Comparing Zoo Enclosures

UNIT

4

Mathematical Concepts

- Area is additive, which means that the area of a figure can be determined by dissecting it into pieces, determining the area of each piece, and then finding the sum of the areas of the pieces.
- The area of a rectangle of height, h unit, and length, l unit, is $h \times l$ unit^2. But, $h \times l$ unit^2 is equal to 1 unit $\times$ hl unit, so that an $h \times l$ rectangle always has the same area as a 1 $\times$ hl rectangle.

Unit Overview

Given unit dimensions, students find and rank order the areas of 6 figures that are either rectangles or are composed of rectangles. For selected figures, students coordinate units of length measure (inches) to show the square units of area measure. And, for each of these figures, students find the perimeter. Then students use dissection with n units of area measure to demonstrate that the area of any figure is equivalent to the area of a 1 unit $\times$ n unit rectangle.

Contents

Materials and Preparation

Read

- **Unit 4**
  Start by reading the unit to learn the content and become familiar with the activities.

Gather

- For each pair of students, worksheets with figures labeled A—F.
- Rulers with inch markings.
- Student math journals

Academic Vocabulary

- Area
- Length
- Width
- Rectangle
- Perimeter
- Unit
- Greater than
- Less than
- Equal to
- Partition

Mathematical Background

Additive Property of Area Measure
Area measure is additive. This means that the measure of the area of a whole figure is equivalent to the sum of the measures of the area of its pieces. It follows that if figure F is wholly contained in figure G, then Area F < Area G.

Area is a Product
The measure of the area of a rectangular figure is a product of the measures of the lengths of its sides. It follows that a line segment has zero area ($0 \times L = 0$). Sweeping a length L along a length M results in an area with measure n ($L \times M$). For a rectangle of dimension a U, b U, sweeping aU along b U results in ab U^2, which is equal to ab U $\times$ 1 U. So, any a U $\times$ b U rectangle's area is equivalent to the area of the corresponding 1 U $\times$ ab U rectangle.

Conservation of Area
Area is preserved by isometries (e.g., translations, turns, reflections)
The area of a figure does not change if it is moved by an isometry. Two congruent figures have equal area.

Dissection
Two figures are equivalent by dissection if one can be cut up into a finite number of pieces and the pieces rearranged to form the other, as demonstrated in Unit 1. If the measure of the area of a rectangle is n square units, this area measure is equivalent to the area of a 1 unit $\times$ n unit rectangle.

Perimeter

The linear distance all the way around the sides of a figure is its perimeter. This distance can be thought of as a path around a figure. Perimeter is measured with units of length, in contrast to area's measure with 2-dimensional units.

Instruction

Whole Group

Individual students each receive diagrams of six figures.

Tell students: These are plans of zoo enclosures. Because different animals have different requirements for space, the number of square feet in each enclosure must be determined. To determine the amount of fencing for each enclosure, the perimeter must also be determined. In these diagrams, one inch represents 1 foot. Let's look at A. Use your ruler and find its length and width (5 *in.* × 8 *in.*). Find the area and the perimeter of each. For A, B, and F, draw the units of area measure.

Individual

Students use the diagrams to find the area and perimeter of each enclosure.

Teacher note. As students solve the problems, rove and get a sense of different solution strategies.

Instruction

Whole Group

Starting with Figures A and F, compare and contrast solution strategies. Then ask students to compare the strategies used in Figures A and F to the strategies they used to solve Figure D. By nesting F within A, it is clear that the area of F < area A. Be sure that students can coordinate units of length measure to represent the units of area measure, so that the use of the formula $l \times w = A$ is understood.

Then move to Figures B, C, and E, and again compare strategies with an eye toward determining how length measures can be used to find the areas of pieces of each figure.

Pairs

For Figure A, what rectangle with a length of 1 inch will have the same area? How could you show that your choice is right?

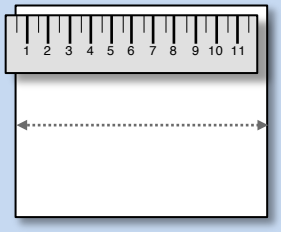
Whole Group

Demonstrate the dissection of Figure A into 40 square units that can be re-arranged to create a 1 unit × 40 unit rectangle.

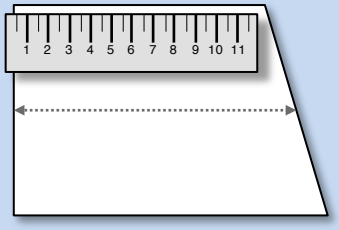
STUDENTS' WAYS OF THINKING

- Students might suggest that after measuring the length of a side of a rectangle, they also need to measure the length across the center of the figure. Typically, this means that they are not relating the figure to properties of a rectangle. This thinking presents an opportunity to help students learn to interpret diagrams. To promote this understanding, the teacher may decide to present a counter case to illustrate the relationship between the length of opposite sides and the resulting structure of the shape, such as that depicted below.

  Students may suggest measuring across the middle of the figure, which, by definition of a rectangle, would also measure 12 inches.

