# Comparing the Areas of Our Hands

UNIT

2

## Mathematical Concepts

- Area is measured as the ratio of the space covered by a figure to that covered by a unit.
- Areas of different figures can be compared by counting units, if the units are identical and tile the plane.
- Units of area measure need not resemble the figure being measured.
- Partial units of area measure can be constructed and represented as fractional measures of a unit.

## Unit Overview

Students compare and order the area of their hands. The unit begins by posing a challenge to students of finding a way to order the area covered by each student's hand from least to greatest. Students typically try to meet this challenge by direct comparison of pairs of hands, but they typically quickly find that this does not work—how does one compensate for thicker pinkies and slimmer thumbs? Most often, this failure of direct comparison motivates the need for a unit of measure. To facilitate comparison, each student traces the outline of her or his hand and makes a cutout of the outline. Motivated by the failure of direct comparison, students are encouraged to construct a unit of area measure. Construction tools include items that resemble the contour of the hand, such as beans and spaghetti, as well as paper ruled with square inch or square cm. During the course of investigation, students usually find that although beans and spaghetti resemble the contour of the hand, they do not yield consistent counts, because they don't tile the plane ("cracks") and are of variable size. Children are guided to consider the virtue of square units that do not resemble the contour of the hand. This in turn promotes attention to the "left-overs" – units that are not contained in the boundaries of the hand—and hence to developing and combining fractional units of area measure. The lesson concludes with ordering the measures of the area of hands.

Contents

# Materials & Preparation





## Read

- **Unit 2** Start by reading the unit to learn the content and become familiar with the activities.
- The Area Measure Construct Map to become familiar with landmarks of student thinking.

## Gather

- Construction paper
- Black felt-tip markers
- Scissors
- Materials that resemble the contour of the hand and that can be contained within the boundaries of a handprint (e.g., beans, spaghetti, buttons)
- Grid paper (1 in. square and 1 cm. square) and/or transparent grid paper.
- Color pencils
- Chart paper

## Prepare

- Have students trace and cut-out their handprint on a piece of construction paper

## Academic Vocabulary

- Area
- Length
- Width
- Unit
- Greater than
- Less than
- Fractional Unit

## Mathematical Background

The areas of 2-dimensional figures are compared by relating each area to the same unit of area measure. Area measure is the ratio of the space covered by the figure to the space covered by a unit of area measure. To enable this comparison, which does not rely on the partitioning and re-allotment of the previous unit, the units should have the following properties:

**Identical units**

Units of area measure should be identical, unless specifically marked, as in "5 square yards and 3 square ft.", although $5\frac{1}{3}$ square yards would be more appropriate in most circumstances.

**Tiling**

The units of area measure should tile the plane. Else, portions of the area will not be measured.

**Iteration**

The unit of area measure can be "re-used" so that it is not necessary to literally cover the entire figure with units of area measure.

**Partial units**

Units can be partitioned to allow for measure of areas that are not whole-number counts of units. Fractional parts of the same unit can be additively combined, as in $\frac{1}{2}\,in.^2 + \frac{1}{2}\,in.^2 = 1\,in.^2$

## Introducing the Problem

### Whole Group

Think about putting your hands in order from biggest to smallest. Talk with your elbow partner:

> Q: What do we mean by biggest?
> Q: What about the hand are we thinking about?
> Q: Whose hand do you think covers the least amount of space? Explain.
> Q: Whose hand do you think covers the most amount of space? Explain.

### Partners

Come up with a way to compare the area of your hands.

### Whole Group

Elicit student strategies, which are most likely based on direct comparison. Working from what students say, select two students for whom direct comparison will be difficult to discern—perhaps because one has thicker thumbs but skinnier fingers than her or his partner.

> Q: What else might we do if we can't compare everyone's hands directly?
> Q: How did we compare the rectangles? How did we know that they all had the same area? (Unit of measure)
> Q: How might working with paper cut-outs of our hands be better than working with our hands?

## Finding a Unit of Measure

### Whole Group

Make a paper cut out of your hand. Then find the measure of its area by counting units. You can use any of these as units or you can make up one of your own. (Indicate the materials that are available.)

### Individual

As students work, pose questions to help students relate the goal of measurement to the unit they choose. Depending on student strategy, these questions may be fruitful for helping students think:

Q: Why did you choose that unit?
Q: Did you measure all of the space?
Q: What did you do about the curved parts?
Q: What did you do when a unit hung over the edge of your hand/finger?
Q: What did you call this space that is only covered by part of a unit?
Q: How did you keep track of all the parts*?*

### Partner

Write down the measure of your hand but don't tell anyone. Then give your handprint to your partner and ask them to find the measure of your hand with your unit.

## Whole Group

**During a whole-class conversation, choose some students to present their partner's strategy and unit, and any problems that they ran into.**

Q: If we all use this unit (e.g., the beans), should we all get the same measurement? Why?
Q: Why didn't we all get the same measurement?

