Sweeping Area

Mathematical Concepts

- Area is generated by sweeping one length along another at some angle greater than 0.
- A measure of the area generated by a sweep can be found by re-structuring the swept area as a set of discrete units.
- The units are products of the measures of length.
- Units can be whole or parts (fractions).
- The area of a rectangle is the product of the measures of the lengths of its base and height (length × width).
- Cavalieri's principle.

Unit Overview

This unit encourages students to spatially structure, and re-structure, 2-dimensional spaces as they generate area by sweeping one length through another and then use thin spaghetti to define units. Area measure is the sum of the units. The lesson begins with sweeping play. Using a ceramic tile, a wet surface, such as shaving cream or finger paint, two rulers aligned on the base and left side of the tile, and a squeegee, students hold the squeegee in different ways to generate shapes that look like squares, rectangles, parallelograms, and circles. Then, using rulers aligned with the sides of the tile, students sweep squares and rectangles of specific dimensions. They use spaghetti to create units of the area of the swept figures in square inches and in squeegee-inches. Students make a conjecture about a formula to find the area of a rectangle. The area of a rectangle and parallelogram of the same height and base are compared. The lesson concludes with an informal investigation of Cavalieri's principle: In comparison to a rectangle with base b and height h, moving the length of the base parallel to the base through h produces a figure with the same area.

UNIT

3

Contents

Materials and Preparation

Read

- **Unit 3**
  Start by reading the unit to learn the content and become familiar with the activities.
- **Area Measure Construct Map**
  Read the area measure construct map to get a sense of the forms of thinking about area targeted by this lesson.

Gather

- Square ceramic tiles thick enough to allow placement of rulers along the sides without interfering with the action of the squeegee.
- 2 foot-rulers for each tile
- Shaving cream (high lubrication) or finger paint.
- 5 *in*. and 8 *in*. long squeegee's (or whatever else is handy)
- Boxes of thin spaghetti
- Paper towels for cleaning.

Prepare

- Pairs of students prepare one ceramic tile by smearing finger paint or shaving cream over its surface

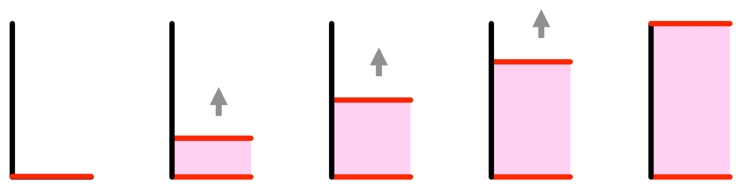
Academic Vocabulary

- Area
- Length
- Width
- Rectangle
- Sweep
- Unit
- Greater than
- Less than
- Equal to
- Split

Sweeping Area

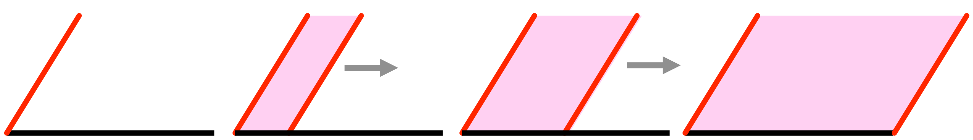
Sweeping

Area is generated by sweeping one length through another at any angle other than 0. In the figure below, a horizontal length is swept through a vertical length, generating an area.

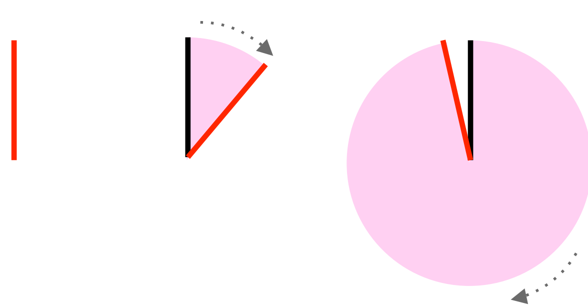


One length (*red*) pulled along the other length (*black*) constructs an area (*pink*).

The lengths can be oriented to create any angle, other than 0.

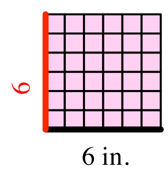


The type of motion can include parallel translation, as in the previous two examples, or rotation, as in the figure below.



Area Measure

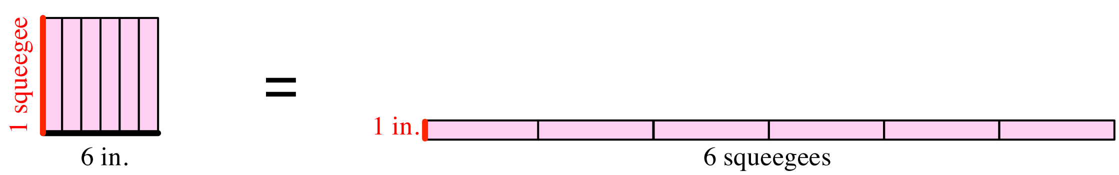
Area measure is a ratio of the space enclosed by a plane figure and a unit of measure. A unit of measure can be generated by sweeping one length through another, as in the figure below, where 1 inch translated through another inch generates a square inch. The area of this figure is 36 times the area of one square inch.