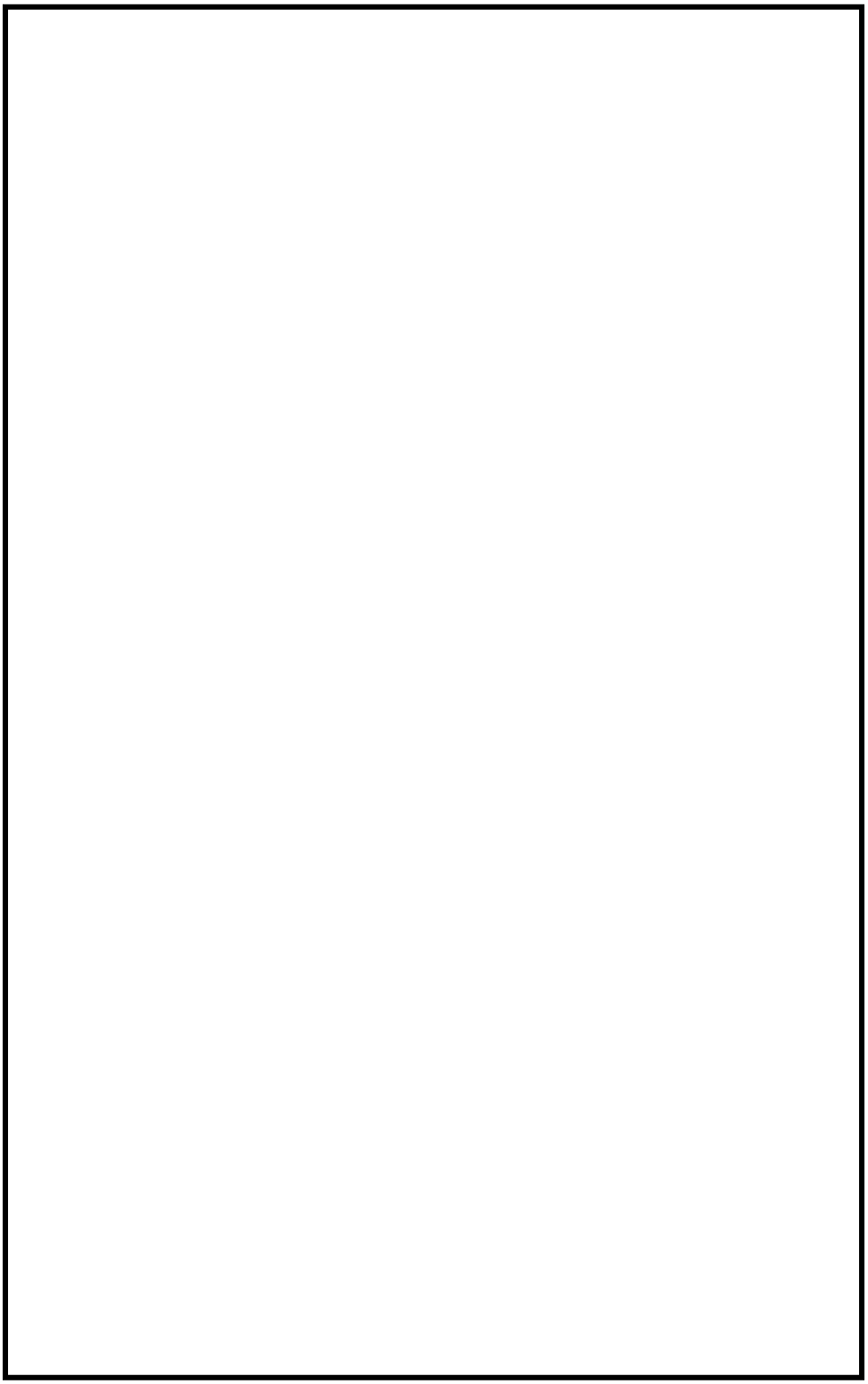


  A counter example shows that if the distance across the middle of the shape did not equal the measure of the long side, then the opposite side of the shape would not be the same length as the original side measured and the resulting shape would also be different—a suggestion made by a second-grade student.



