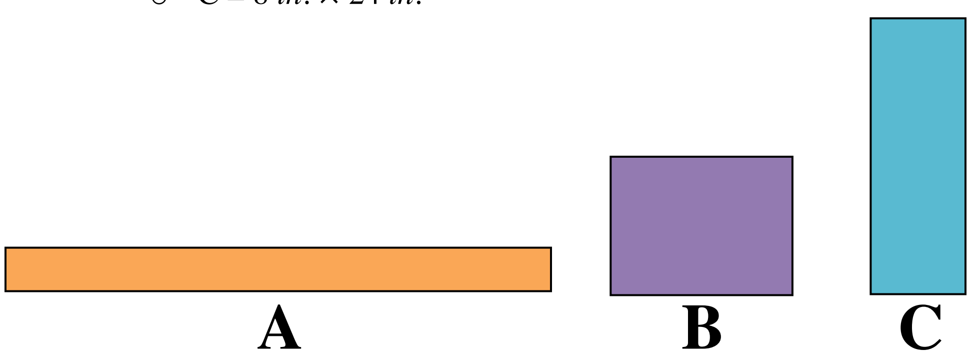


Figure 1. Rectangles A, B, and C

- Unit squares consistent with the dimensions of the smaller rectangles (1 $in. \times 1$ $in.$)
- Markers
- Chart paper for drawing figures and recording dimensions
- Scotch tape or markers for affixing rectangles to the board
- Teacher math-journal (for note-taking)
- Camera (optional, for documenting student activity)
- Student math journals

Materials and Preparation — Area Unit 1

Prepare

- Three large rectangles that the whole class can see and refer to.
- Three rectangles for each pair of students
- 12 identical unit squares or 6 identical unit rectangles for each pair.
- Space for pairs to work together relatively independently
- 1 T-chart per pair labeled “Area” and “Perimeter”
- Make markers available

Academic Vocabulary

▪ Area	Unit
▪ Length	Greater than
▪ Width	Less than
▪ Rectangle	Equal to
▪ Perimeter	Split
▪ Congruent	Length

Mathematical Background Area Unit 1

The equality of area of 2-dimensional figures is established by demonstrating that they are additively congruent or by counting the same number of identical area units.

Additive Congruence/Dissection

If a 2-dimensional figure can be folded and its partitions rearranged so that they completely cover a second figure, the two figures are said to be *additively congruent*, and they have the same area. Another way to say this mathematically is that they are equivalent by dissection. One can be cut up into a finite number of pieces, and these pieces can be rearranged to form the second figure.

Area Measure

Area measure is a ratio of the space enclosed by a plane figure and a unit of measure. Even though two figures may look very different, they may still have the same area, covering the same amount of space.

Unit of Area Measure

Units of area measure (typically squares or rectangles) are consistent-sized partitions of 2-dimensional space that can be counted and composed to quantify the area enclosed by a figure. Measuring all of the figures with the same unit is a way to compare their areas.

Perimeter

The linear distance all the way around a figure is its perimeter. This distance can be thought of as a path around a figure. Perimeter is measured with units of length, in contrast to area's measure with 2-dimensional units.

Whole Group

The teacher presents the three rectangles to the class and pairs of students (or each student) receive a scaled copy of the set. For whole group, the teacher attaches each rectangle to the board with magnets or tape so students can make comparisons among them. She tells students she is making a quilt or rug by cutting and sewing together remnants (pieces). Each piece costs the same amount, so she wants to buy the piece that covers the most space, so she gets the most for her money. She explains that the three pieces of paper represent the 3 sizes the material is sold in. She asks students to help figure out which piece to buy to get the most material.

Teacher Note: The teacher facilitates the discussion, asks clarifying questions about student statements, asks for student reasoning or definitions, and juxtaposes students' ideas to promote mathematical argument around the structure and measurement of the three rectangles. Students are permitted to fold the pieces but <u>not</u> to use rulers or other tools. The aim is to support strategies of additive congruence (dissection) — meaning that students split the area into parts and rearrange the parts to establish the relative amounts of space covered by each rectangle.

Discussion Questions

Q: Which piece of material covers the greatest amount of space? Why do you think so?
Q: How can we compare the space without cutting it?
Q: If there is a proposal that they all cover the same amount of space: How can three rectangles that look so different cover the same amount of space? How could I demonstrate that they do?

Pair Work

Students work in pairs to consider which of the three rectangles covers the most space. As students work in pairs, the teacher roams and prompts student thinking:

> Q: If students develop a consistent partition to establish that the rectangles have the same measure:
> Is there another one that would work?
> Q: If students have developed a rectangle as a unit of measure and no other solution seems to use a square, ask at least one pair:
> Is there a square that might work for comparing the space enclosed by the rectangles?

Whole Group

Allow students to explore these various methods (without cutting or physically disassembling the rectangles) and have them share and compare strategies. After students have shared their strategies, ask them to consider which methods would work all of the time or with any set of rectangles. If student strategies have not included a unit (e.g. a rectangle), then introduce this idea to them and ask what its advantages might be. It is also helpful to compare how different partitions lead to different units. For example, the space covered by rectangle A can have a measure of 12 (1 *in*. x 1 *in*.) squares, or 6 (1 *in*. by 2 *in*.) rectangles or 3 (1 *in*. by 4 *in*.) rectangles. *Point out that although the amount of space covered (its area) is not changing, its measure is*.

Exploration and Extension

After students create a unit of measure and agree to privilege that unit, give everyone 12 identical units (if squares, or 6 if 1 x 2 rectangles). Ask students to find at least 5 different ways the same amount of space can be covered. For each configuration of units, students should record the perimeter and the area. Which configuration has the least perimeter? The most? Why? Record findings in math journal or on worksheet.

Formative Assessment

Administer the formative assessment and select contrasting student responses to create further opportunities for learning about area measure, especially the difference between units of length measure (perimeter) and units of area measure.

STUDENTS' WAYS OF THINKING

- Students may fail to specify attributes, simply insisting instead that one or another rectangle is "bigger." If so, ask students to clarify what they mean by "big."

- Students may focus on one dimension, such as "tallest" or "longest" to justify their choice. If so, promote the idea that although one rectangle is longer, another may be wider, and that both length and width need to be considered together.

- Students may suggest a number of different methods for comparing the area of the rectangles. They may initially try to superimpose one onto another. If so, you may ask, "But what about the part of Rectangle X that isn't covered by Rectangle Y?"

- Students most typically fold subsections of one rectangle and use congruence to compare that section to a section of another rectangle (equality of subsections. Note that the subdivisions do not need to be squares. See Typical Student Strategies (next page).

- Students may match unequal subsections to parts on each of the other rectangles until all the space is accounted for. They may need support to account for all the parts as they superimpose them on each other. See Typical Student Strategies.

- As students reconfigure the space by folding and comparing one piece to another, units may emerge. Folds may create an array of identical squares or rectangles that can be used as units to find the measure of any space. Teachers should support the action of unit creation and use it to discuss "privileging a partition." See Typical Student Strategies.

Typical Student Strategies for Partitioning and Re-arranging Equal Parts

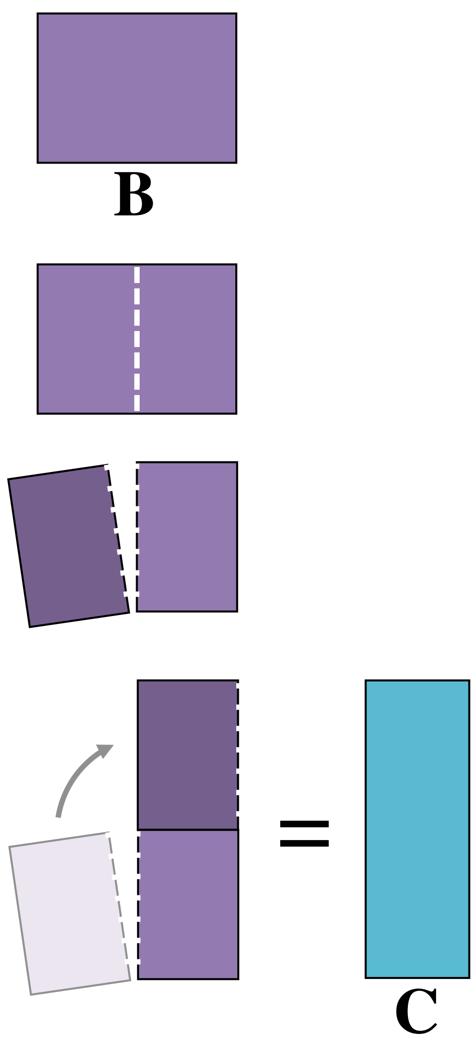
Rectangle **B** rearranged to rectangle **C**

$\frac{1}{2}\mathbf{B} = \frac{1}{2}\mathbf{C}$

$2\left(\frac{1}{2}\mathbf{B}\right) = 2\left(\frac{1}{2}\mathbf{C}\right)$

$\mathbf{B} = \mathbf{C}$

B $= 4 \times 3$, so the half split is 2×3. When $\frac{1}{2}$ B (or 2×3) is slid up and over the other half of B, the space is rearranged to rectangle C, 2×6.



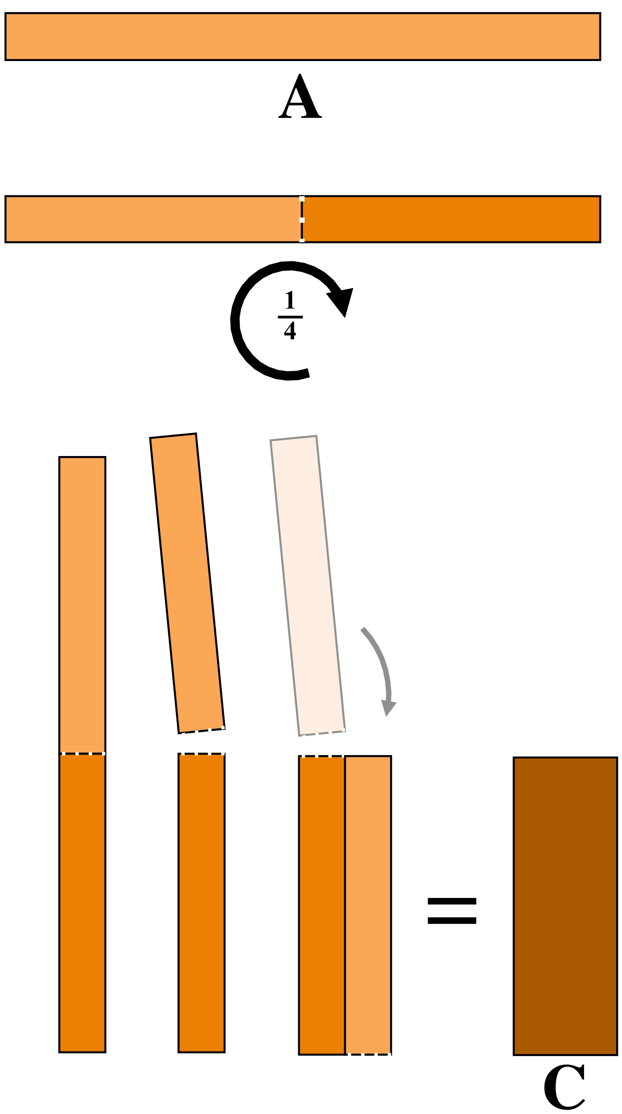
Rectangle **A** rearranged to rectangle **C**

$\frac{1}{2}\mathbf{A} = \frac{1}{2}\mathbf{C}$

$2\,(\frac{1}{2}\mathbf{A}) = 2\,(\frac{1}{2}\mathbf{C})$

$\mathbf{A} = \mathbf{C}$

$\mathbf{A} = 12 \times 1$, so the half split is 6×1. When **A** is rotated $\frac{1}{4}$ turn right and $\frac{1}{2}$ **A** is slid over and down, the space is rearranged to rectangle **C**, 2×6.



Rectangle **A** rearranged to rectangle **B**

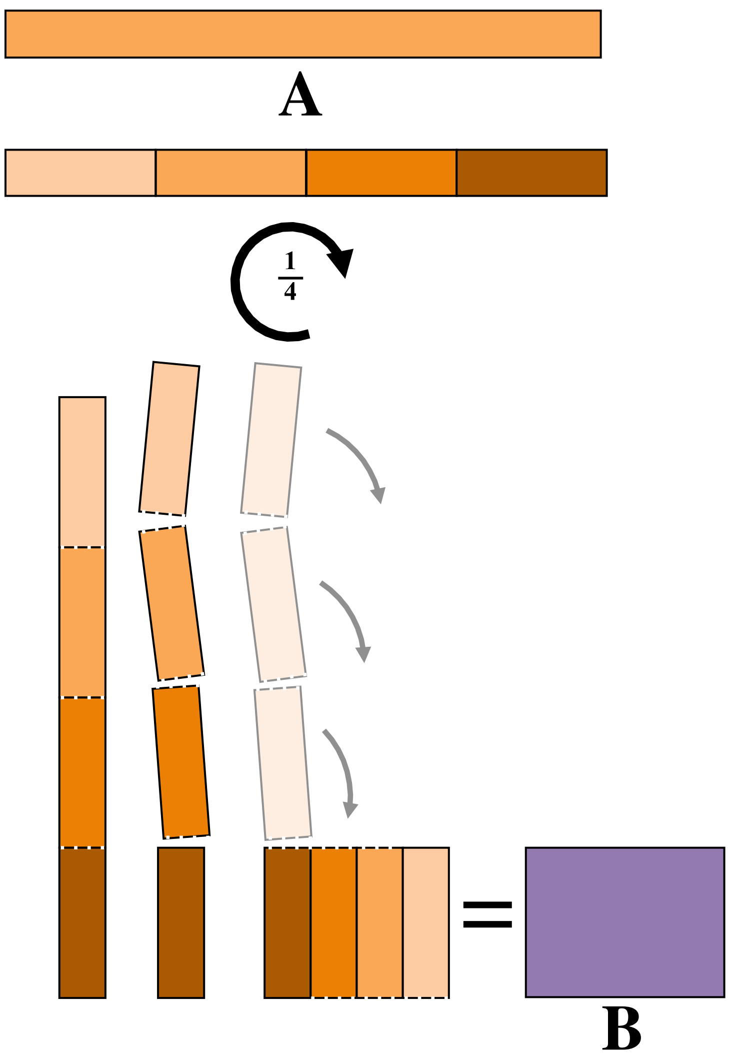
$\frac{1}{4}\mathbf{A} = \frac{1}{4}\mathbf{B}$

$4\left(\frac{1}{4}\mathbf{A}\right) = 4\left(\frac{1}{4}\mathbf{B}\right)$

$\mathbf{A} = \mathbf{B}$

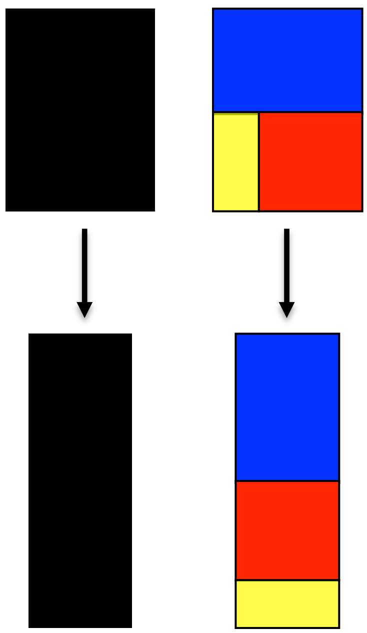
$\mathbf{A} = 12 \times 1, so \frac{1}{4}\mathbf{A} = 3 \times 1$

When A is rotated $\frac{1}{4}$ turn right, and each forth is slid so the long side is aligned with the next fourth, A is rearranged to rectangle B, 4×3.