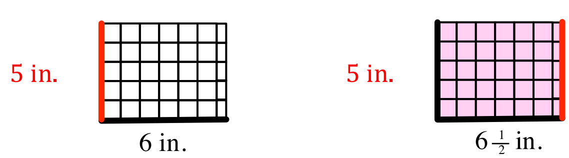
6 in. X 6 in. = 36 sq. in.

Area measured in a length × length unit (1 square inch).

The units of length measure can differ. In the figure below, the measure of the figure is 6 squeegee-inch, where one side of the figure is measured in squeegee units (the side is one squeegee length), and the other side is measured in inches. The area of this figure is 6 times the area of one squeegee-inch. Sweeping 1 inch along 6 squeegees will produce the same area and the same area measure.

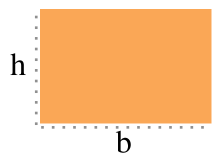


The units of area measure can be split. For example, 5 in. swept through a length of $6\frac{1}{2}\ in$, results in an area measure of $32\frac{1}{2}\ in^2$.



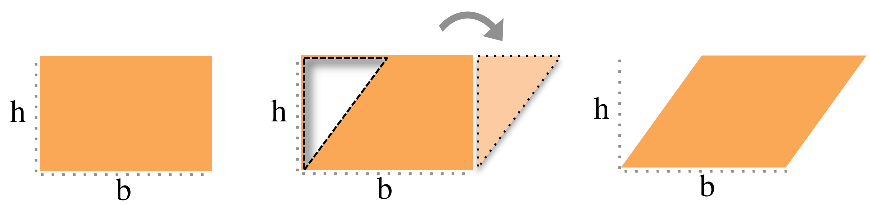
Area Measure of a Rectangle

The area measure of a rectangle is length × width, or base, *b*, × height, *h*. This follows from defining area as the translation (sweep) of one length through another.



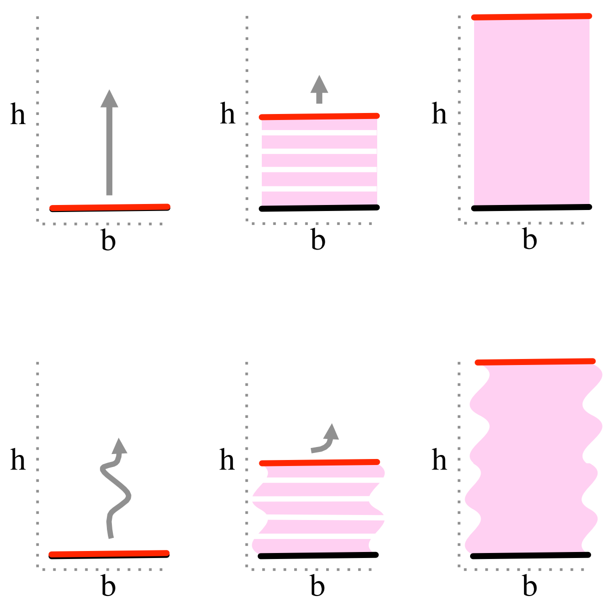
Area Measure of a Parallelogram

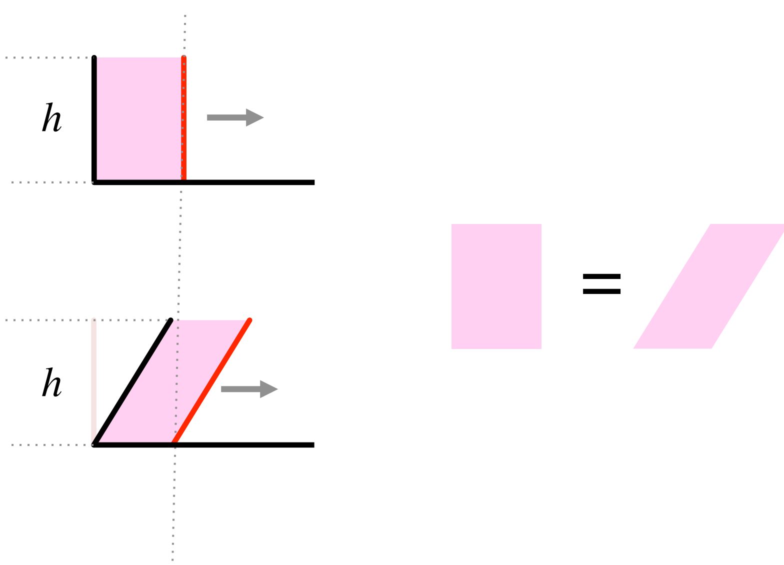
The area of a parallelogram is the same as that of a rectangle with the same height and base. A = h × b. The figure below suggests why this must be so.