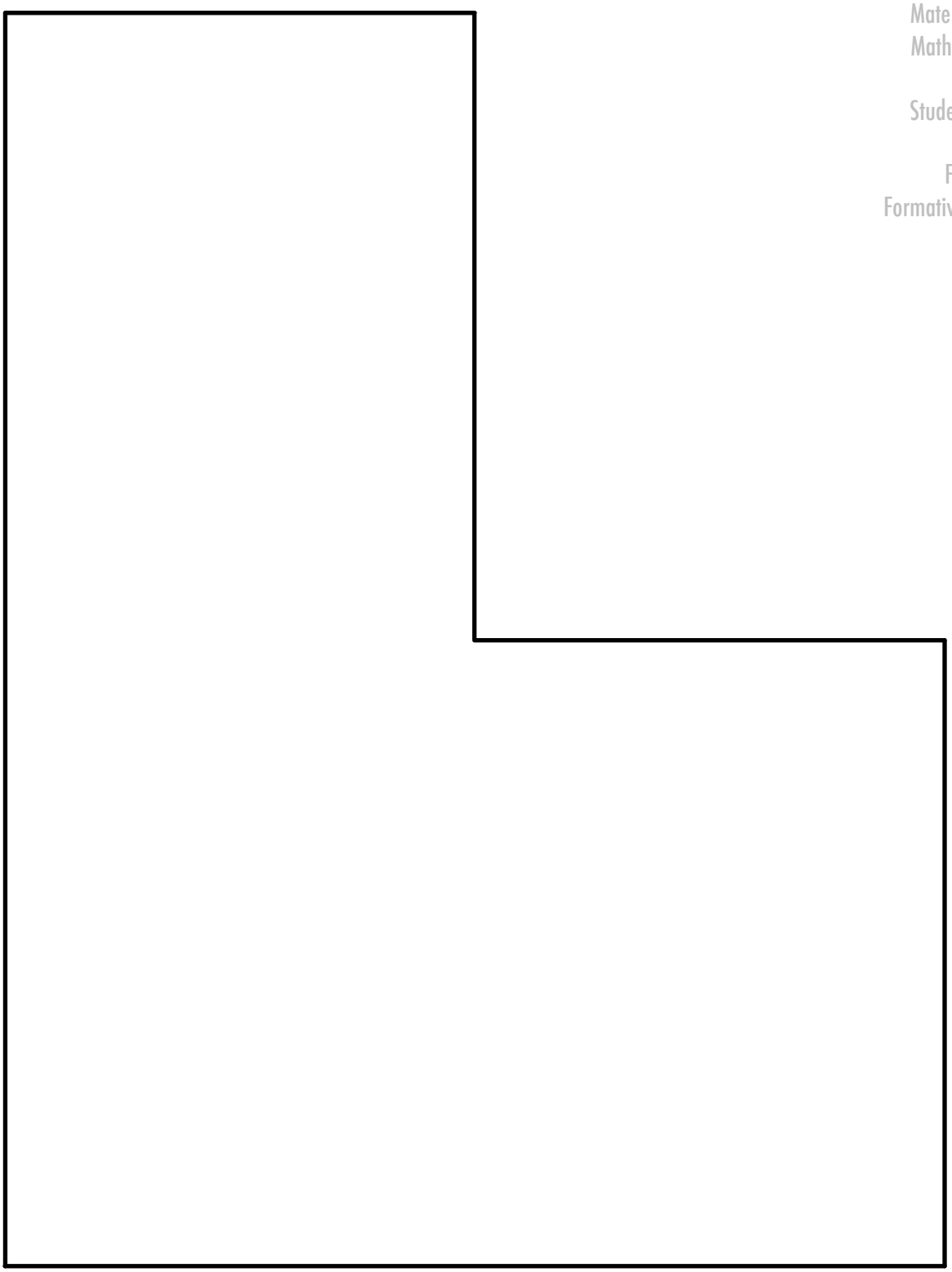
Blackline Masters for Figures A-F

A (5 *in.* × 8 *in.*)