Additive Congruence Using Unequal Parts

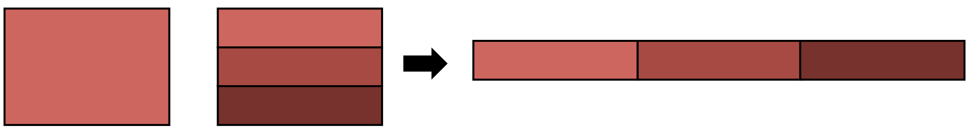
Equivalence of B (3 × 4) and C (6 × 2) established via dissection but with unequal parts.



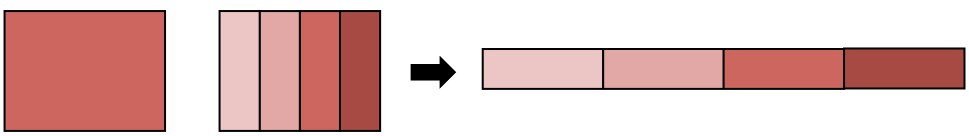
Measurement Congruence

Structuring a Square Measurement Unit from Rectangular Folds

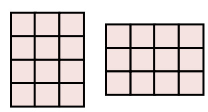
Fold thirds of **B** (12 in. x 16 in.) horizontally (each is 4 in. x 16 in.) and use the $\frac{1}{3}$ of **B** to measure, for example, **A**.



Then fold fourths of **B** vertically (each is 12 in. x 4 in.) using a composition of 2-splits to measure, for example, **A**.



Unfold rectangle **B**. It is composed of 12 squares (each is 4 in. x 4 in.). Notice that rotating **B** does not affect its area.

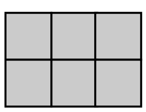


Formative Assessment — Area Unit 1

NAME:______________________________

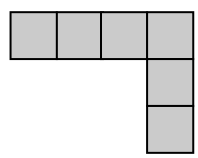
1. Use your ruler in the space below to draw an inch and a square inch.

2. Each of these shapes was made by putting together square inches (pretend that each square is 1 inch on every side). What is the area of each figure? What is its perimeter?



Area = ____in^2

Perimeter = ____in



Area = ____in^2

Perimeter = ____in

3. The area of a rectangle is measured with square inches and with square centimeters. One inch is about $2\frac{1}{2}$ times as long as one centimeter.

   Circle the statement that is true.

   a. The measure of the area of the rectangle is greater with square inches than with square centimeters.
   b. The measure of the area of the rectangle is greater with square centimeters than with square inches.
   c. The measure of the area of the rectangle is not affected by the choice of square inch or square centimeter.

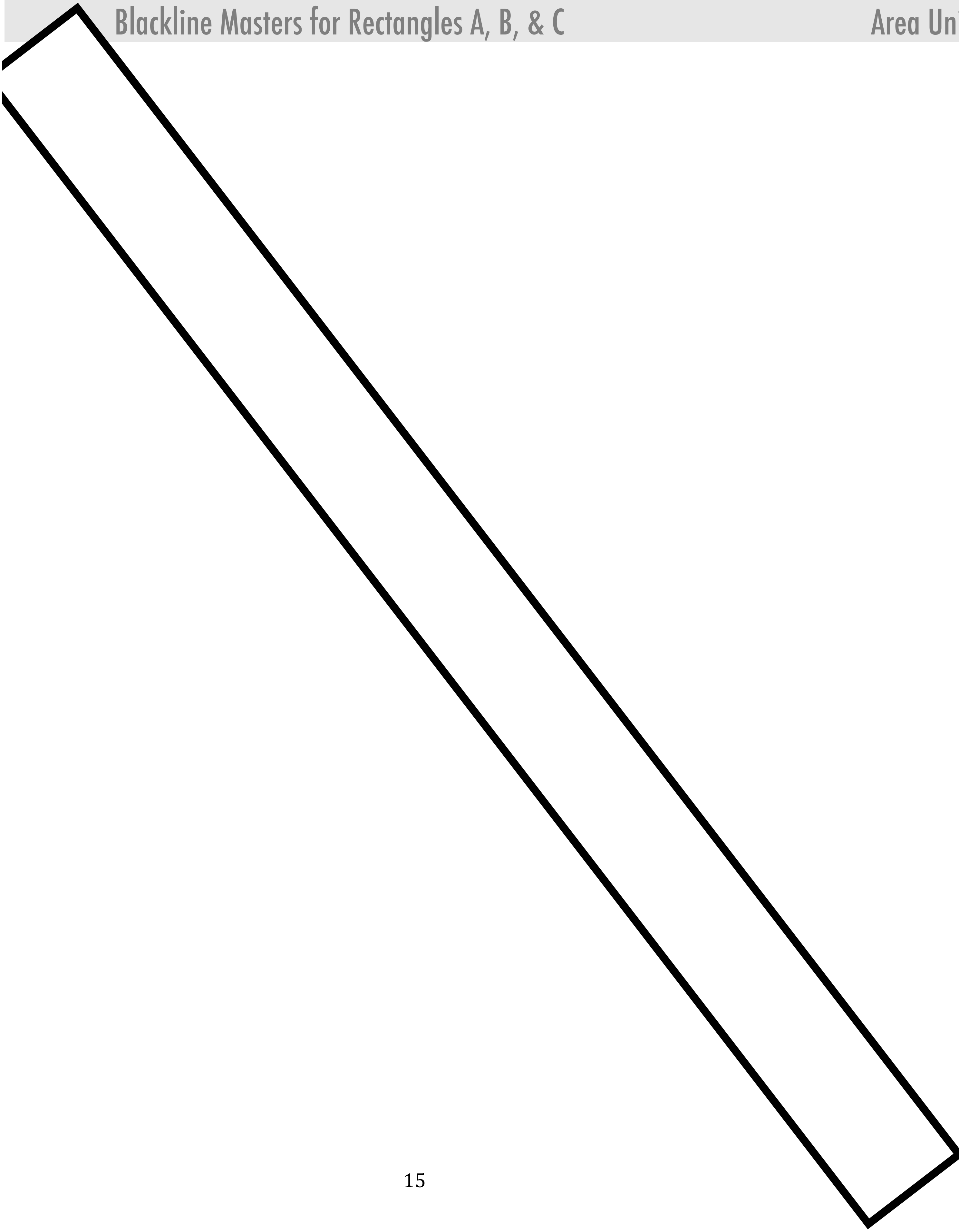
Tell why:

Formative Assessment Record — Area Unit 1

Indicate the levels of mastery demonstrated by circling those for which there is clear evidence:

Item	**Level** Circle highest level of performance	**Description**	**Notes**
Item 1 Constructing and Differentiating Inch and Square Inch.	**ToAM 3D** Recognize/construct suitable units.	Distinguishes between inch and square inch.	
	NL	Cannot construct the distinctions.	
Item 2 Using counts of units for area, edges of squares for perimeter.	**ToAM 3D** Recognize/construct suitable units.	Area as 6 $in.^2$ and perimeters as 10 *in.*, 14 in.	
	Other Describe		
Item 3 Extension: Constructing and Differentiating fractional length and area.	**ToAM 3E** Flexible conceptions of appropriate units of area unit, including recognition of inverse relation between unit and area magnitudes.	Chooses b and explains that each square centimeter covers less space and so it takes more of them to cover the area of the rectangle.	

Blackline Masters for Rectangles A, B, & C Area Unit 1



Blackline Masters for Rectangles A, B, & C

Comparing the Areas of Our Hands

UNIT

2

Mathematical Concepts

- Area is measured as the ratio of the space covered by a figure to that covered by a unit.
- Areas of different figures can be compared by counting units, if the units are identical and tile the plane.
- Units of area measure need not resemble the figure being measured.
- Partial units of area measure can be constructed and represented as fractional measures of a unit.

Unit Overview

Students compare and order the area of their hands. The unit begins by posing a challenge to students of finding a way to order the area covered by each student's hand from least to greatest. Students typically try to meet this challenge by direct comparison of pairs of hands, but they typically quickly find that this does not work—how does one compensate for thicker pinkies and slimmer thumbs? Most often, this failure of direct comparison motivates the need for a unit of measure. To facilitate comparison, each student traces the outline of her or his hand and makes a cutout of the outline. Motivated by the failure of direct comparison, students are encouraged to construct a unit of area measure. Construction tools include items that resemble the contour of the hand, such as beans and spaghetti, as well as paper ruled with square inch or square cm. During the course of investigation, students usually find that although beans and spaghetti resemble the contour of the hand, they do not yield consistent counts, because they don't tile the plane ("cracks") and are of variable size. Children are guided to consider the virtue of square units that do not resemble the contour of the hand. This in turn promotes attention to the "left-overs" – units that are not contained in the boundaries of the hand—and hence to developing and combining fractional units of area measure. The lesson concludes with ordering the measures of the area of hands.

Contents

Materials & Preparation — Area Unit 2

Read

- **Unit 2** Start by reading the unit to learn the content and become familiar with the activities.
- The Area Measure Construct Map to become familiar with landmarks of student thinking.

Gather

- Construction paper
- Black felt-tip markers
- Scissors
- Materials that resemble the contour of the hand and that can be contained within the boundaries of a handprint (e.g., beans, spaghetti, buttons)
- Grid paper (1 in. square and 1 cm. square) and/or transparent grid paper.
- Color pencils
- Chart paper

Prepare

- Have students trace and cut-out their handprint on a piece of construction paper

Academic Vocabulary

- Area
- Length
- Width
- Unit
- Greater than
- Less than
- Fractional Unit

Mathematical Background

The areas of 2-dimensional figures are compared by relating each area to the same unit of area measure. Area measure is the ratio of the space covered by the figure to the space covered by a unit of area measure. To enable this comparison, which does not rely on the partitioning and re-allotment of the previous unit, the units should have the following properties:

Identical units

Units of area measure should be identical, unless specifically marked, as in "5 square yards and 3 square ft.", although $5\frac{1}{3}$ square yards would be more appropriate in most circumstances.

Tiling

The units of area measure should tile the plane. Else, portions of the area will not be measured.

Iteration

The unit of area measure can be "re-used" so that it is not necessary to literally cover the entire figure with units of area measure.

Partial units

Units can be partitioned to allow for measure of areas that are not whole-number counts of units. Fractional parts of the same unit can be additively combined, as in $\frac{1}{2}\,in.^2 + \frac{1}{2}\,in.^2 = 1\ in.^2$

Introducing the Problem

Whole Group

Think about putting your hands in order from biggest to smallest. Talk with your elbow partner:

> Q: What do we mean by biggest?
> Q: What about the hand are we thinking about?
> Q: Whose hand do you think covers the least amount of space? Explain.
> Q: Whose hand do you think covers the most amount of space? Explain.

Partners

Come up with a way to compare the area of your hands.

Whole Group

Elicit student strategies, which are most likely based on direct comparison. Working from what students say, select two students for whom direct comparison will be difficult to discern—perhaps because one has thicker thumbs but skinnier fingers than her or his partner.

> Q: What else might we do if we can't compare everyone's hands directly?
> Q: How did we compare the rectangles? How did we know that they all had the same area? (Unit of measure)
> Q: How might working with paper cut-outs of our hands be better than working with our hands?

Finding a Unit of Measure

Whole Group

Make a paper cut out of your hand. Then find the measure of its area by counting units. You can use any of these as units or you can make up one of your own. (Indicate the materials that are available.)

Individual

As students work, pose questions to help students relate the goal of measurement to the unit they choose. Depending on student strategy, these questions may be fruitful for helping students think:

Q: Why did you choose that unit?
Q: Did you measure all of the space?
Q: What did you do about the curved parts?
Q: What did you do when a unit hung over the edge of your hand/finger?
Q: What did you call this space that is only covered by part of a unit?
Q: How did you keep track of all the parts?

Partner

Write down the measure of your hand but don't tell anyone. Then give your handprint to your partner and ask them to find the measure of your hand with your unit.

Whole Group

During a whole-class conversation, choose some students to present their partner's strategy and unit, and any problems that they ran into.

> Q: If we all use this unit (e.g., the beans), should we all get the same measurement? Why?
> Q: Why didn't we all get the same measurement?

Teacher note. Try to raise the problem of obtaining reliable measurement when the unit does not tile the plane (leaves space unmeasured) and/or is not identical. A count of units of measure of the same handprint is often revealing.

If no student used the grid paper, ask: Could we use this as a unit of measure by re-drawing our handprint on it? (Elicit opinions and rationales).

Partial Units, Inch or Centimeter Square

Whole Group

Some of us think that this grid paper could be used as a unit of measure and some of us aren't so sure. Let's try it out. Trace your hand on the large grid paper.

Individual/Partner

Using the grid paper, find the measure of the area of your hand.

As students work, the teacher can ask individuals or small groups:

> Q: When we use the grid paper, what is the unit we are using to measure?
> Q: What problems that we had does the grid paper help us solve?
> Q: What are some problems that you are finding when you use the grid paper?

Teacher note. Help students see the need to work with partial units of measure.

Whole Group check-in

Select student strategies for presentation. Look for a range of solutions to the problem of partial units. If no student has done so, raise the prospect of identifying and labeling part-units with fractions. To support visualization, use chart paper or other means, and invite students to generate as many different $\frac{1}{2}$ partitions of the square that they can. *Teacher note*. Be sure to include partitions along a diagonal and "checker board" partitions into quarters, with any 2 quarters shaded.

Repeat for $\frac{1}{4}$ and $\frac{1}{3}$.

Individual-Partner

Using what we have just talked about, finish up your measurement of the area of your hand.

Whole Group

Select student strategies for presentation. Focus on whole-number only solutions, solutions with matching parts, and solutions that combine fractional parts. Be sure to emphasize the nature of units, especially the need to use identical units that tile the plane. Also emphasize that constructing partitions of units means that units do not need to resemble the shape being measured.

Teacher note. This is a great opportunity to re-consider fraction addition, both adding fractions of the same partition (split) (such as halves and such as thirds), and using common partitions (such as fourths, to find sums of halves and fourths, or sixths, to find sums of thirds and halves, etc).

What if we change the unit of measure?

Whole Group

Show students the cm. square grid. If we use this unit to measure, predict whether or not the measure of the area of your hand will increase, decrease, or remain the same. Talk with your elbow partner and see if you agree.

Q: If you think it will remain the same, why do you think so?
Q: If you think it will decrease, why do you think so?
Q: If you think it will increase, why do you think so?

Individual

Use the cm. square grid paper to find the area of your hand.

Whole Group

What happened and why? What might be some advantages to using the smaller cm. squares to measure the area of your hand?

Using the Measurements

Whole Group

Let's use our inch square or our cm. square measurements to order the size of our hands. Talk with your elbow partner about how we should do that. (Order the measurements on the board from greatest to least)

Partner

Using our measurements, what was the difference in area between the largest and the smallest hand?

When you look at all the measurements, what do you notice?

Whole Group Summary

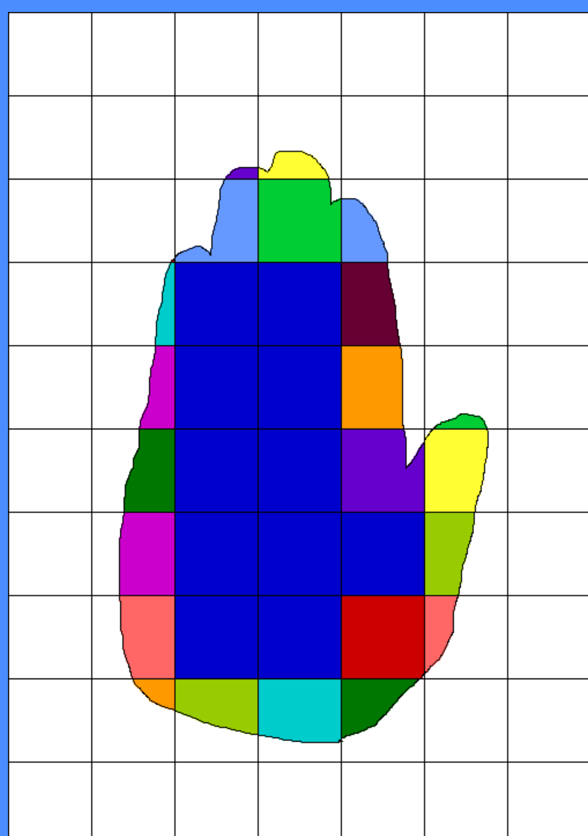
Conduct a conversation that makes explicit the use of measurement units to compare the areas of shapes, even when they are not rectangular, but do have an inside and an outside. Raise the functional value of using identical units that tile the plane. Be sure to re-visit the role of partial units and how they can be combined. Conclude with what students noticed about the measurements when they viewed them collectively as an ordered list.