Cavalieri's principle

The area of a rectangle is the product of base × height. Sweeping the length of the base in parallel to the base through the same height produces the same area, as shown below. This is another way of justifying the equivalence in area between a rectangle and parallelogram if both have the same height.





Instruction

Whole Group

The teacher presents the ceramic tile coated with shaving cream or finger paint and demonstrates the placement of two rulers that share a common zero point. S/he uses the squeegee to create a rectangle, emphasizing how to use the rulers to determine the dimensions of the rectangle. Then students are challenged to create their own rectangles, squares, parallelograms and circles.

Teacher Note: Use the terminology of sweeping and mention that by sweeping one length through another, area is created.

Partners

Students work in pairs to create figures. As students work, the teacher roves the classroom to remind students that any orientation of the squeegee is fine and to challenge students to create new figures.

Whole Group

Students walk around to see the figures created. The teacher leads a discussion to compare how students created different figures and how they decided to classify a construction as a square, rectangle, parallelogram or circle.

Partners

Students work in pairs to create the following figures, to find the area measure of each, and use spaghetti to show the corresponding units of measure:

(1) 1 *in.* × 1 *in.*
(2) 4 *in.* × 8 *in.* rectangle
(3) 1 squeegee × 8 *in.* rectangle
(4) 1 squeegee × 1 squeegee square
(5) (length of squeegee in inches) × (length of squeegee in inches)
(6) 3 *in.* × 3 *in.* square

Whole Group

Students compare solutions for each problem. An Elmo or other means is used to show the resulting units of measure. Square inch and squeegee-inch are highlighted.

Teacher Note. Many students initially believe that the units of area measure must be squares and must be of the same unit of measure. The squeegee-inch unit is rectangular and is composed as the product of the units. Contrasting cases of the same square measured in squeegees and in inches further clarifies that the same figure can have different measures of area, depending upon the nature of the unit. Note too that the inverse relation between the size of the unit and the resulting area measure.

Partners

Students work in pairs to construct the following figures, to find the area measure of each, and use spaghetti to show the units of area measure.

(7) $3\ in. \times 8\frac{1}{2}\,in.$ rectangle

(8) $3\frac{1}{2}\,in. \times 7\ in.$ rectangle

(9) $1\ squeegee \times 4\frac{1}{2}\,in.$ rectangle

(10) $4\frac{1}{2}\,in. \times 4\frac{1}{2}\,in.$ square

(11) Is there an expression for the area of a rectangle involving the lengths of its sides that will always work?

Whole Group

Students compare solutions for each problem. An Elmo or other means is used to show the resulting units of measure. Fractions of square inch and squeegee-inch are highlighted. The formula length × width for a rectangle is justified by demonstration of sweeping and reference back to (1) – (10).

Pair Work

Compare the area of these two figures. The first is made by sweeping 4 *in.* through 6 *in.* at an angle of 90 degrees. The second is made by sweeping 4 *in.* through 6 *in.* at an angle that is less than 90 degrees.

Whole Group

Students compare solutions and rationale.

Teacher note. At an opportune time, have students cut-out the rectangle and parallelogram in the student worksheet. Then lead a discussion of how the parallelogram can be dissected and transformed into a rectangle that is congruent with the rectangle in the worksheet.

Whole Group

The teacher uses the Elmo or other means to demonstrate Cavalieri's principle.

Pair Work

Directions to students: Position the squeegee at the base of the tile so that there is room for it to move to the right and left. Move the squeegee so that it stays parallel to the base of the tile, through a height of 5 inches. Compare this area with that of a rectangle with length of one squeegee and a height of 5 inches.

Whole Group

Selected pairs of students explain why they think the area is the same or different.

Teacher note. Many students will spontaneously explain that there is compensation or conservation of area when the squeegee is moved. For example, when it is translated to the right, area is generated in that direction, but not in the other direction. So the total area is not changing.