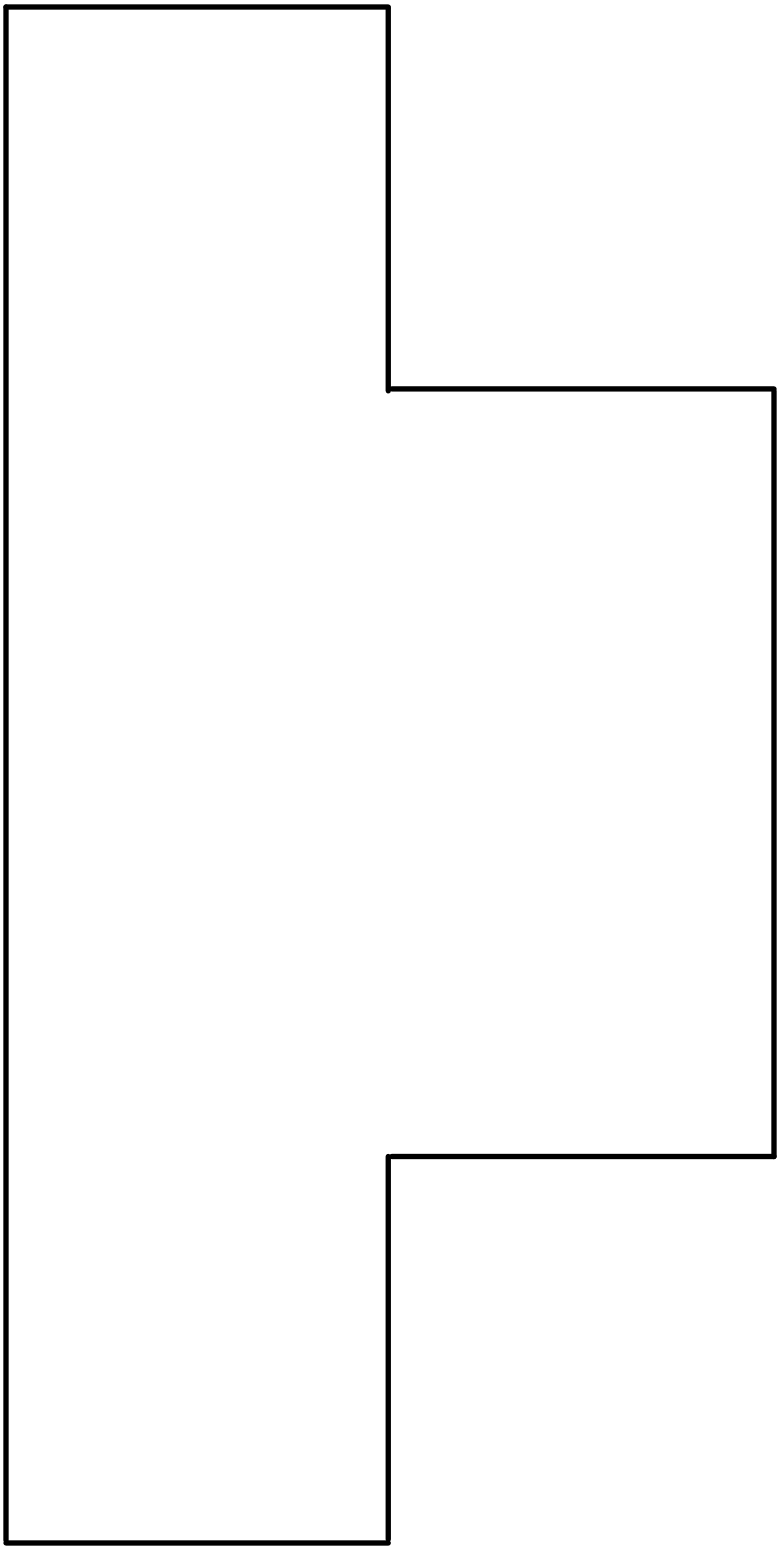
Blackline Masters for Figures A-F

B ($3\ in.\ \times\ 8\ in.\ +\ 3\ in.\ \times\ 4\ in.$)



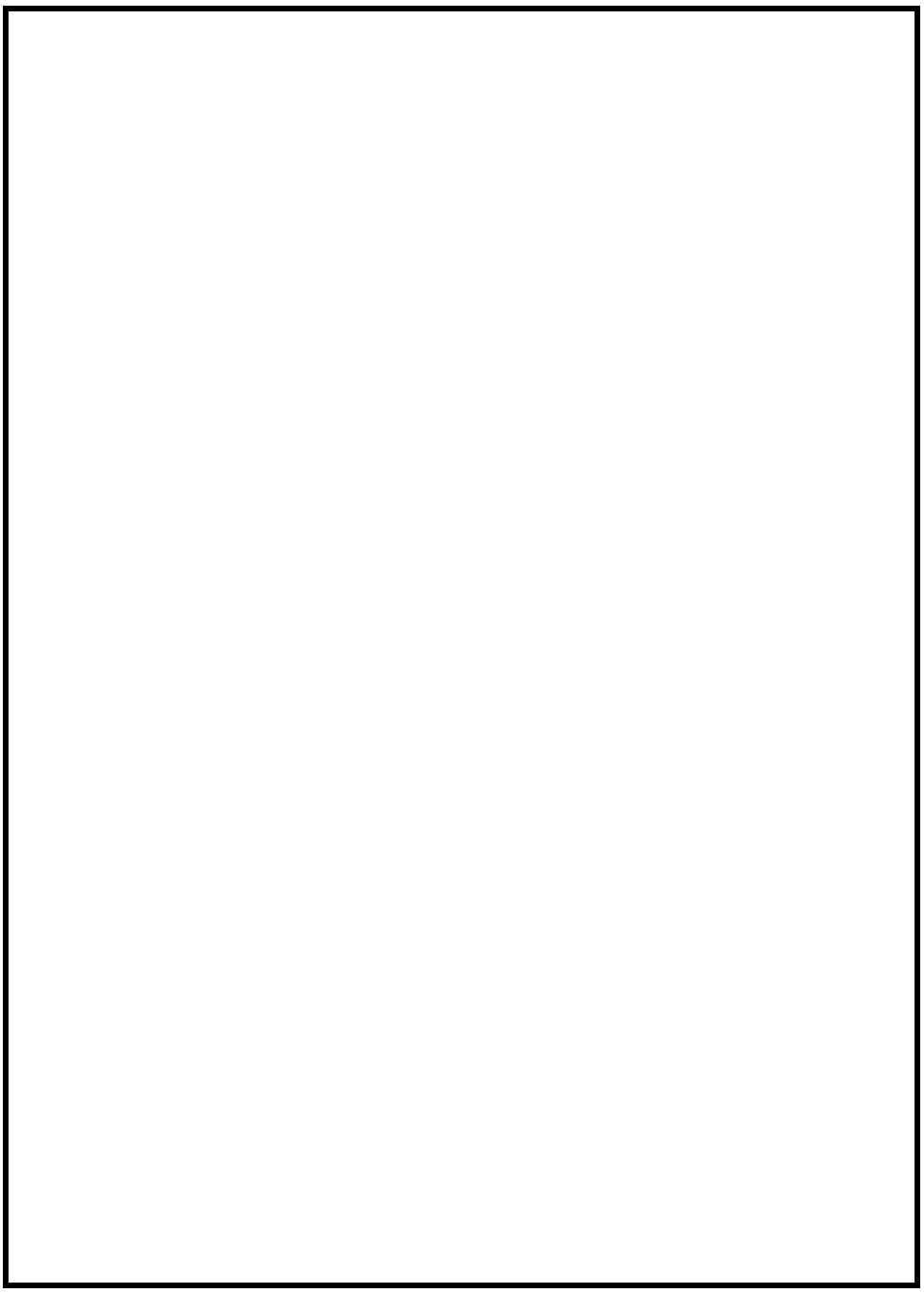
Blackline Masters for Figures A-F

C ($2\ in. \times 8\ in. + 2\ in. \times 4\ in.$)