STUDENTS' WAYS OF THINKING

Students' choices of appropriate units of area measure are often guided by criteria of literal resemblance and boundary. Literal resemblance refers to a disposition to select or make units of measure that resemble the figure being measured. So, for handprints, beans and spaghetti are often considered as potential units that look like the contour of the hand, as displayed below.

STUDENTS' WAYS OF THINKING

Boundary refers to the need to have units of measure fill the space enclosed by the figure so students are often reluctant to select a unit that is not contained within the figure. When students overcome this reluctance and use units that are not fully enclosed by the figure, this creates the need to consider partial units and to consider how to count these units to create a measure. In the illustration below students used a strategy of matching parts: They indicated parts of units that when combined would produce approximately one square inch by using the same color for those parts.



STUDENTS' WAYS OF THINKING

Students may choose from several different strategies to account for all the fractional parts.

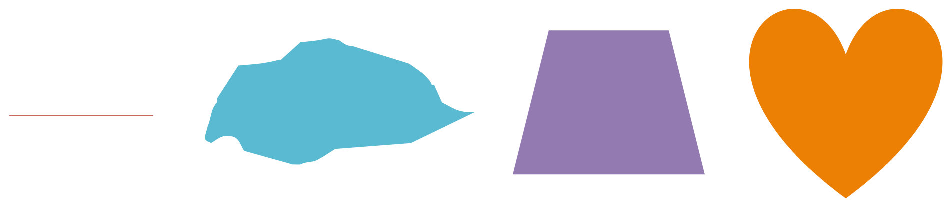
- *Matching parts-* students may simply indicate parts of two different sections of the hand that "look like" they would add up to a whole square (as displayed).
- *Adding like fractions-* students may join together fractional parts with the same partition (i.e., count the number of ½ square units and add them together, then finding all the ¼ square units and adding them together)
- *Adding fractions with different denominators-* students with a strong understanding of equivalence (i.e., $\frac{2}{4} = \frac{1}{2}$) and addition may choose to combine fractions with different partitions and add them (i.e., $\frac{1}{2} + \frac{3}{4} \rightarrow 1\frac{1}{4} + \frac{2}{8} \rightarrow 1\frac{1}{2} + \frac{1}{3} \rightarrow 1\frac{5}{6}$)

Formative Assessment — Area Unit 2

Name:_______________________________________

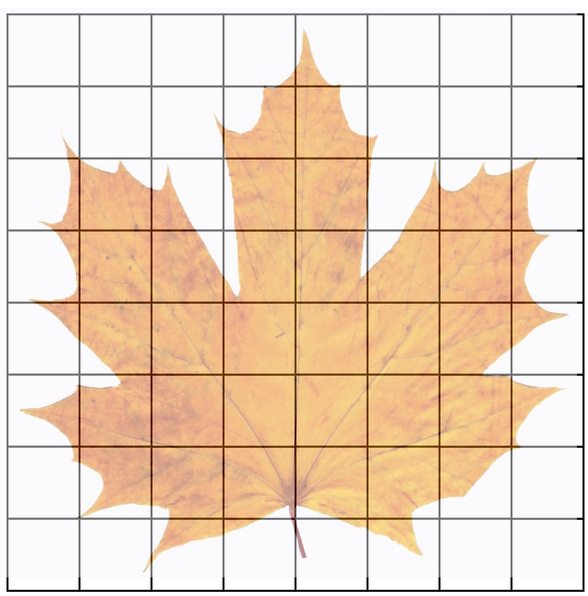
1. From the following, circle the shapes that have an area:

A: B: C: D:



2. Use the square as the unit of measure. What is the area of the leaf?

Show your work.



Formative Assessment Record Area Unit 2

Indicate the levels of mastery demonstrated by circling those for which there is clear evidence:

Item	**Level** Circle highest level of performance	**Description**	**Notes**
Item 1 Area as Space Enclosed by Figure	**ToAM 1A**	Circles all figures. May include line if justifies by saying that it has some small thickness.	
	NL	Circles only the trapezoid (thinks that area is only for figures without curves).	
Item 2 Using counts of units for area.	**ToAM 3F**	Partitions unit and combines like partitions.	
	ToM 3C	Counts only whole units contained within contour of the hand.	
	Other Describe		

Sweeping Area

UNIT

3

Mathematical Concepts

- Area is generated by sweeping one length along another at some angle greater than 0.
- A measure of the area generated by a sweep can be found by re-structuring the swept area as a set of discrete units.
- The units are products of the measures of length.
- Units can be whole or parts (fractions).
- The area of a rectangle is the product of the measures of the lengths of its base and height (length × width).
- Cavalieri's principle.

Unit Overview

This unit encourages students to spatially structure, and re-structure, 2-dimensional spaces as they generate area by sweeping one length through another and then use thin spaghetti to define units. Area measure is the sum of the units. The lesson begins with sweeping play. Using a ceramic tile, a wet surface, such as shaving cream or finger paint, two rulers aligned on the base and left side of the tile, and a squeegee, students hold the squeegee in different ways to generate shapes that look like squares, rectangles, parallelograms, and circles. Then, using rulers aligned with the sides of the tile, students sweep squares and rectangles of specific dimensions. They use spaghetti to create units of the area of the swept figures in square inches and in squeegee-inches. Students make a conjecture about a formula to find the area of a rectangle. The area of a rectangle and parallelogram of the same height and base are compared. The lesson concludes with an informal investigation of Cavalieri's principle: In comparison to a rectangle with base b and height h, moving the length of the base parallel to the base through h produces a figure with the same area.

Contents

Materials and Preparation

Area Unit 3

Read

- **Unit 3**
  Start by reading the unit to learn the content and become familiar with the activities.
- **Area Measure Construct Map**
  Read the area measure construct map to get a sense of the forms of thinking about area targeted by this lesson.

Gather

- Square ceramic tiles thick enough to allow placement of rulers along the sides without interfering with the action of the squeegee.
- 2 foot-rulers for each tile
- Shaving cream (high lubrication) or finger paint.
- 5 *in*. and 8 *in*. long squeegee's (or whatever else is handy)
- Boxes of thin spaghetti
- Paper towels for cleaning.

Prepare

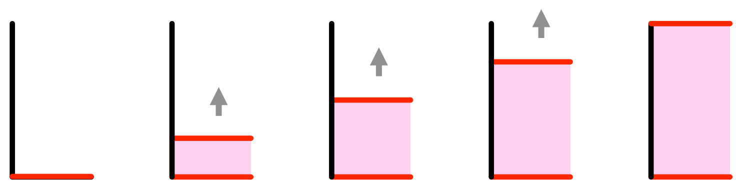
- Pairs of students prepare one ceramic tile by smearing finger paint or shaving cream over its surface

Academic Vocabulary

• Area	• Unit
• Length	• Greater than
• Width	• Less than
• Rectangle	• Equal to
• Sweep	• Split

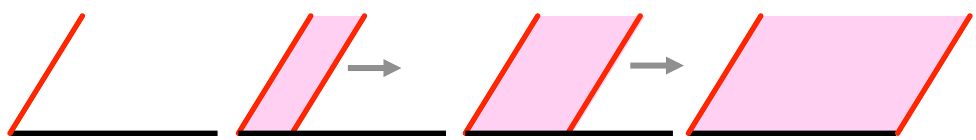
Sweeping

Area is generated by sweeping one length through another at any angle other than 0. In the figure below, a horizontal length is swept through a vertical length, generating an area.

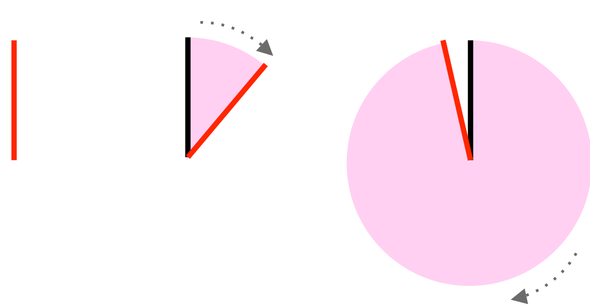


One length (*red*) pulled along the other length (*black*) constructs an area (*pink*).

The lengths can be oriented to create any angle, other than 0.

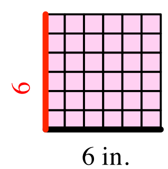


The type of motion can include parallel translation, as in the previous two examples, or rotation, as in the figure below.



Area Measure

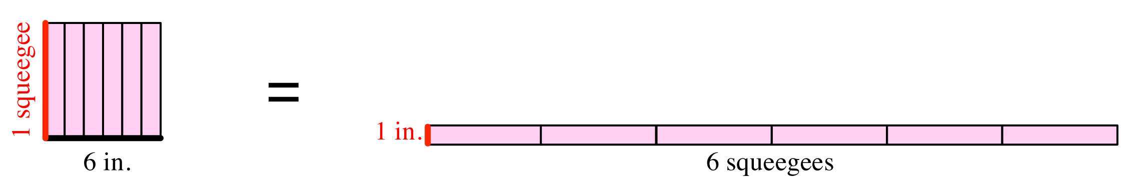
Area measure is a ratio of the space enclosed by a plane figure and a unit of measure. A unit of measure can be generated by sweeping one length through another, as in the figure below, where 1 inch translated through another inch generates a square inch. The area of this figure is 36 times the area of one square inch.



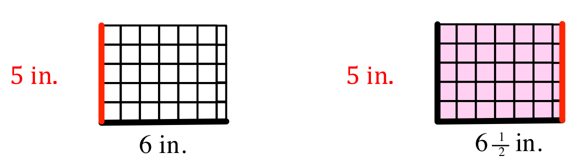
6 in. X 6 in. = 36 sq. in.

Area measured in a length × length unit (1 square inch).

The units of length measure can differ. In the figure below, the measure of the figure is 6 squeegee-inch, where one side of the figure is measured in squeegee units (the side is one squeegee length), and the other side is measured in inches. The area of this figure is 6 times the area of one squeegee-inch. Sweeping 1 inch along 6 squeegees will produce the same area and the same area measure.

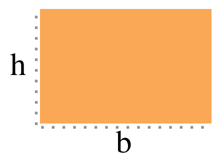


The units of area measure can be split. For example, 5 in. swept through a length of $6\frac{1}{2}\,in$, results in an area measure of $32\frac{1}{2}\,in^2$.



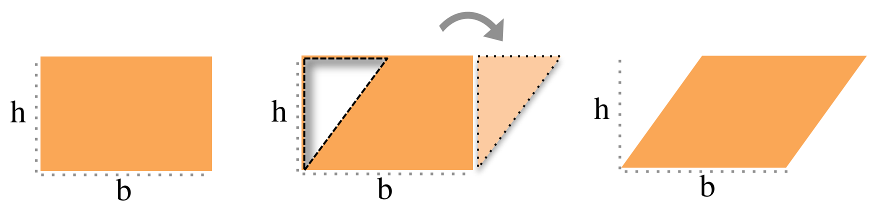
Area Measure of a Rectangle

The area measure of a rectangle is length × width, or base, b, × height, h. This follows from defining area as the translation (sweep) of one length through another.



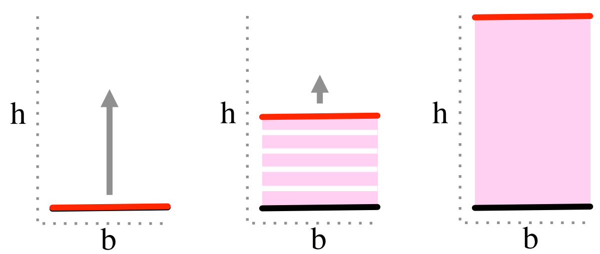
Area Measure of a Parallelogram

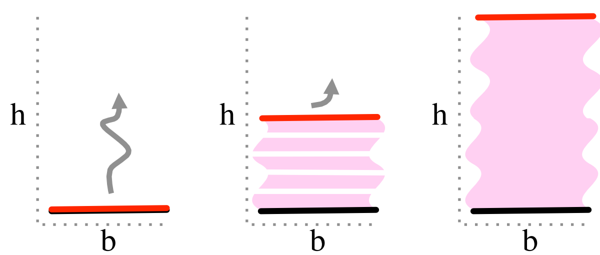
The area of a parallelogram is the same as that of a rectangle with the same height and base. A = h × b. The figure below suggests why this must be so.

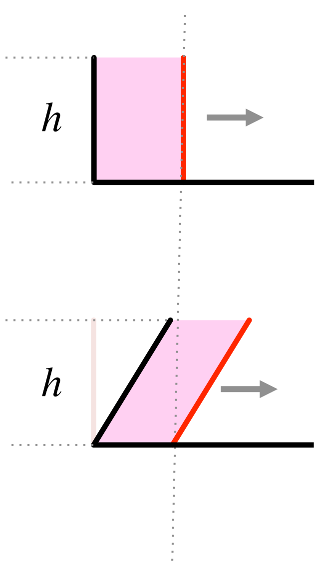


Cavalieri's principle

The area of a rectangle is the product of base × height. Sweeping the length of the base in parallel to the base through the same height produces the same area, as shown below. This is another way of justifying the equivalence in area between a rectangle and parallelogram if both have the same height.







Whole Group

The teacher presents the ceramic tile coated with shaving cream or finger paint and demonstrates the placement of two rulers that share a common zero point. S/he uses the squeegee to create a rectangle, emphasizing how to use the rulers to determine the dimensions of the rectangle. Then students are challenged to create their own rectangles, squares, parallelograms and circles.

Teacher Note: Use the terminology of sweeping and mention that by sweeping one length through another, area is created.

Partners

Students work in pairs to create figures. As students work, the teacher roves the classroom to remind students that any orientation of the squeegee is fine and to challenge students to create new figures.

Whole Group

Students walk around to see the figures created. The teacher leads a discussion to compare how students created different figures and how they decided to classify a construction as a square, rectangle, parallelogram or circle.

Partners

Students work in pairs to create the following figures, to find the area measure of each, and use spaghetti to show the corresponding units of measure:

(1) 1 *in.* × 1 *in.*
(2) 4 *in.* × 8 *in.* rectangle
(3) 1 squeegee × 8 *in.* rectangle
(4) 1 squeegee × 1 squeegee square
(5) (length of squeegee in inches) × (length of squeegee in inches)
(6) 3 *in.* × 3 *in.* square

Whole Group

Students compare solutions for each problem. An Elmo or other means is used to show the resulting units of measure. Square inch and squeegee-inch are highlighted.

Teacher Note. Many students initially believe that the units of area measure must be squares and must be of the same unit of measure. The squeegee-inch unit is rectangular and is composed as the product of the units. Contrasting cases of the same square measured in squeegees and in inches further clarifies that the same figure can have different measures of area, depending upon the nature of the unit. Note too that the inverse relation between the size of the unit and the resulting area measure.

Partners

Students work in pairs to construct the following figures, to find the area measure of each, and use spaghetti to show the units of area measure.

(7) $3\ in. \times 8\frac{1}{2}\ in.$ rectangle

(8) $3\frac{1}{2}\ in. \times 7\ in.$ rectangle

(9) $1\ squeegee \times 4\frac{1}{2}\ in.$ rectangle

(10) $4\frac{1}{2}\ in. \times 4\frac{1}{2}\ in.$ square

(11) Is there an expression for the area of a rectangle involving the lengths of its sides that will always work?