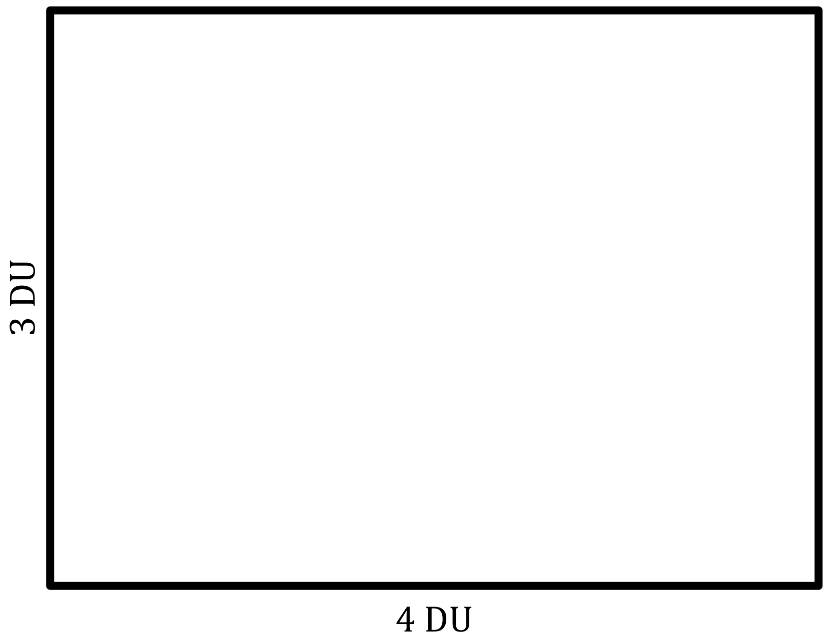
Formative Assessment

Administer the formative assessment and select contrasting student responses to create further opportunities for learning about area measure, especially rectangular area as a product of lengths, structuring rectangles and squares to reveal units of area measure, accounting for partial units of area measure, and justifying why the area of a rectangle and a parallelogram with the same base and height must be the same.

Mathematical Concepts
Unit Overview
Materials and Preparation
Sweeping Area
Area Measure
Area Measure Rectangle
Area Measure Parallelogram
Cavalieri's Principle
Instruction
Formative Assessment

Name:____________________________________

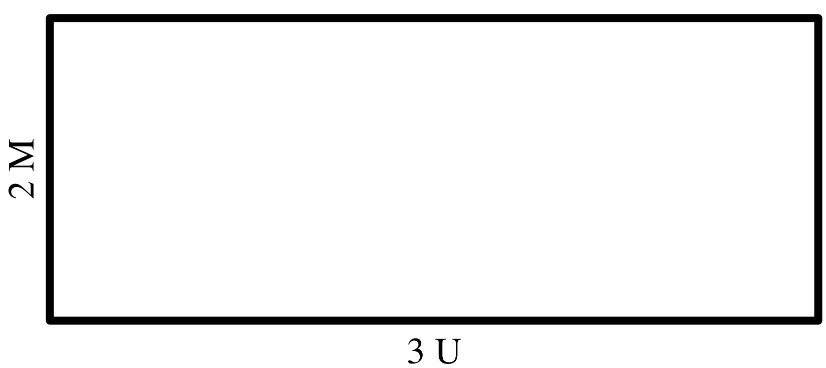
1. What is the area of this rectangle? Show the units by drawing them.



2. What is the area of a $2\frac{1}{2}\,in. \times 4\ in.$ rectangle? What is its perimeter? Show how you found out.

3. Using your ruler, draw $\frac{1}{4}\, in.^2$

4. The lengths of the sides of this rectangle are measured in units of *M* and *U*, as shown below. What is the area of the rectangle? Show the units of area. What is the perimeter?



Worksheet Area Unit 3

Name:____

Formative Assessment Record — Area Unit 3

Indicate the levels of mastery demonstrated by circling those for which there is clear evidence:

Item	Level Circle highest level of performance	Description	Notes
Item 1 Finding the area of a $3\ in. \times 4\ in.$ rectangle and showing $12\ in.^2$	**ToMAA 4A** Given an area, partition into arrays of units by coordinating linear measurements of the shape.	3-splits one side, 4-splits the other side, coordinates splits to show 12 DU^2	
	ToAM3B Find and compare areas by counting identical units used to tile.	Cannot coordinate lengths to generate square units but generates some other unit that is used consistently to cover.	
	NL	Cannot partition region systematically.	
Item 2 Finding an area of $2\frac{1}{2} in. \times 4\ in.$ rectangle	**ToAM3F** Partition to find and compare areas using half-units and other two-splits.	Area as $10\ in.^2$ and perimeter as $13\ in.$	
	Other Describe		
Item 3 Draw $\frac{1}{4} sq. in.$	**ToAM 3F** Partition to find and compare areas using half-units and other two-splits.	Draws a unit $\frac{1}{2} in. \times \frac{1}{2} in.$ or 1 in. $\times \frac{1}{4}$ in.	
	ToAM3D Recognize/construct suitable units.	Draws a unit $\frac{1}{4} in. \times \frac{1}{4} in.$	
	Other Describe		
Item 4 $2M \times 3U$	**ToAM4E** Find and compare areas with dimensions given in unlike units (e.g., length in cm, width in inches).	Draws 6 rectangular MU units. Notes that the perimeter is 4M + 6U	
	ToAM3D Recognize/construct suitable units.	Attempts to use a unit of area other than an MU	
	Other Describe		