*Teacher note*. Try to raise the problem of obtaining reliable measurement when the unit does not tile the plane (leaves space unmeasured) and/or is not identical. A count of units of measure of the same handprint is often revealing.

**If no student used the grid paper, ask: Could we use this as a unit of measure by re-drawing our handprint on it?** (Elicit opinions and rationales).

## Partial Units, Inch or Centimeter Square

### Whole Group

Some of us think that this grid paper could be used as a unit of measure and some of us aren't so sure. Let's try it out. Trace your hand on the large grid paper.

### Individual/Partner

**Using the grid paper, find the measure of the area of your hand.**

As students work, the teacher can ask individuals or small groups:

> Q: When we use the grid paper, what is the unit we are using to measure?
> Q: What problems that we had does the grid paper help us solve?
> Q: What are some problems that you are finding when you use the grid paper?

*Teacher note*. Help students see the need to work with partial units of measure.

## Whole Group check-in

**Select student strategies for presentation. Look for a range of solutions to the problem of partial units.** If no student has done so, raise the prospect of identifying and labeling part-units with fractions. To support visualization, use chart paper or other means, and invite students to generate as many different $\frac{1}{2}$ partitions of the square that they can.

*Teacher note*. Be sure to include partitions along a diagonal and "checker board" partitions into quarters, with any 2 quarters shaded.

Repeat for $\frac{1}{4}$ and $\frac{1}{3}$.

## Individual-Partner

**Using what we have just talked about, finish up your measurement of the area of your hand.**

## Whole Group

**Select student strategies for presentation**. Focus on whole-number only solutions, solutions with matching parts, and solutions that combine fractional parts. Be sure to emphasize the nature of units, especially the need to use identical units that tile the plane. Also emphasize that constructing partitions of units means that units do not need to resemble the shape being measured.

*Teacher note*. This is a great opportunity to re-consider fraction addition, both adding fractions of the same partition (split) (such as halves and such as thirds), and using common partitions (such as fourths, to find sums of halves and fourths, or sixths, to find sums of thirds and halves, etc).

## What if we change the unit of measure?

### Whole Group

**Show students the cm. square grid. If we use this unit to measure, predict whether or not the measure of the area of your hand will increase, decrease, or remain the same.** Talk with your elbow partner and see if you agree.

Q: If you think it will remain the same, why do you think so?
Q: If you think it will decrease, why do you think so?
Q: If you think it will increase, why do you think so?

### Individual

**Use the cm. square grid paper to find the area of your hand.**

### Whole Group

**What happened and why?** What might be some advantages to using the smaller cm. squares to measure the area of your hand?

## Using the Measurements

### Whole Group

**Let's use our inch square or our cm. square measurements to order the size of our hands**. Talk with your elbow partner about how we should do that. (Order the measurements on the board from greatest to least)

### Partner

Using our measurements, what was the difference in area between the largest and the smallest hand?

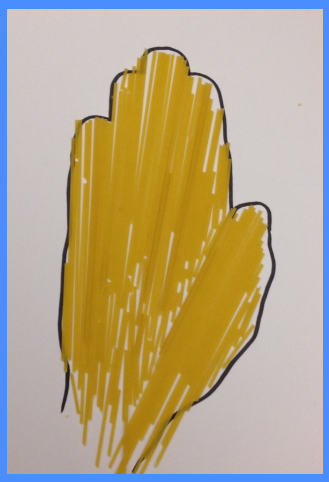
When you look at all the measurements, what do you notice?

### Whole Group Summary

Conduct a conversation that makes explicit the use of measurement units to compare the areas of shapes, even when they are not rectangular, but do have an inside and an outside. Raise the functional value of using identical units that tile the plane. Be sure to re-visit the role of partial units and how they can be combined. Conclude with what students noticed about the measurements when they viewed them collectively as an ordered list.

# STUDENTS' WAYS OF THINKING

Students' choices of appropriate units of area measure are often guided by criteria of literal resemblance and boundary. Literal resemblance refers to a disposition to select or make units of measure that resemble the figure being measured. So, for handprints, beans and spaghetti are often considered as potential units that look like the contour of the hand, as displayed below.