Whole Group

Students compare solutions for each problem. An Elmo or other means is used to show the resulting units of measure. Fractions of square inch and squeegee-inch are highlighted. The formula length × width for a rectangle is justified by demonstration of sweeping and reference back to (1) – (10).

Instruction

Pair Work

Compare the area of these two figures. The first is made by sweeping 4 *in.* through 6 *in.* at an angle of 90 degrees. The second is made by sweeping 4 *in.* through 6 *in.* at an angle that is less than 90 degrees.

Whole Group

Students compare solutions and rationale.

Teacher note. At an opportune time, have students cut-out the rectangle and parallelogram in the student worksheet. Then lead a discussion of how the parallelogram can be dissected and transformed into a rectangle that is congruent with the rectangle in the worksheet.

Whole Group

The teacher uses the Elmo or other means to demonstrate Cavalieri's principle.

Pair Work

Directions to students: Position the squeegee at the base of the tile so that there is room for it to move to the right and left. Move the squeegee so that it stays parallel to the base of the tile, through a height of 5 inches. Compare this area with that of a rectangle with length of one squeegee and a height of 5 inches.

Whole Group

Selected pairs of students explain why they think the area is the same or different.

Teacher note. Many students will spontaneously explain that there is compensation or conservation of area when the squeegee is moved. For example, when it is translated to the right, area is generated in that direction, but not in the other direction. So the total area is not changing.

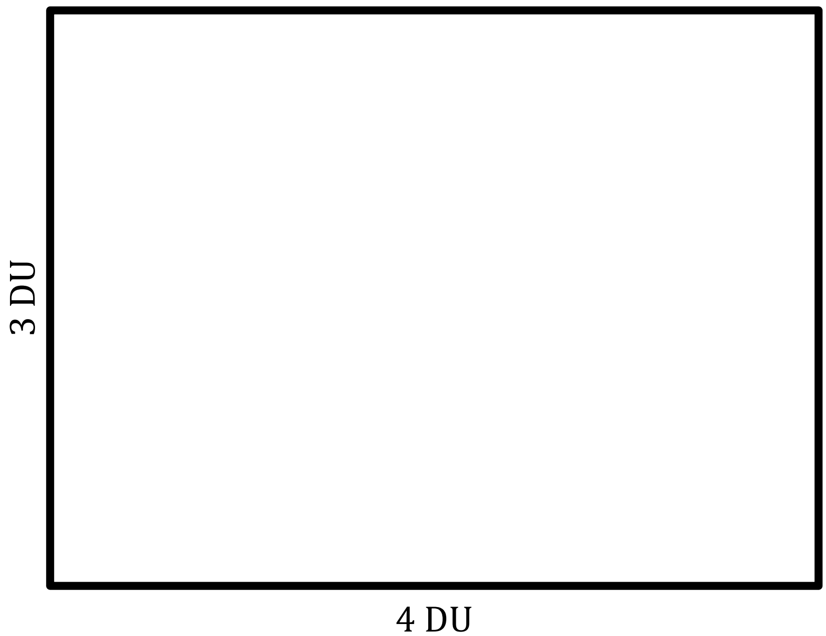
Formative Assessment

Administer the formative assessment and select contrasting student responses to create further opportunities for learning about area measure, especially rectangular area as a product of lengths, structuring rectangles and squares to reveal units of area measure, accounting for partial units of area measure, and justifying why the area of a rectangle and a parallelogram with the same base and height must be the same.

Formative Assessment — Area Unit 3

Name:_____________________________________

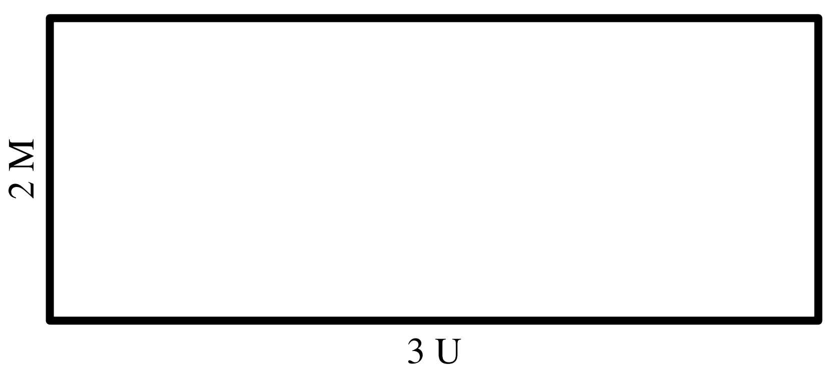
1. What is the area of this rectangle? Show the units by drawing them.



2. What is the area of a $2\frac{1}{2}\,in. \times 4\,in.$ rectangle? What is its perimeter? Show how you found out.

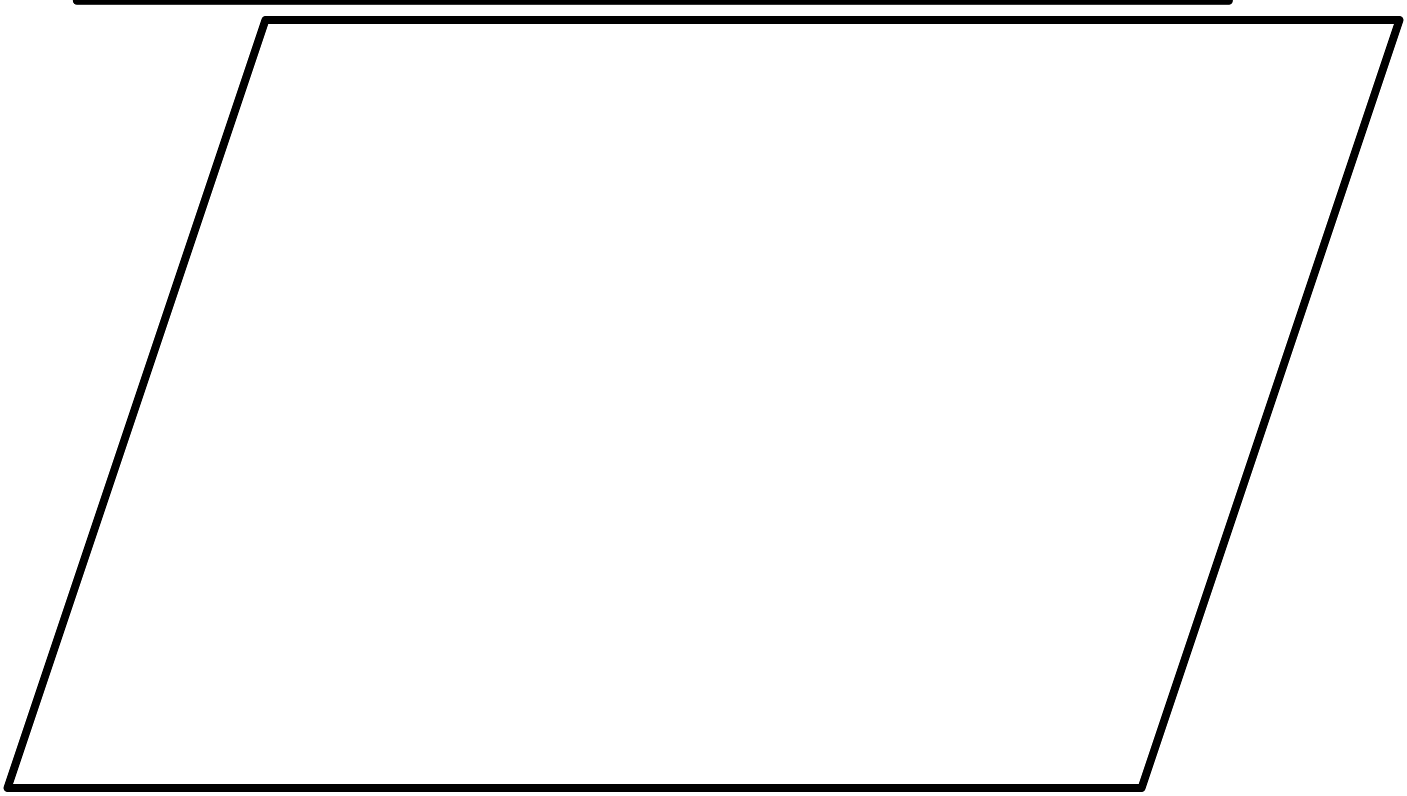
3. Using your ruler, draw $\frac{1}{4}\,in.^2$

4. The lengths of the sides of this rectangle are measured in units of *M* and *U*, as shown below. What is the area of the rectangle? Show the units of area. What is the perimeter?



Worksheet Area Unit 3

Name:____



Formative Assessment Record Area Unit 3

Indicate the levels of mastery demonstrated by circling those for which there is clear evidence:

Item	Level Circle highest level of performance	Description	Notes
Item 1 Finding the area of a $3\ in. \times 4\ in.$ rectangle and showing $12\ in.^2$	**ToMAA 4A** Given an area, partition into arrays of units by coordinating linear measurements of the shape.	3-splits one side, 4-splits the other side, coordinates splits to show 12 DU^2	
	ToAM3B Find and compare areas by counting identical units used to tile.	Cannot coordinate lengths to generate square units but generates some other unit that is used consistently to cover.	
	NL	Cannot partition region systematically.	
Item 2 Finding an area of $2\frac{1}{2} in. \times 4\ in.$ rectangle	**ToAM3F** Partition to find and compare areas using half-units and other two-splits.	Area as $10\ in.^2$ and perimeter as $13\ in.$	
	Other Describe		
Item 3 Draw $\frac{1}{4} sq. in.$	**ToAM 3F** Partition to find and compare areas using half-units and other two-splits.	Draws a unit $\frac{1}{2} in. \times \frac{1}{2} in.$ or 1 in. $\times \frac{1}{4}$ in.	
	ToAM3D Recognize/construct suitable units.	Draws a unit $\frac{1}{4} in. \times \frac{1}{4} in.$	
	Other Describe		
Item 4 $2M \times 3U$	**ToAM4E** Find and compare areas with dimensions given in unlike units (e.g., length in cm, width in inches).	Draws 6 rectangular MU units. Notes that the perimeter is 4M + 6U	
	ToAM3D Recognize/construct suitable units.	Attempts to use a unit of area other than an MU	
	Other Describe		

Comparing Zoo Enclosures

UNIT

4

Mathematical Concepts

- Area is additive, which means that the area of a figure can be determined by dissecting it into pieces, determining the area of each piece, and then finding the sum of the areas of the pieces.
- The area of a rectangle of height, h unit, and length, l unit, is $h \times l$ unit^2. But, $h \times l$ unit^2 is equal to 1 unit × hl unit, so that an $h \times l$ rectangle always has the same area as a $1 \times hl$ rectangle.

Unit Overview

Given unit dimensions, students find and rank order the areas of 6 figures that are either rectangles or are composed of rectangles. For selected figures, students coordinate units of length measure (inches) to show the square units of area measure. And, for each of these figures, students find the perimeter. Then students use dissection with n units of area measure to demonstrate that the area of any figure is equivalent to the area of a 1 unit × n unit rectangle.

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Read

- **Unit 4**
  Start by reading the unit to learn the content and become familiar with the activities.

Gather

- For each pair of students, worksheets with figures labeled A—F.
- Rulers with inch markings.
- Student math journals

Academic Vocabulary

- Area
- Length
- Width
- Rectangle
- Perimeter

Unit
Greater than
Less than
Equal to
Partition

Additive Property of Area Measure

Area measure is additive. This means that the measure of the area of a whole figure is equivalent to the sum of the measures of the area of its pieces. It follows that if figure F is wholly contained in figure G, then Area F < Area G.

Area is a Product

The measure of the area of a rectangular figure is a product of the measures of the lengths of its sides. It follows that a line segment has zero area ($0 \times L = 0$). Sweeping a length L along a length M results in an area with measure $n\ (L \times M)$. For a rectangle of dimension a U, b U, sweeping aU along b U results in ab U^2, which is equal to ab U × 1 U. So, any a U × b U rectangle's area is equivalent to the area of the corresponding 1 U × ab U rectangle.

Conservation of Area

Area is preserved by isometries (e.g., translations, turns, reflections)
The area of a figure does not change if it is moved by an isometry. Two congruent figures have equal area.

Dissection

Two figures are equivalent by dissection if one can be cut up into a finite number of pieces and the pieces rearranged to form the other, as demonstrated in Unit 1. If the measure of the area of a rectangle is n square units, this area measure is equivalent to the area of a 1 unit × n unit rectangle.

Perimeter

The linear distance all the way around the sides of a figure is its perimeter. This distance can be thought of as a path around a figure. Perimeter is measured with units of length, in contrast to area's measure with 2-dimensional units.

Whole Group

Individual students each receive diagrams of six figures.

Tell students: These are plans of zoo enclosures. Because different animals have different requirements for space, the number of square feet in each enclosure must be determined. To determine the amount of fencing for each enclosure, the perimeter must also be determined. In these diagrams, one inch represents 1 foot. Let's look at A. Use your ruler and find its length and width (5 *in*. × 8 *in*.). Find the area and the perimeter of each. For A, B, and F, draw the units of area measure.

Individual

Students use the diagrams to find the area and perimeter of each enclosure.

Teacher note. As students solve the problems, rove and get a sense of different solution strategies.

Whole Group

Starting with Figures A and F, compare and contrast solution strategies. Then ask students to compare the strategies used in Figures A and F to the strategies they used to solve Figure D. By nesting F within A, it is clear that the area of F < area A. Be sure that students can coordinate units of length measure to represent the units of area measure, so that the use of the formula $l \times w = A$ is understood.

Then move to Figures B, C, and E, and again compare strategies with an eye toward determining how length measures can be used to find the areas of pieces of each figure.

Pairs

For Figure A, what rectangle with a length of 1 inch will have the same area? How could you show that your choice is right?

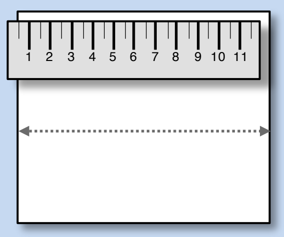
Whole Group

Demonstrate the dissection of Figure A into 40 square units that can be re-arranged to create a 1 unit × 40 unit rectangle.

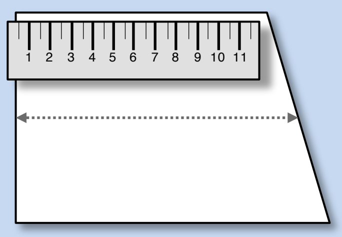
STUDENTS' WAYS OF THINKING

- Students might suggest that after measuring the length of a side of a rectangle, they also need to measure the length across the center of the figure. Typically, this means that they are not relating the figure to properties of a rectangle. This thinking presents an opportunity to help students learn to interpret diagrams. To promote this understanding, the teacher may decide to present a counter case to illustrate the relationship between the length of opposite sides and the resulting structure of the shape, such as that depicted below.

  Students may suggest measuring across the middle of the figure, which, by definition of a rectangle, would also measure 12 inches.