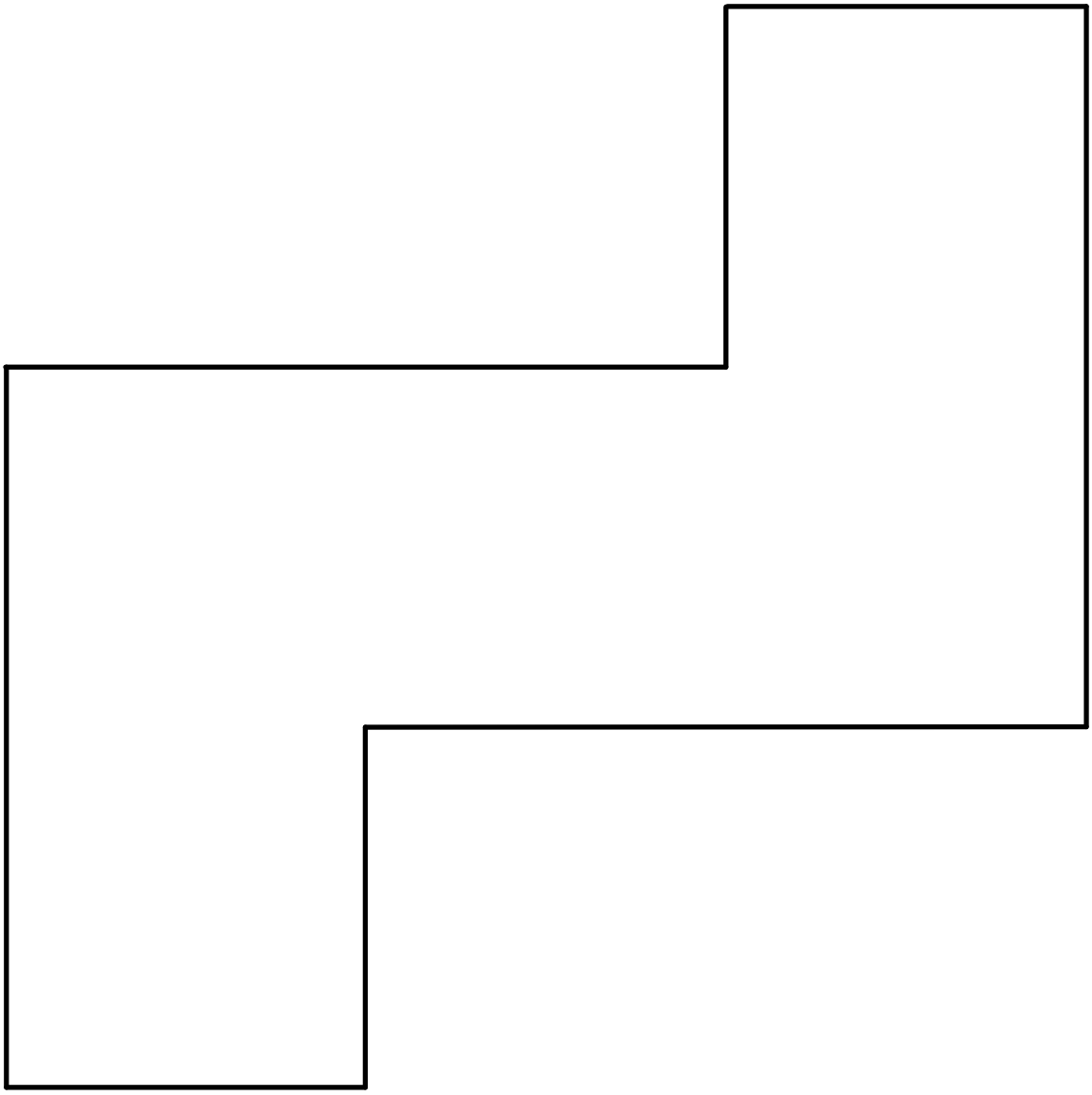
Blackline Masters for Figures A-F

D (7 *in.* × 5 *in.*)