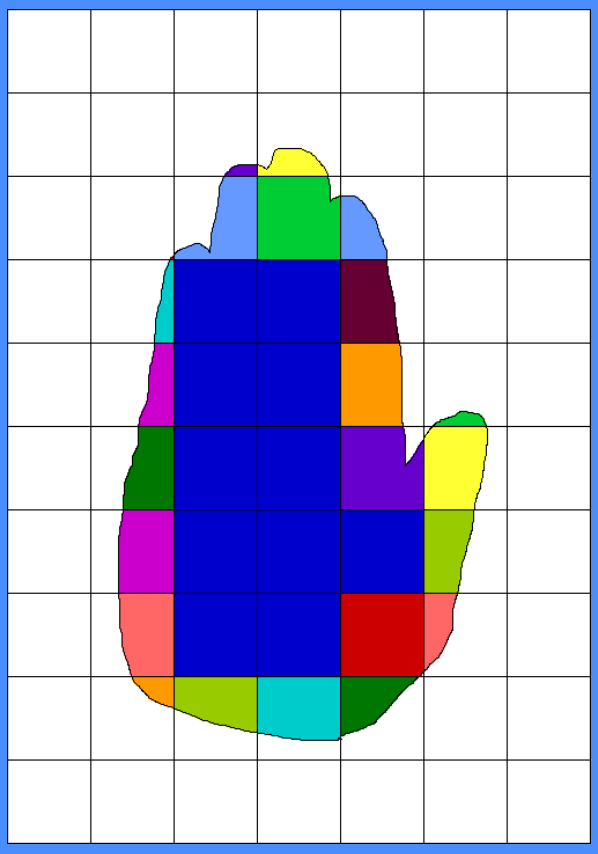
# Students' Ways of Thinking





## STUDENTS' WAYS OF THINKING

Boundary refers to the need to have units of measure fill the space enclosed by the figure so students are often reluctant to select a unit that is not contained within the figure. When students overcome this reluctance and use units that are not fully enclosed by the figure, this creates the need to consider partial units and to consider how to count these units to create a measure. In the illustration below students used a strategy of matching parts: They indicated parts of units that when combined would produce approximately one square inch by using the same color for those parts.



# Students' Ways of Thinking





## STUDENTS' WAYS OF THINKING

Students may choose from several different strategies to account for all the fractional parts.

- *Matching parts*- students may simply indicate parts of two different sections of the hand that "look like" they would add up to a whole square (as displayed).

- *Adding like fractions*- students may join together fractional parts with the same partition (i.e., count the number of ½ square units and add them together, then finding all the ¼ square units and adding them together)

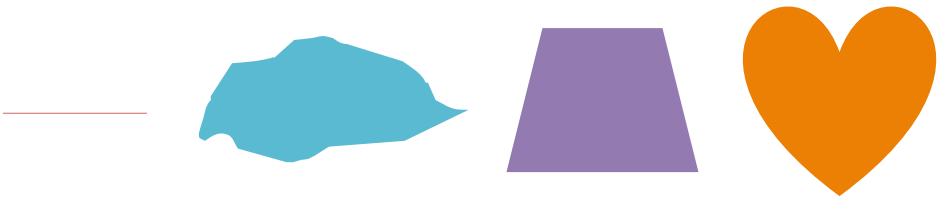
- *Adding fractions with different denominators*- students with a strong understanding of equivalence ( i.e., $\frac{2}{4} = \frac{1}{2}$ ) and addition may choose to combine fractions with different partitions and add them ( i.e., $\frac{1}{2} + \frac{3}{4} \rightarrow 1\frac{1}{4} + \frac{2}{8} \rightarrow 1\frac{1}{2} + \frac{1}{3} \rightarrow 1\frac{5}{6}$ )

## Formative Assessment — Area Unit 2

Name:_____

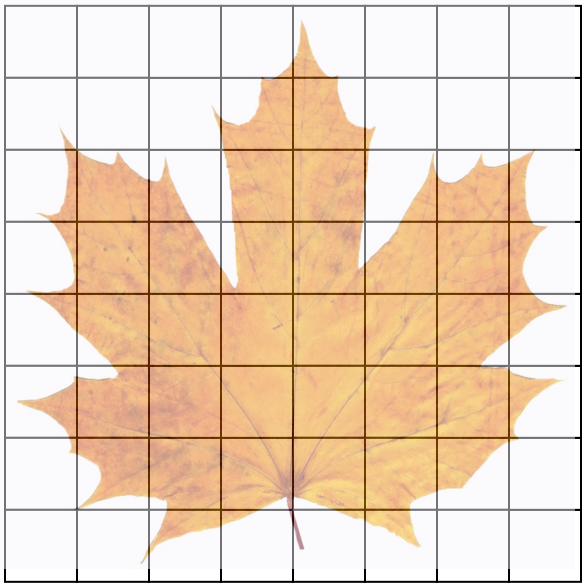
1. From the following, circle the shapes that have an area:

A: B: C: D:



2. Use the square as the unit of measure. What is the area of the leaf?

Show your work.



# Formative Assessment Record — Area Unit 2

Indicate the levels of mastery demonstrated by circling those for which there is clear evidence:

| Item | Level<br>Circle highest level of performance | Description | Notes |
|---|---|---|---|
| **Item 1**<br>Area as Space Enclosed by Figure | **ToAM 1A** | Circles all figures. May include line if justifies by saying that it has some small thickness. | |
| | **NL** | Circles only the trapezoid (thinks that area is only for figures without curves). | |
| | | | |
| **Item 2**<br>Using counts of units for area. | **ToAM 3F** | Partitions unit and combines like partitions. | |
| | **ToM 3C** | Counts only whole units contained within contour of the hand. | |
| | **Other**<br>Describe | | |