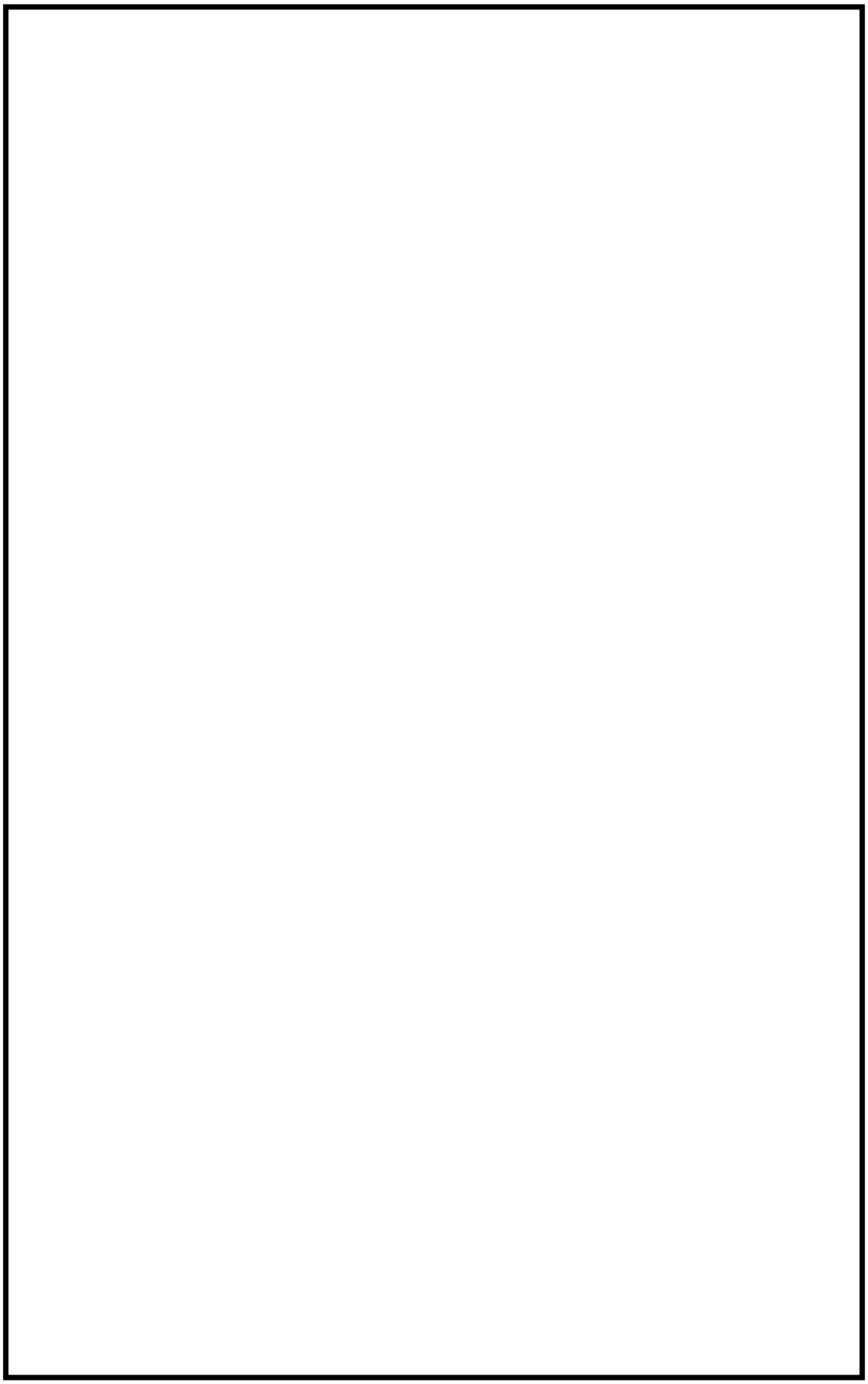


  A counter example shows that if the distance across the middle of the shape did not equal the measure of the long side, then the opposite side of the shape would not be the same length as the original side measured and the resulting shape would also be different—a suggestion made by a second-grade student.

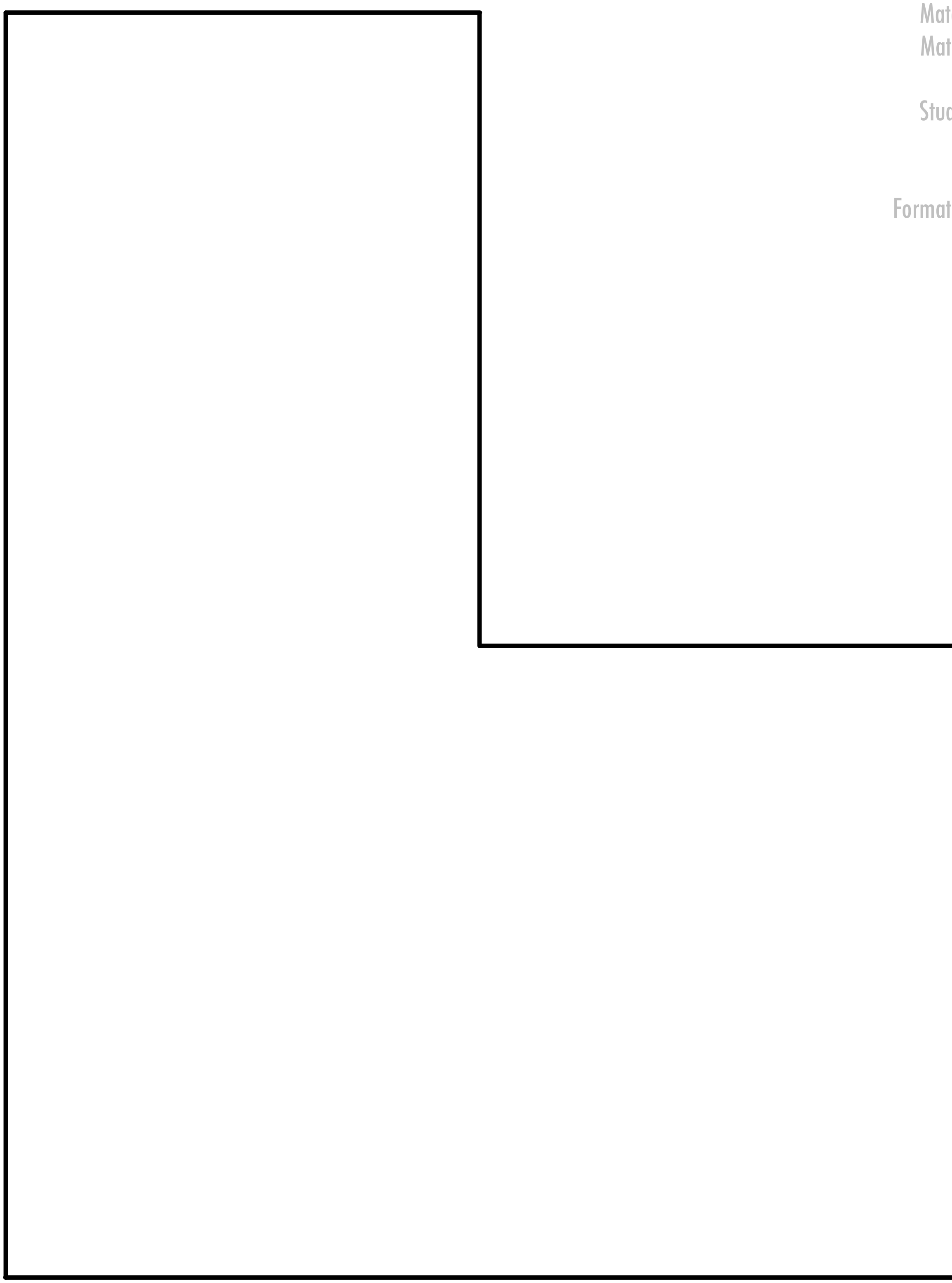


Blackline Masters for Figures A-F

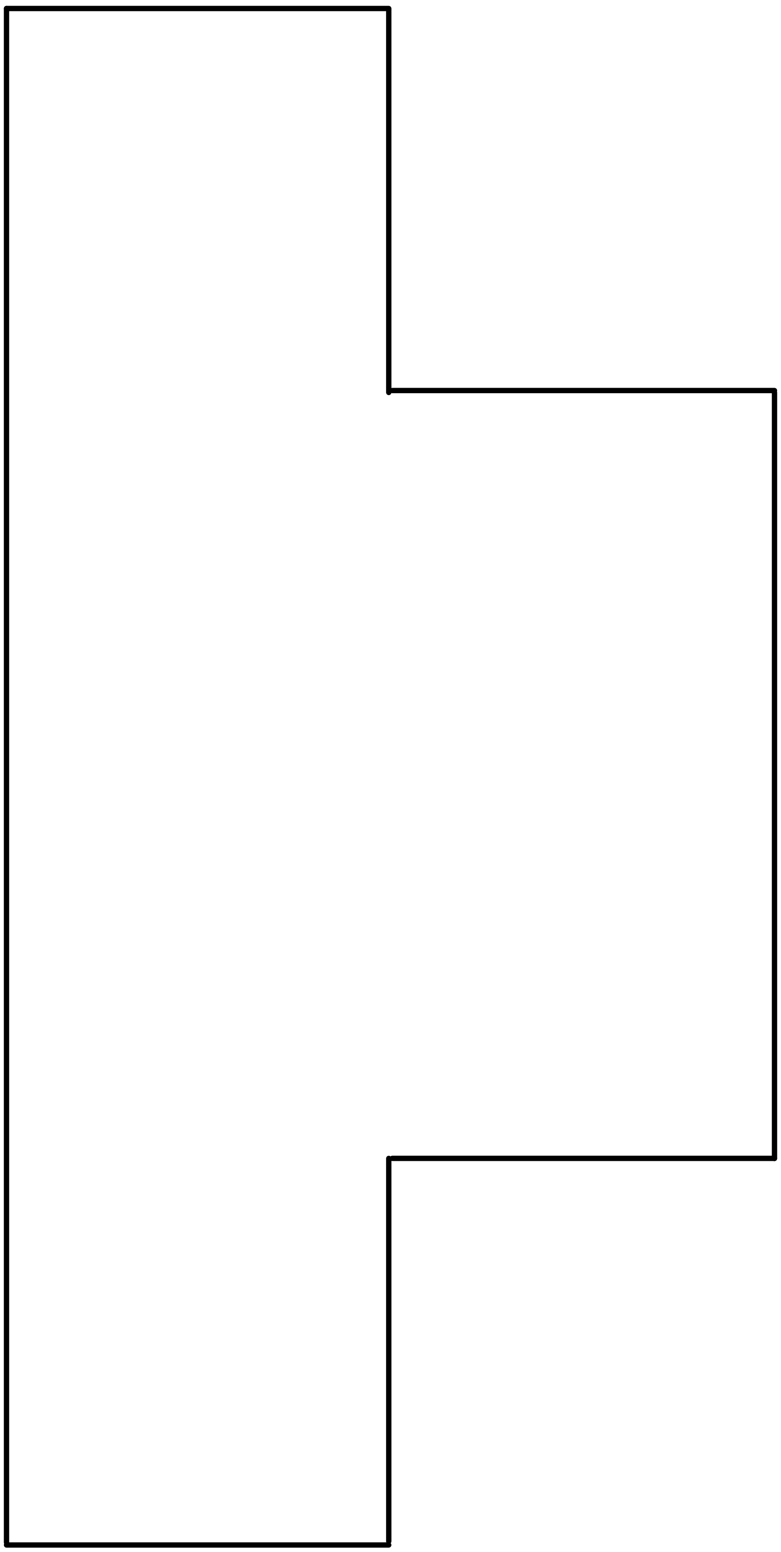
A (5 *in*. × 8 *in*.)



B $(\mathbf{3}\ \boldsymbol{in.} \times \mathbf{8}\ \boldsymbol{in.} + \mathbf{3}\ \boldsymbol{in.} \times \mathbf{4}\ \boldsymbol{in.})$



C $(2\ in. \times 8\ in. + 2\ in. \times 4\ in.)$



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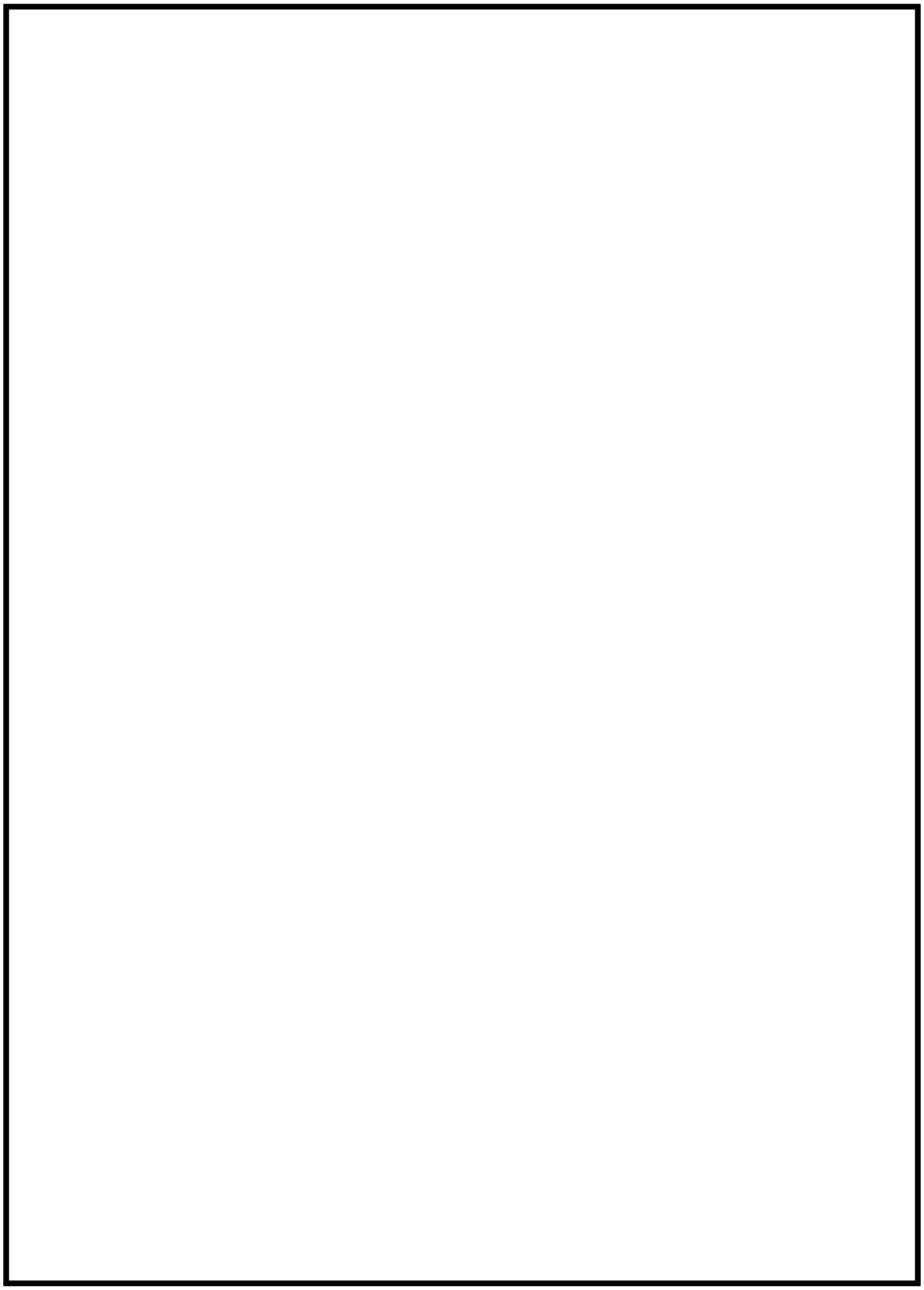
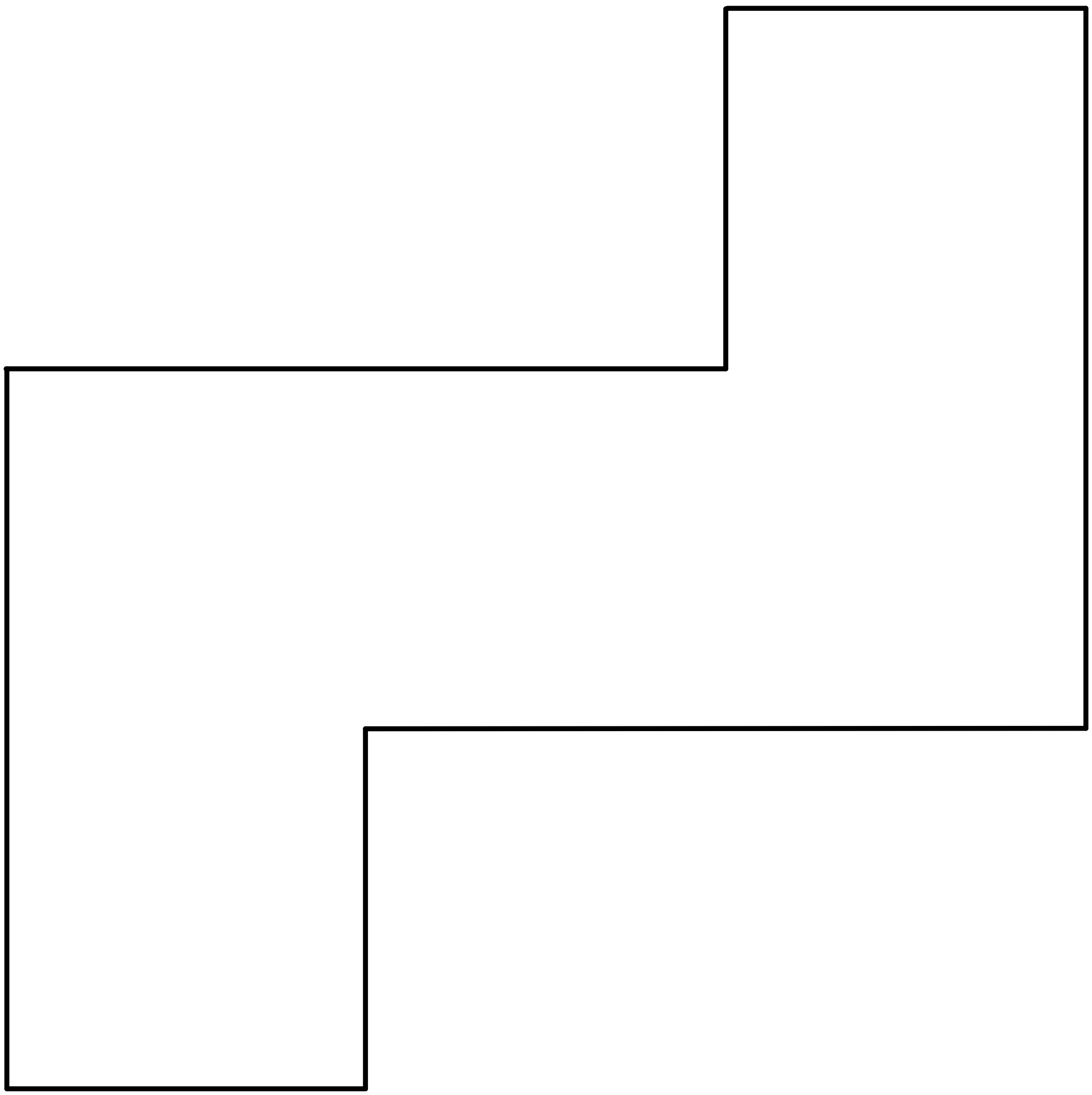
D (7 *in*. × 5 *in*.)