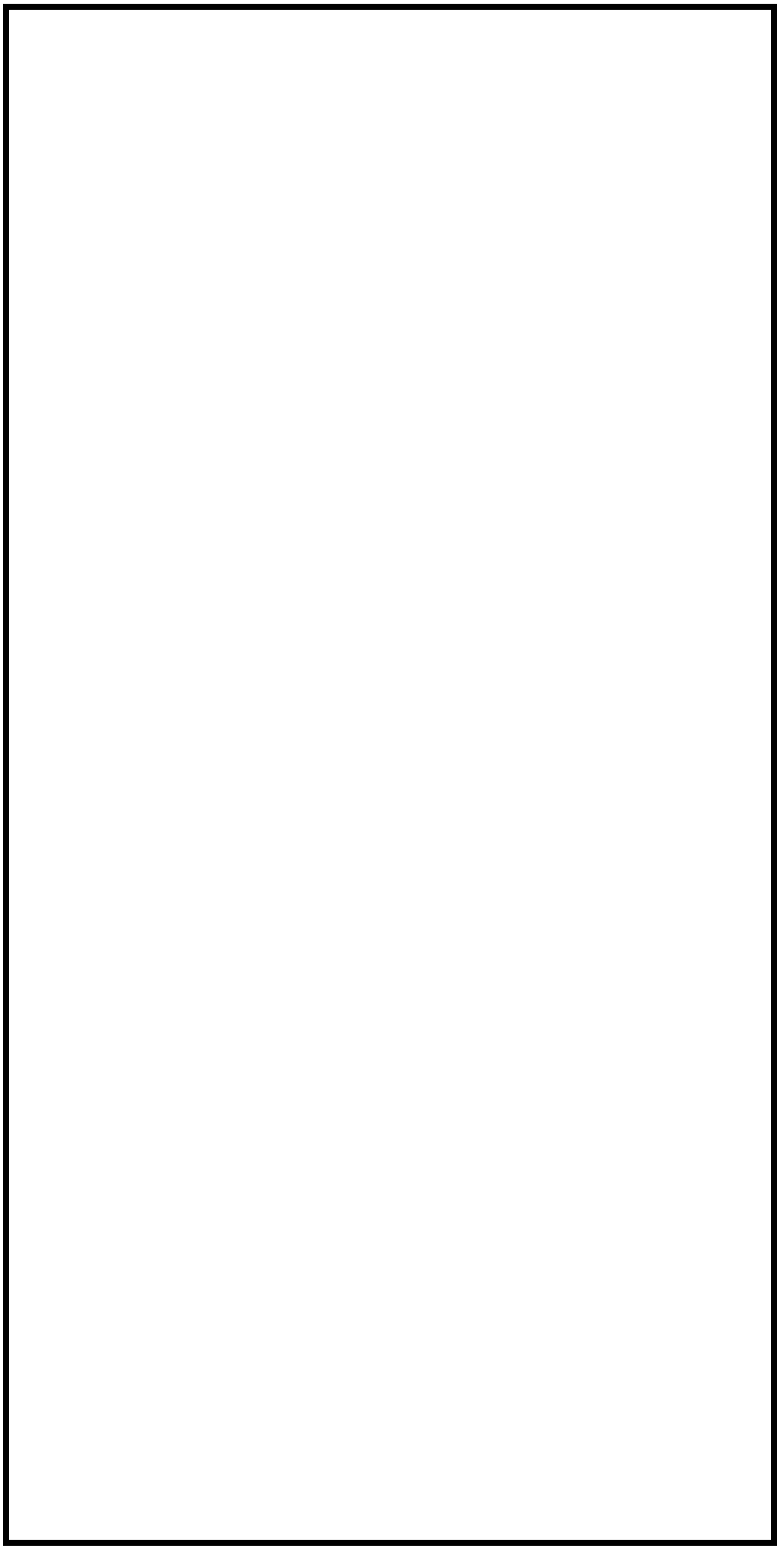
Blackline Masters for Figures A-F

E $(\mathbf{6}\ \boldsymbol{in.} \times \mathbf{2}\ \boldsymbol{in.}) + \mathbf{2}(\mathbf{2}\ \boldsymbol{in.} \times \mathbf{2}\ \boldsymbol{in.})$



Blackline Masters for Figures A-F

F (4 *in.* × 8 *in.*)



Formative Assessment

Formative Assessment

Administer the formative assessment and select contrasting student responses to create further opportunities for learning about area measure, especially the difference between units of length measure (perimeter) and units of area measure.

Formative Assessment

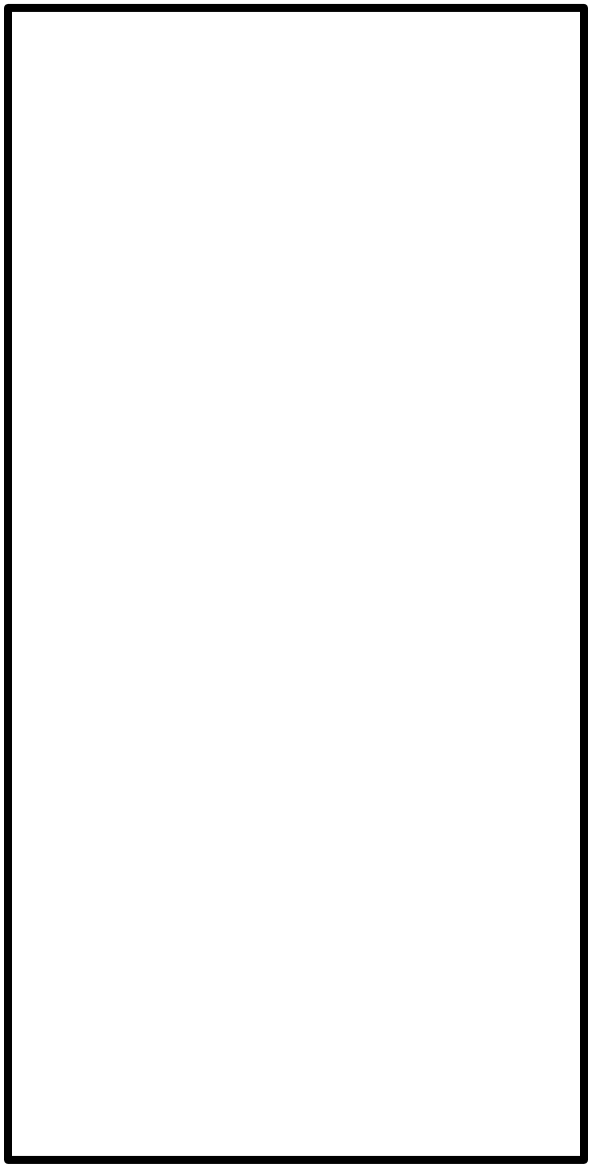
Area Unit 4

NAME:_____

1. What is the area of this rectangle? What is its perimeter? Show the units of area measure. Try this one without using your ruler.