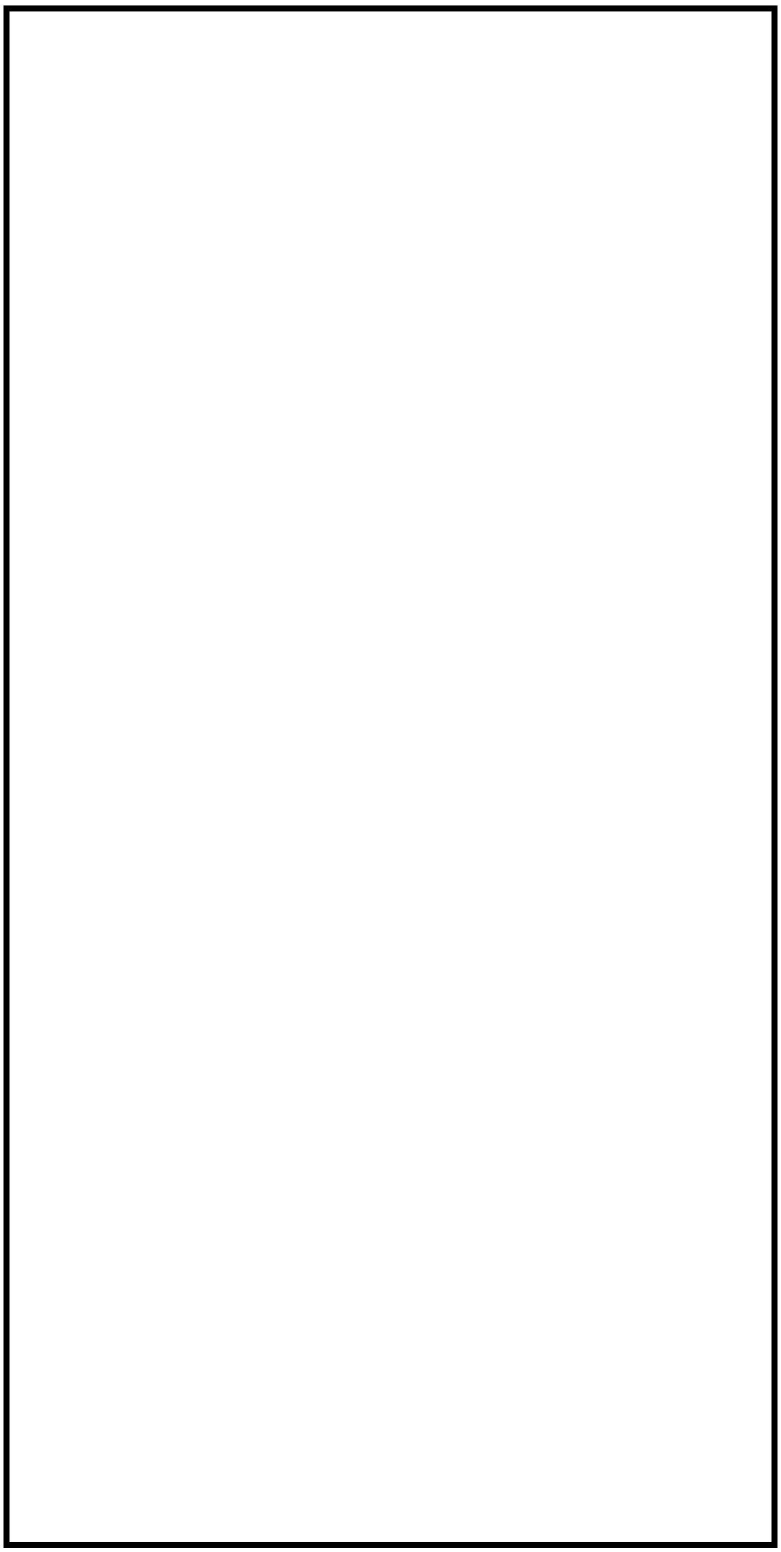


E $(\mathbf{6}\ \boldsymbol{in}. \times \mathbf{2}\ \boldsymbol{in}.) + \mathbf{2}(\mathbf{2}\ \boldsymbol{in}. \times \mathbf{2}\ \boldsymbol{in}.)$



Blackline Masters for Figures A-F

F (4 *in.* × 8 *in.*)

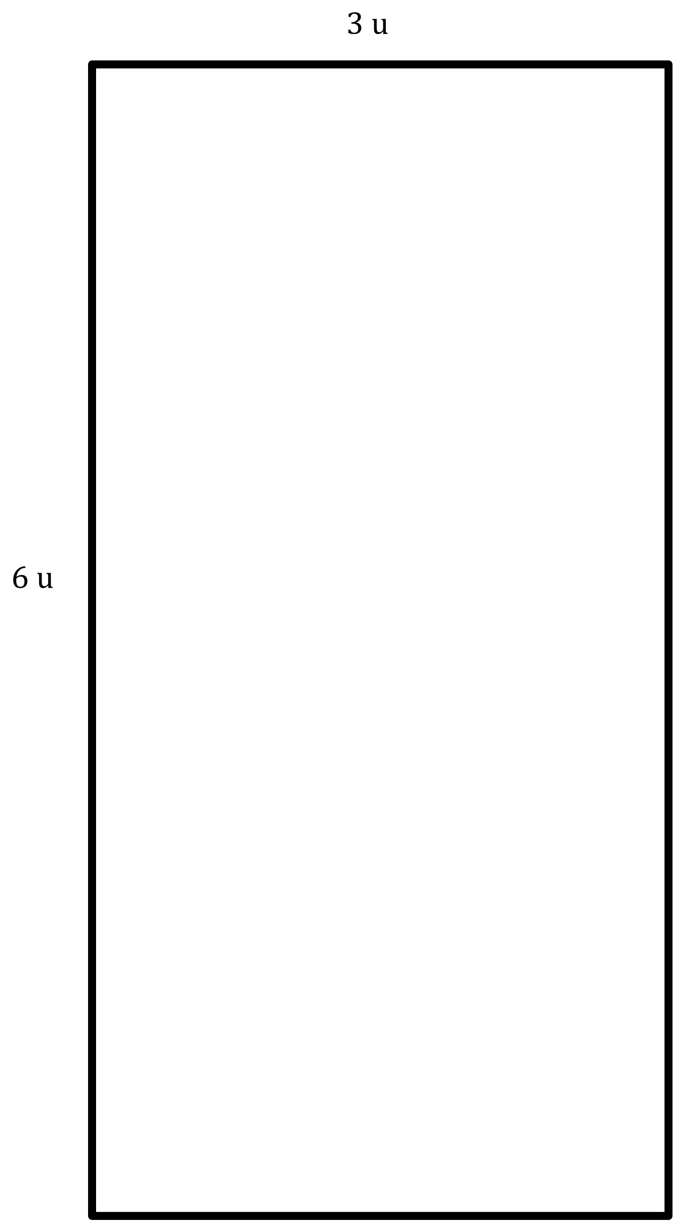


Formative Assessment

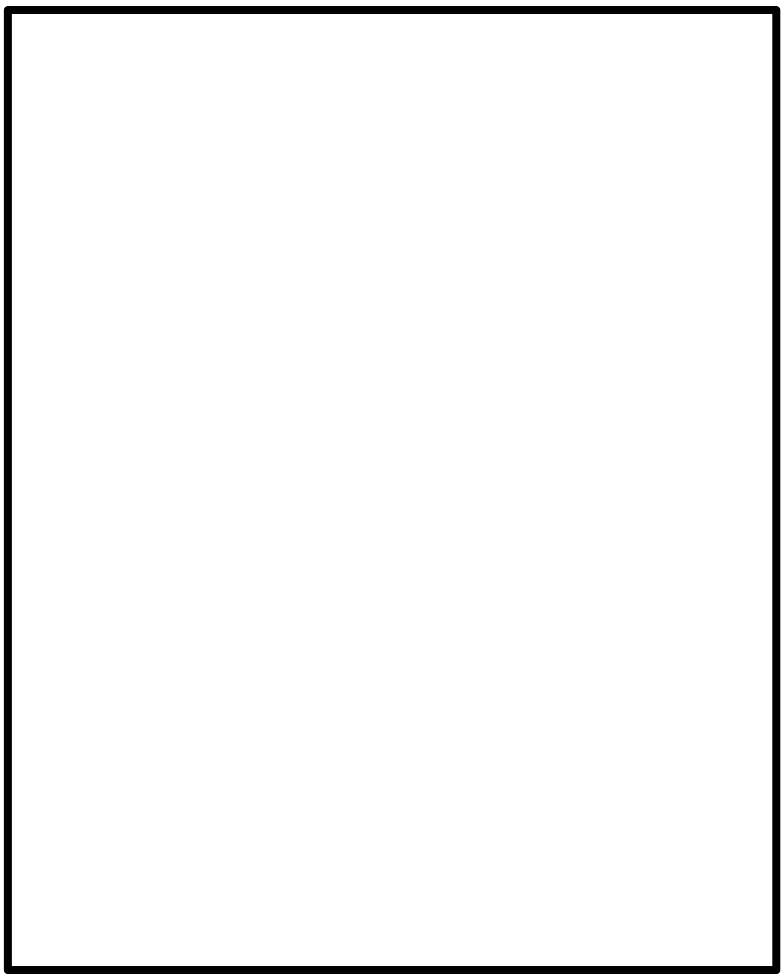
Administer the formative assessment and select contrasting student responses to create further opportunities for learning about area measure, especially the difference between units of length measure (perimeter) and units of area measure.

NAME:______________________________

1. What is the area of this rectangle? What is its perimeter? Show the units of area measure. Try this one without using your ruler.



2. What is the area of this rectangle? What is its perimeter? Show the units of area measure. You can use your ruler.



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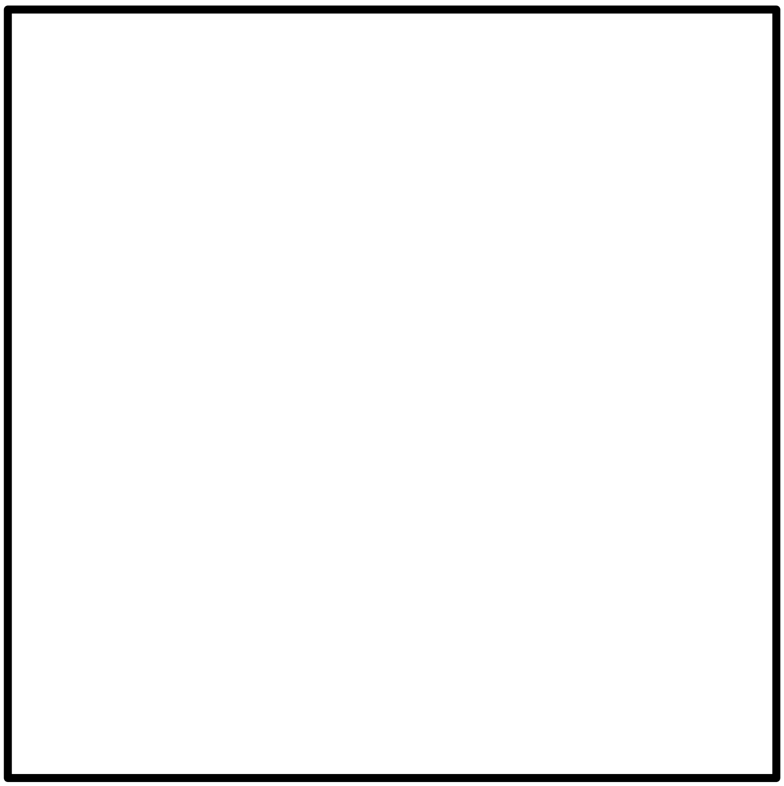
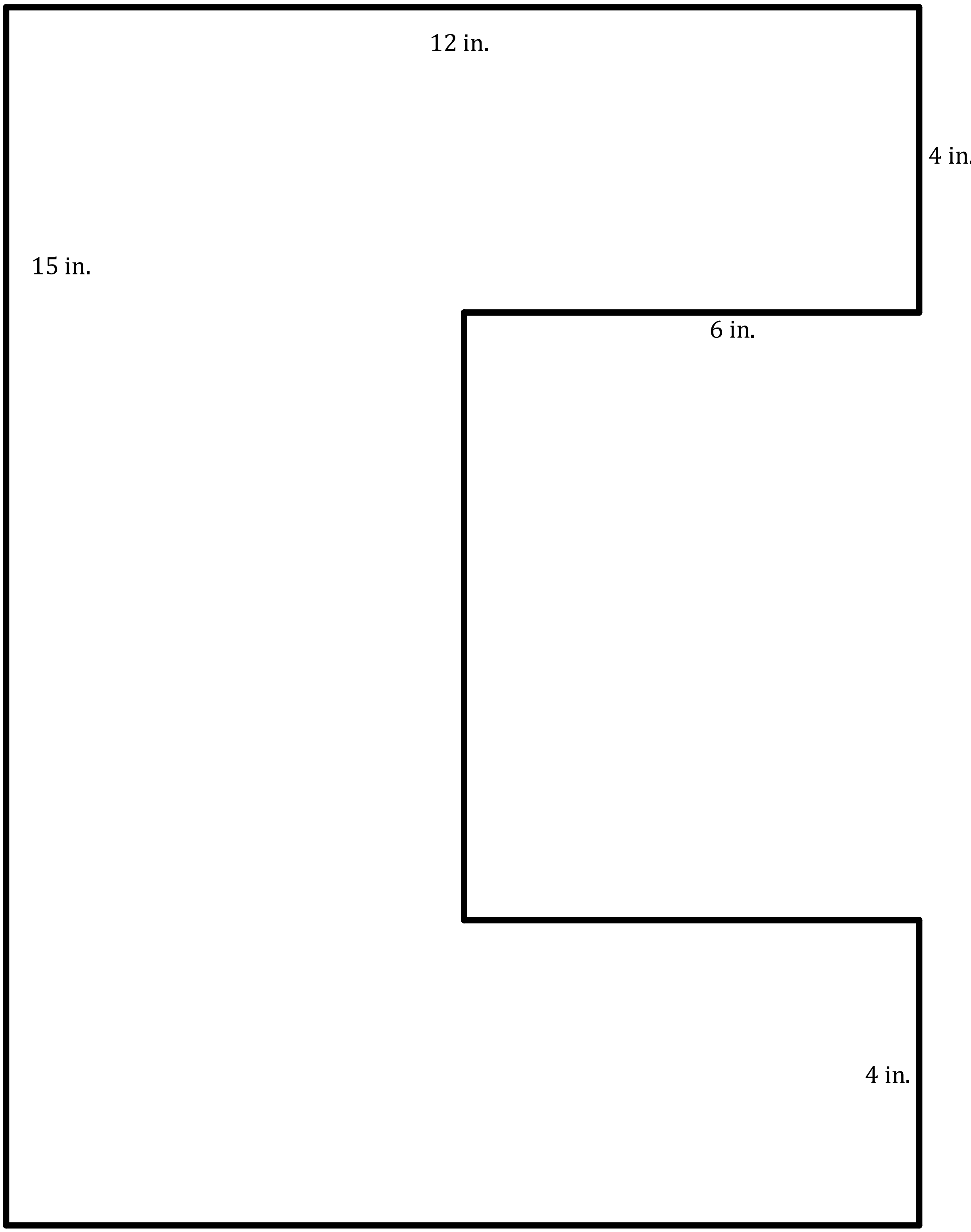
3. Here is a square with a side length of 4 inches. What is the length of a rectangle with the same area if one side of the rectangle is 1 inch? How could you show someone that your choice of the side of the rectangle is correct?



4. What is the area of this figure? What is its perimeter? (Notice that the diagram represents inches and is not really 12 inches or 15 inches long.)



Formative Assessment — Area Unit 4

Indicate the levels of mastery demonstrated by circling those for which there is clear evidence:

Item	**Level** Circle highest level of performance	**Description**	**Notes**
Item 1 Finding the area of a $3\ u \times 6\ u$ rectangle and showing $18\ u.^2$	**ToMAA 4A** Given an area, partition into arrays of units by coordinating linear measurements of the shape.	3-splits one side, 3-splits the other side, followed by a 2-split or 2-splits, followed by a 3 split, coordinates splits to show 18 in.^{2}	
	ToAM3B Find and compare areas by counting identical units used to tile.	Cannot coordinate lengths to generate square units but generates some other unit that is used consistently to cover.	
	NL	Cannot partition region systematically.	
Item 2 Finding an area of $5\ in. \times 4\frac{1}{2}\ in.$ rectangle	**ToAM3F** Partition to find and compare areas using half-units and other two-splits.	Area as 22 1/2 $in.^2$ and perimeter as 19 *in*.	
	Other Describe		
Item 3 Establish equivalence of area of figure to $1 \times n$ rectangle	**ToMAA 4A** Given an area, partition into arrays of units by coordinating linear measurements of the shape.	Shows that 1 in. x 16 in. rectangle has same area as 4 in. x 4 in. square.	
Item 4 Find the area of a figure by finding the sum of the areas of its parts. Differentiate perimeter from area.	**ToMAA 4A** Given an area, partition into arrays of units by coordinating linear measurements of the shape.	Uses length measures on figure and properties of rectangles to find area, either by subtraction (area of 15 x 12 rectangle- 7 x 6 rectangle) or by partitioning and finding areas of smaller rectangles, then summing. Note strategy.	

Justifying and Using Formulas to Find Area Measure of Triangle, Regular Polygon, and Circle

UNIT

5

Mathematical Concepts

- The measure of the area of a triangle, $\frac{1}{2}b \times h$, is $\frac{1}{2}$ the product of the length of a side and its corresponding height. The height is the shortest distance between the opposite vertex and the side chosen as the base, or the line containing the side chosen as the base if the opposite vertex is not over the base.
- The measure of the area of a regular polygon, $\frac{1}{2}\ P\times\ a$, is $\frac{1}{2}$ the product of its perimeter, P, and its apothem, a. The apothem is the shortest distance from the center of a regular polygon to a side.
- The measure of the area of a circle, πr^2, is the product of π and the square of the circle's radius.

Unit Overview

In light of previous consideration of the area of a parallelogram as $b \times h$, the area of a triangle partitioning a parallelogram into 2 congruent triangles is established as $\frac{1}{2}$ the area of the parallelogram, or $\frac{1}{2}b \times h$. Since any triangle can be $\frac{1}{2}$ turn-rotated about one of its sides and composed to form a parallelogram, the formula is general. Working from this foundation, the area of a regular hexagon is considered by partitioning it into six congruent triangles. By summing the area of each of these six congruent triangles, the area of the hexagon is established as $\frac{1}{2}P \times a$, where P represents the perimeter of the hexagon and a the height of any of the six congruent triangles. As the number of sides of a regular polygon increases without limit, the polygon increasingly resembles a circle. Considering a circle as a regular n-gon with an infinite number of sides, the perimeter (the circumference of the circle) is π x d and its apothem is the radius of the circle. This results in the familiar formula for the area of a circle, π x r^2, because by analogy to the formula for the area of a regular polygon, the area of the circle is $\frac{1}{2}$x π x $(2r)\times r$.

Contents

Materials and Preparation

Read

- **Unit 5**
  Start by reading the unit to learn the content and become familiar with the activities.

Gather

- Rulers with inch markings or with centimeter markings
- Patty paper

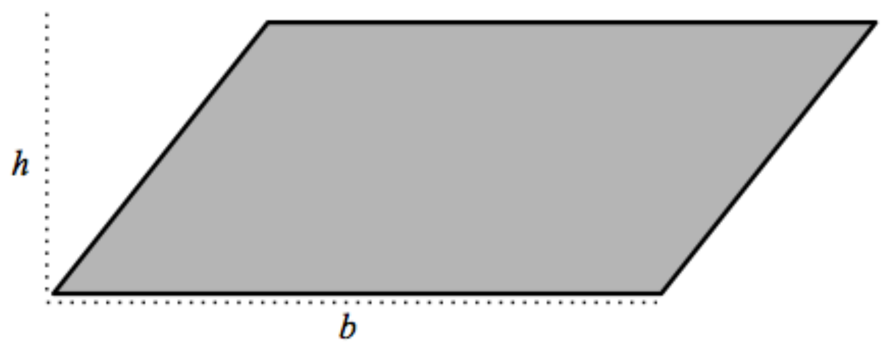
Academic Vocabulary

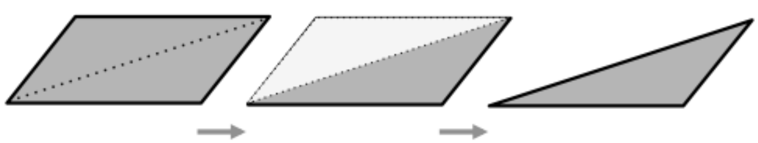
- Area
- Length
- Width
- Rectangle
- Perimeter
- Regular Polygon
- Pi
- Unit
- Greater than
- Less than
- Equal to
- Partition
- Apothem
- Radius

Mathematical Background

Area of a Triangle

Recall that in Unit 4, we established the measure of the area of a parallelogram as the product, *base* × *height*, which was justified by considering the area of a rectangle as $b \times h$, and by Cavalieri's Principle, the area of a parallelogram must also be $b \times h$. Because a parallelogram can be formed by composing a triangle and its $\frac{1}{2}$-turn rotated image, the area of a triangle must be $\frac{1}{2}b \times h$, as shown below. Recall h is the distance perpendicular to the selected base from the opposite vertex.



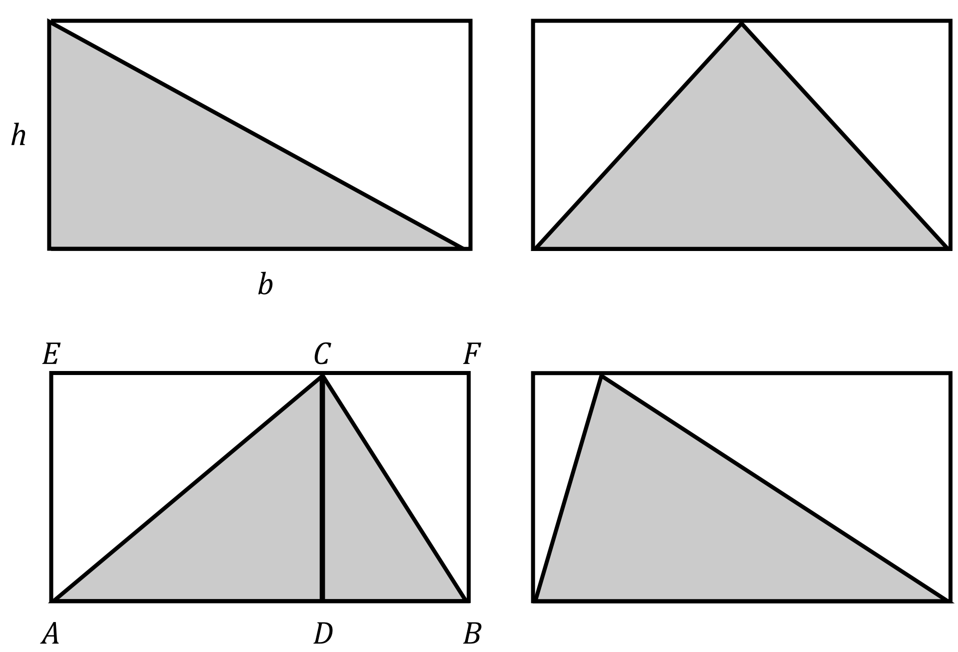


Another justification for the measure of the area of a triangle can be generated by considering the area measure of a rectangle, $b \times h$. Consider a rectangle as composed of two right triangles, as depicted by the upper left panel in the figure below. Selecting the midpoint of the hypotenuse of the shaded triangle and rotating about that point $\frac{1}{2}$ turn demonstrates that the triangles are congruent. This suggests that the area of a triangle is $\frac{1}{2}\ base \times height$. Consider other potential triangles, such as those displayed in the remaining panels of the figure.

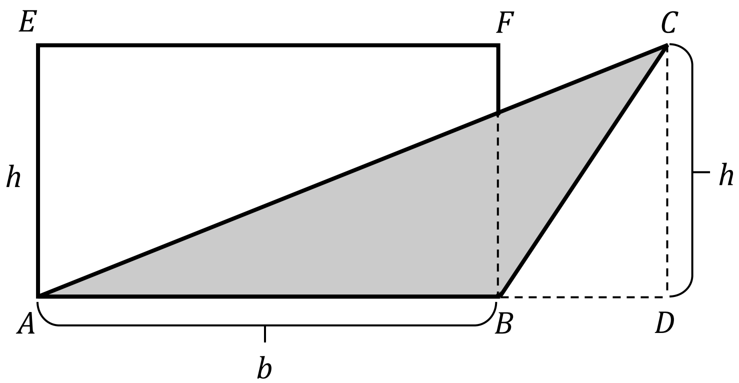
For example, in the lower left panel:

- the area of Δ ABC $=$ area Δ ADC $+$ area Δ CDB;
- the area of Δ ADC $= \frac{1}{2}$ area AECD;
- and the area of Δ CDB $= \frac{1}{2}$ area DCFB;

hence, the area of Δ ABC $= \frac{1}{2}$area AEFB $= \frac{1}{2} b \times h$.



For further conviction, consider cases of triangles not enclosed in the rectangle.

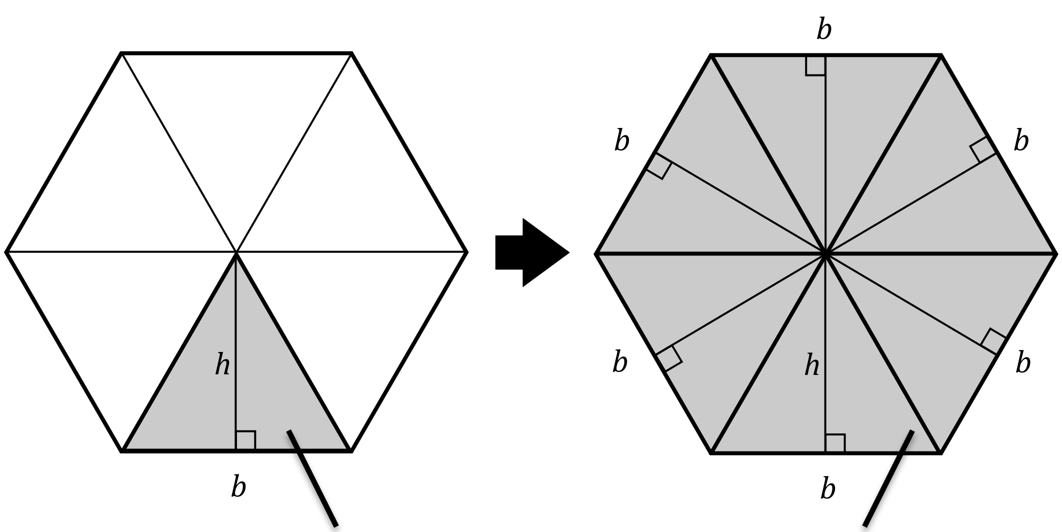


Area of Δ ABC $=$ Area (Δ ADC) $-$ Area (Δ BCD)

This difference is $\frac{1}{2}$ area (AECD) – $\frac{1}{2}$ area (DCFB), which is the same as $\frac{1}{2}$ area (AEFB), and therefore Δ ABC $= \frac{1}{2}\ b \times h$.

Area of a Regular Hexagon

Given the measure of the area of a triangle, a strategy for finding the area of any polygon is to dissect it into triangles. The area measure of the polygon is then the sum of the measures of the areas of its constituent triangles. Because the triangles for regular polygons are congruent, the area of a regular polygon can be found as illustrated below for a regular hexagon. The altitude or height of the equilateral triangle is also called its apothem.



Area of Triangle $= \frac{1}{2} \times b \times h$

Area of Hexagon $= 6 \times$ *Area of Triangle*
$= 6 \times (\frac{1}{2} \times b \times h)$
$= (6 \times b) \times \frac{1}{2} \times h$
$= P \times \frac{1}{2} \times h$
$= \frac{1}{2}\ P \times h$

Area of a Circle

Consider that as the number of sides of a regular polygon grows, the polygon approximates a circle more and more closely. As the number of sides grows without limit, the perimeter of the polygon and the circumference of the circle become indistinguishable. This means that we can use our knowledge of the relation between the circumference and the diameter of a circle to obtain a measure of the circumference. The measure of the length of the circumference, *C*, of a circle (its perimeter) is πd, where π is about 3.14 and *d* is the diameter of the circle. The apothem of the *n*-gon and radius, *r*, of the circle are now also indistinguishable, so the area of the circle is $\frac{1}{2}\ C \times r$. But $C = \ \pi$ x $(2r)$, leading to $\frac{1}{2}$ x $\pi\ x\ (2r) \times r$, so the area of the circle is π x r^2, often abbreviated as πr^2.