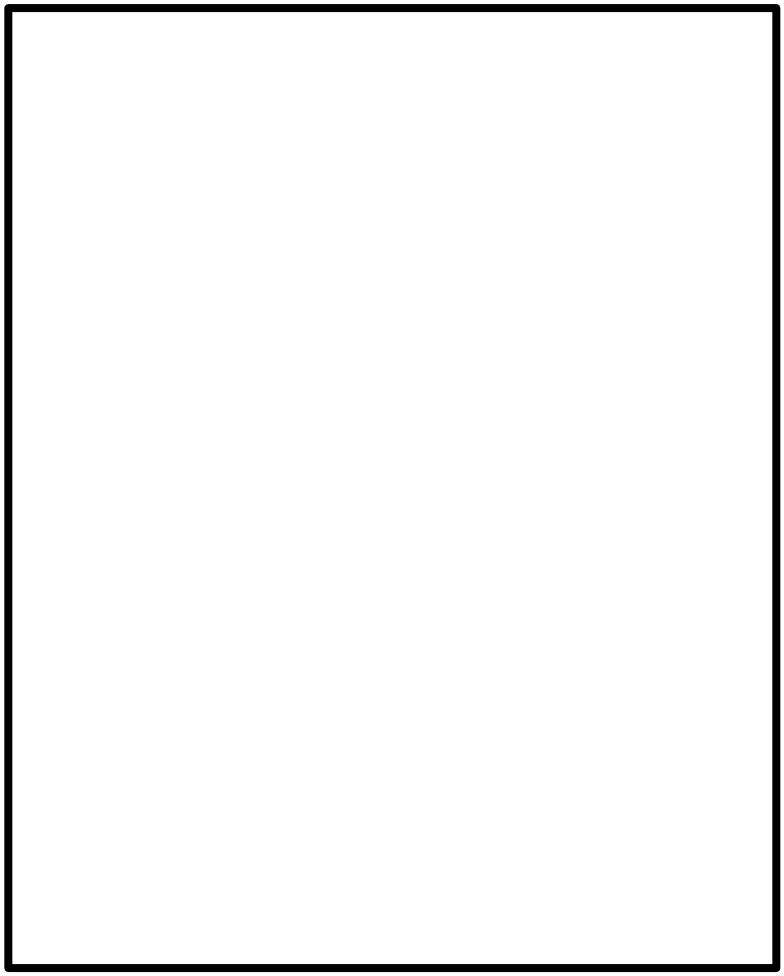
3 u

6 u



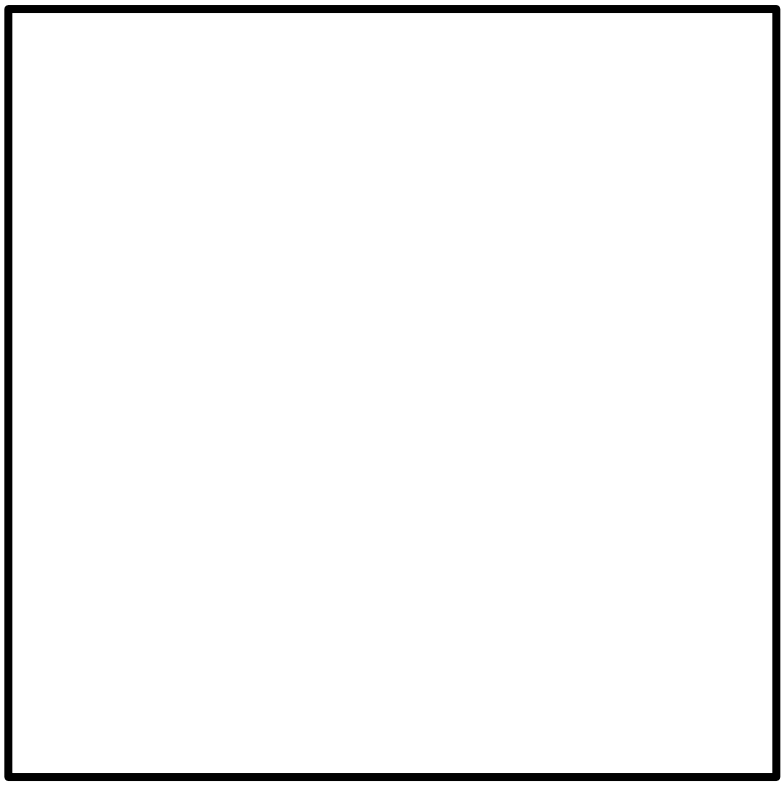
Formative Assessment

2. What is the area of this rectangle? What is its perimeter? Show the units of area measure. You can use your ruler.

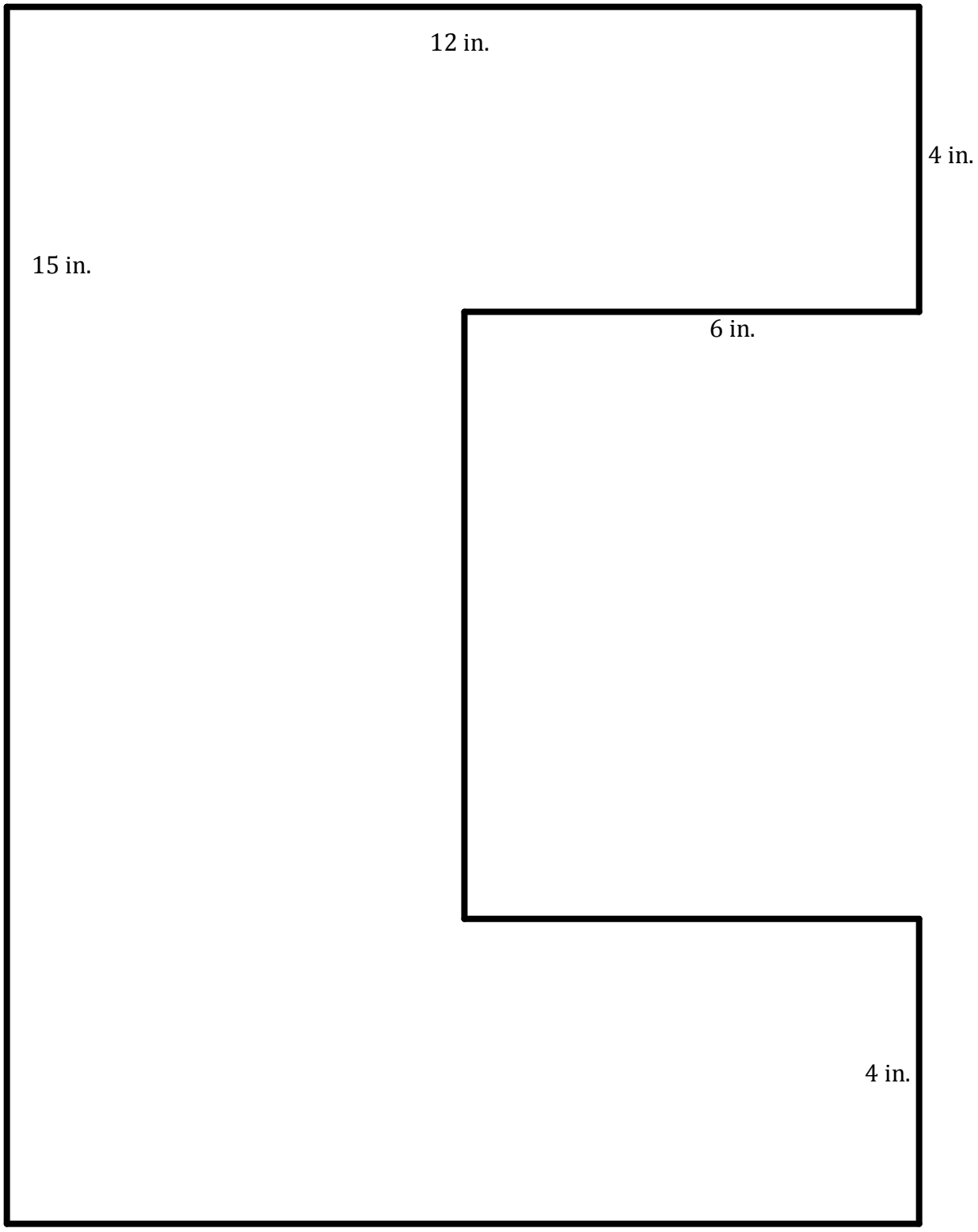


Formative Assessment

3. Here is a square with a side length of 4 inches. What is the length of a rectangle with the same area if one side of the rectangle is 1 inch? How could you show someone that your choice of the side of the rectangle is correct?



4. What is the area of this figure? What is its perimeter? (Notice that the diagram represents inches and is not really 12 inches or 15 inches long.)



Formative Assessment Area Unit 4

Indicate the levels of mastery demonstrated by circling those for which there is clear evidence:

Item	Level Circle highest level of performance	Description	Notes
Item 1 Finding the area of a $3\ u \times 6\ u$ rectangle and showing 18 $u.^2$	**ToMAA 4A** Given an area, partition into arrays of units by coordinating linear measurements of the shape.	3-splits one side, 3-splits the other side, followed by a 2-split or 2-splits, followed by a 3 split, coordinates splits to show 18 in.^{2}	
	ToAM3B Find and compare areas by counting identical units used to tile.	Cannot coordinate lengths to generate square units but generates some other unit that is used consistently to cover.	
	NL	Cannot partition region systematically.	
Item 2 Finding an area of $5\ in. \times 4\frac{1}{2}\ in.$ rectangle	**ToAM3F** Partition to find and compare areas using half-units and other two-splits.	Area as 22 1/2 $in.^2$ and perimeter as 19 $in.$	
	Other Describe		
Item 3 Establish equivalence of area of figure to $1 \times n$ rectangle	**ToMAA 4A** Given an area, partition into arrays of units by coordinating linear measurements of the shape.	Shows that 1 in. x 16 in. rectangle has same area as 4 in. x 4 in. square.	
Item 4 Find the area of a figure by finding the sum of the areas of its parts. Differentiate perimeter from area.	**ToMAA 4A** Given an area, partition into arrays of units by coordinating linear measurements of the shape.	Uses length measures on figure and properties of rectangles to find area, either by subtraction (area of 15 x 12 rectangle- 7 x 6 rectangle) or by partitioning and finding areas of smaller rectangles, then summing. Note strategy.	