Individual

Each student receives a copy of the parallelogram worksheet, a piece of patty paper and a ruler.

Find the area of the parallelogram.

Whole Group

Share and compare solution strategies.

Teachers note: Be sure to elicit justifications for strategies used, especially area measure as $h \times b$, where h is height and b is the length of the base.

Individual

Partition the parallelogram along one of its diagonals into two congruent triangles. Use patty paper to trace the outline of one triangle. Then use the midpoint of the diagonal to $\frac{1}{2}$ turn the triangle. What do you notice? If the area of the parallelogram is $b \times h,$ what must the area of the triangle be?

Draw a triangle, however you like. Trace it with the patty paper. Then $\frac{1}{2}$ turn it about one of its sides to construct a parallelogram. Make a conclusion about the area of a triangle and its measure.

Whole Group

Share and compare the results of the investigation of triangles and parallelograms.

Teacher note. Help students understand that if we accept the measure of area of a parallelogram, then by using a transformation ($\frac{1}{2}$ turn), for any triangle we can establish its area measure. Recall that all parallelograms have $\frac{1}{2}$ turn symmetry, and that all parallelograms can be dissected into 2 triangles. An alternative pathway for arriving at the same conclusion can be established by starting with a rectangle, as shown in Mathematical Background.

Individual

Find the areas of the triangles depicted in the Worksheet, Area of a Triangle.

Teacher note. You could assign each student a different side of each triangle and then rely on the Whole Group discussion to highlight how to find the height of each corresponding side.

Whole Group

Share and compare solutions to the problem of identifying the height associated with each side.

Teacher note. Emphasize how to find the height of a triangle from any side of a triangle. Challenge students to come up with a method for finding the height (by extending the side) when the opposite angle is not over the side. Within measurement error, the area measures should be identical for each triangle, no matter which side is selected as its base.

Partner

Construct a regular hexagon using a protractor and a ruler, or construct a triangle (what kinds of triangles work?) and use one or more transformations of the triangle to construct the hexagon.

Teacher note. You might split the class so that some portion attempts to first construct a triangle and then use patty paper to transform it to form a regular hexagon, while the remainder of the class constructs the hexagon as a path. If the starting point is a triangle, the sum of the interior vertex angles at the center of the hexagon composed of 6 congruent triangles must be 360 degrees. (See the geometry lessons on tiling). Only equilateral triangles will meet this criterion, although this will likely be more visible if other types of triangles are attempted. A path perspective on the construction of a regular hexagon consists of 6 moves of the same length and 6 turns, each of 60 degrees, because the total turn is 360 degrees, and hence $360 \div 6 = 60$. Similarly, an equilateral triangle is constructed as a path consisting of 3 congruent lengths and turn angles of 120 degrees.

Whole Group

Compare constructions with an eye toward establishing how the method results in a regular hexagon.

Partner

Invent a strategy for finding the area measure of the regular hexagon.

Whole Group

The teacher selects student strategies that use some form of dissection, such as partitioning into 2 trapezoids or 6 triangles. Other strategies that may not work but which will lead to productive discussions are also highlighted.

Partner

Partners use a dissection to find the area measure of the hexagon.

Whole Group

Share and compare solution strategies.

Teacher note: Be sure to include strategies that make use of congruent triangles. When students indicate that they understand this approach, have them try it out on another hexagon. This is a great opportunity to revisit the path perspective of shape by having students use a ruler and a protractor to construct a regular hexagon. (Recall that a regular hexagon is composed of 6 congruent lengths and 6 turn angles of 60 degrees. Then try to help students develop a general formula for the area of a regular hexagon: $\frac{1}{2}\ P \times apothem$ (altitude or height of each triangle).

This relies on: $\frac{1}{2}$ x h x b_1 $+ \frac{1}{2}$ x h x b_2 $+ \frac{1}{2}$ x h x b_3 $+ \frac{1}{2}$ x h x b_4 $+$ $\frac{1}{2}$ x h x b_5 $+ \frac{1}{2}$ x h x b_6

Note that each b is multiplied by $\frac{1}{2}$ x h. By the distributive property of multiplication over addition, we can express this as:

$\frac{1}{2}$ x $h \times (b_1 + b_2 + b_3 + b_4 + b_5 + b_6)$

An equivalent expression is then $Area = \frac{1}{2}$ x $h \times P$, where the perimeter, P, is the sum of the lengths of the bases of the hexagon.

Partner

What is the relationship between the length of the circumference of a circle and its diameter?

Directions to students: **Construct three different circles** with a pencil and string on paper, or use a compass, or trace the outline of a cylindrical container for this construction. For each circle, use your ruler to find the measure of its diameter and the measure of its circumference (use rope caulk to trace the outline of the circumference and then unfold it and use your ruler to find its length, or use a long paper strip to trace the circumference). Record values of diameter and circumference for each circle.

Whole Group

Create a table of values of diameter, circumference, and how many diameter lengths are in each circumference (C/D). Find the median or mean value of C/D. It should be close to 3.14.

Teacher note: Demonstrate $C = \pi \times D$ for a prototypical circle by first cutting a length of rope caulk, or a length of string, or a paper strip, that is congruent with the diameter. Then use it to predict how long the circumference of the circle will be, iterating the rope caulk or string-length or paper strip, that many times. If students are used to implicit multiplication by juxtaposing letters, then C = πD.

Partner

Predict and test. Construct a new circle with a diameter of your choice. Using the diameter, predict the length of the circumference.

Then find the area of the circle as $\frac{1}{2}$ x $P \times a$, where $P = \pi \times 2r$ and $a = r$.

Whole Group

Students share solution strategies.

Teacher note: Help students arrive at Area = $\pi \times r \times r = \pi r^2$. Help students understand the formula as a generalization of the formula for a regular polygon. Recall that the area of a regular n-gon is $\frac{1}{2} aP$ and now a $= r$ and P = $\pi 2r$, so $\frac{1}{2}$ $r\pi$2r = $\frac{1}{2}$ $2rr\pi = \pi r^2$.

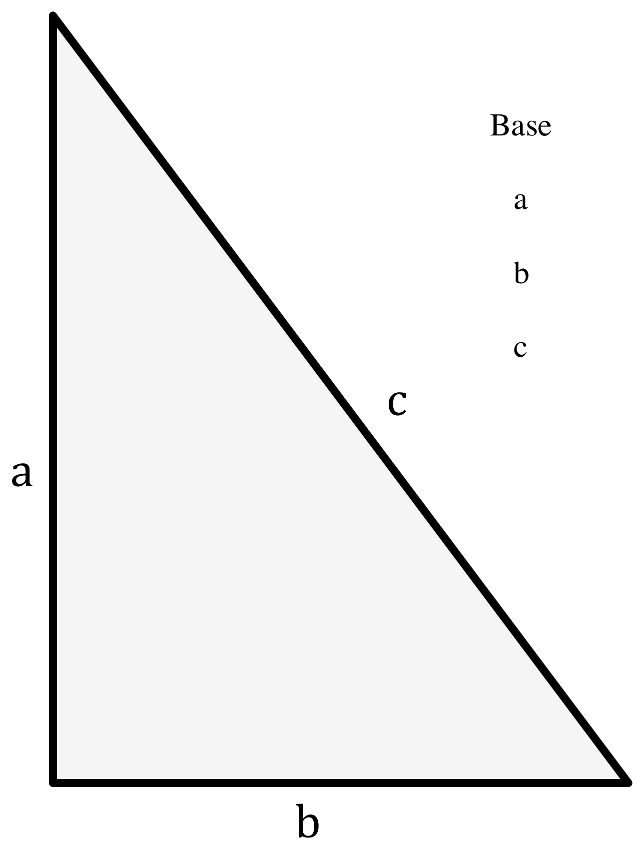
Formative Assessment

Administer the formative assessment and select contrasting student responses to create further opportunities for learning about area measure, especially the difference between units of length measure (perimeter) and units of area measure.

Name:____

1. A triangle has three sides. Explain why the formula for the area measure of a triangle is $\frac{1}{2}\ b \times h$ and not $\frac{1}{3}\ b \times h$. Your explanation can include a drawing.

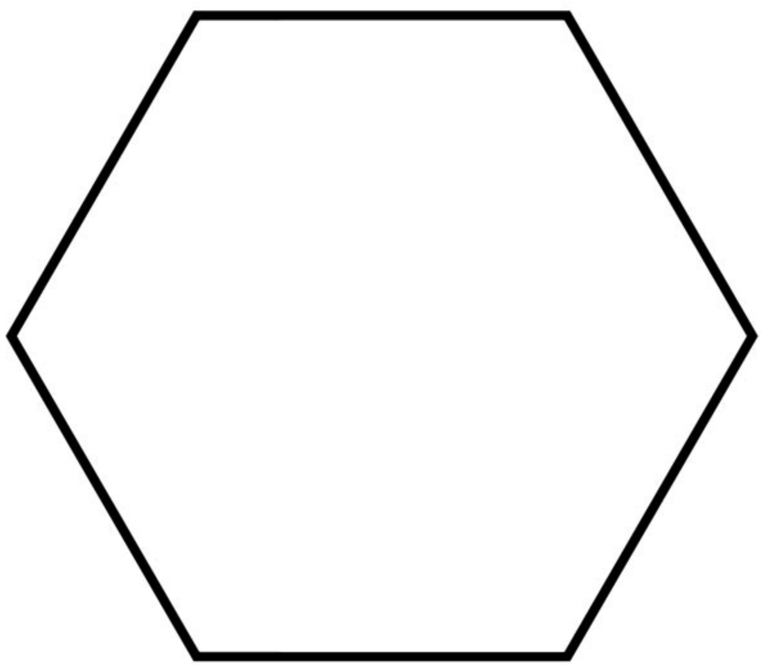
2. In the triangle below, find the heights for each side in the formula $A = \frac{1}{2}\ b \times h$. Draw the height when side c is the base.



Base	Height
a	_____
b	_____
c	_____

What is the area of this triangle?

3. The measure of the area of a regular hexagon is $\frac{1}{2} P \times a$, where P is the perimeter of the hexagon and a is its apothem (the shortest distance from the center of the hexagon to the opposite side). Find the area measure of this regular hexagon. Show your work.



4. Explain why the area measure of a regular hexagon is $\frac{1}{2} P \times a$, where P is the perimeter of the hexagon and a is its apothem.

5. The length of the circumference of a circle is ______ times as along as its diameter.

6. If the radius of a circle is 5 cm., what is its circumference? It's area?

7. Explain how the thinking about the formula for the area of a regular polygon, $A = \frac{1}{2} P \times a$, justifies the formula for the area of a circle, $A = \pi r^2$.

Formative Assessment Record — Area Unit 5

Item	Level Circle highest level of performance	Description Circle each criterion of performance met by student	Notes
Item 1 Explain/justify formula for area of a triangle.	**ToAM 5B** Generate, use, and explain area formula for a triangle.	Justifies by appeal to area of a parallelogram or rectangle as b x h and shows or says that triangle formed by splitting along the diagonal is $\frac{1}{2}\ b \times h$.	
	NL	Cannot justify.	
Item 2 Identify potential bases and associated heights of a right triangle.	**ToAM 5B** Generate, use, and explain area formula for a triangle.	Identifies height of base a as side length b. Identifies height of base b as side a. Draws height of base c as a perpendicular line segment through opposite vertex (the vertex formed by the intersection of sides a, b) Uses formula to find area.	
	Other Describe		
Item 3 Find area of regular hexagon	**ToAM 5C** Generate, use, and explain area formula for other polygons (e.g., hexagon).	Finds perimeter of hexagon. Finds apothem of hexagon. Area as $\frac{1}{2}\ P \times a$. Divides hexagon into 6 eq. triangles and finds area of one, finds area as 6 × that area.	
Item 4 Explain formula for area of a regular hexagon.	**ToAM 5C** Generate, use, and explain area formula for other polygons (e.g., hexagon).	Dissects hexagon into 6 eq. triangles Area of a triangle as $\frac{1}{2}\ b \times h$. Sum of bases is Perimeter, so $A = \frac{1}{2}\ P \times h$ (or a) [or equivalent expression]	
Item 5 C = π x D	**ToAM 5D** Generate, use, and explain area formula for circle (characterized as *n*-gon).	The length of the circumference of a circle is (3 or 3 1/10 or other reasonable approximation) times as long as its diameter.	
Item 6 Find circumference, area of circle with radius 5 cm.	**ToAM 5D** Generate, use, and explain area formula for circle (characterized as *n*-gon).	Understands that C = 2*r*. Finds area as 25 π or reasonable approximation.	
Item 7 Explain formula for area measure of a circle.	**ToAM 5D** Generate, use, and explain area formula for circle (characterized as *n*-gon).	Suggests perimeter of an n-gon and circumference are equivalent if n sides keeps increasing (or come closer and closer together). Suggests radius and apothem (or height of eq triangle in an *n*-gon) are equivalent. Establishes that since C = 2*r*π, then $\frac{1}{2}\ C \times r = \pi r \times r$ or πr^2	

Worksheet Area of a Parallelogram

NAME:____

Find the area of this parallelogram:

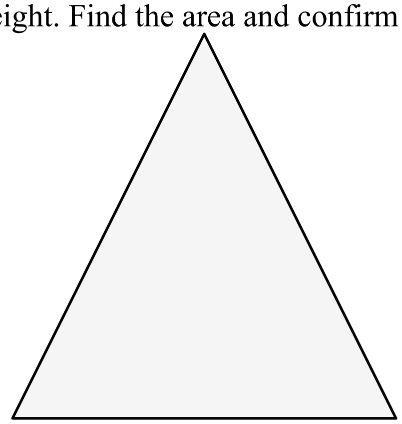
Area = _________

Worksheet Area of a Triangle — Area Unit 5

NAME:____

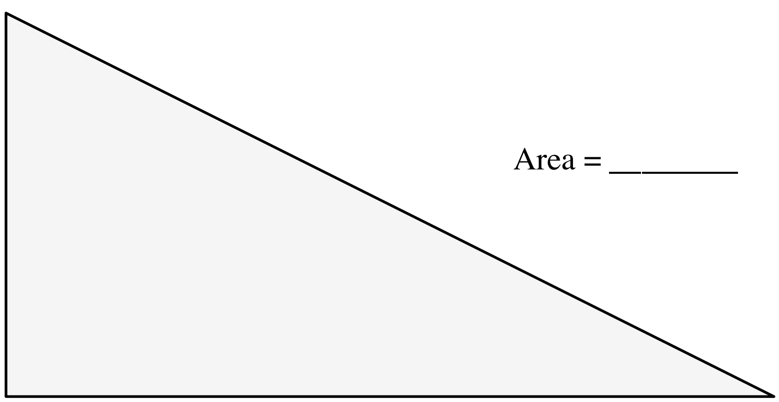
Find the area of each triangle by choosing one side as the base, and finding its height. Then choose a different side as the base and find its height. Find the area and confirm that they are the same measure.

1.



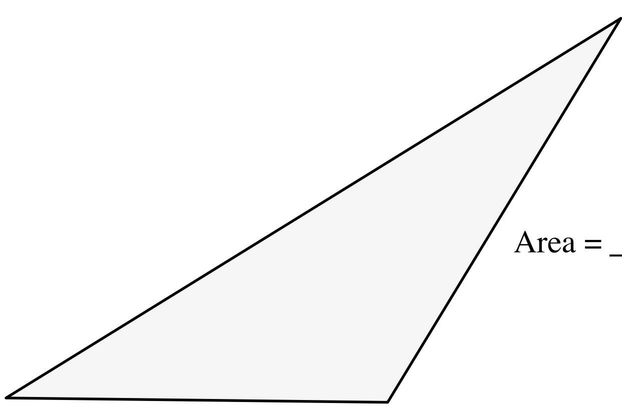
Area = ________

2.



Area = ________

3.



Area = ________