

## Connections to Conventions

### Mathematical Concepts

- A linear measure unit can be iterated to create a measuring tool that is a unit of units.
- Measurement tools utilize equivalent units and partitions.
- Standard units facilitate comparisons.
- Constructing and analyzing conventional units.
- Making connections between the concepts of linear measurement, non-standard units and conventional system (inches, feet and yards).
- Using and reading a ruler (from both zero point and point of origin).
- Extending understanding and mathematically describing the results of measuring with a measuring tool.

### Unit Overview

Starting with units that are 2-inches long and a paperstrip that is 12 inches long, the students will make a measuring tool by iterating their unit to create a ruler and label it. The class will analyze a set of measuring tools (selected purposefully by the teacher) to examine issues related to understanding and labeling a measuring tool.

Next students will use units that are  $1\frac{1}{2}$ -inches long and a paper strip that is 18 inches long, the students will make a measuring tool by iterating their unit 12 times to create unit of units and label it. Then they will measure select items that force the students to partition their units for more accuracy and to continue to press on the ideas of fractional units, point of origin, iteration and the use of ruler.

Finally, students will make comparisons between their measuring tool and the U.S. Customary Ruler. How are they alike and different? What big ideas is this ruler based on?

## U n i t

# 5

## Contents

Mathematical Concepts	1
Unit Overview	1
Materials & Preparation	2 & 3
Instruction	
Introducing the Unit	4
Exploring Compositions & Conventions	5
Making a 12 Unit Ruler	8
Comparison to US Customary	12

## Materials & Preparation

## Connections to Conventions Unit 5

### Read

- Unit 5**  
Start by reading the unit to learn the content and become familiar with the activities.
- Sample Student Thinking**  
Reread the Student Thinking boxes to anticipate the kinds of ideas and discussions you will likely see during instruction.

### Gather

- Student math journals
- Teacher journal for note-taking
- Paper strips that 12 inches long (1 per student) and paper strips that are 2 inches long (2 per student)
- Paper strips that are 18 inches long (1 per student) and paper strips that are 1 ½-inches long (two per student)
- Extra paperstrip units for the teacher to use as a demonstration
- 12 inch rulers with markings to fourths only
- Chart paper, markers

### Prepare

- Prepare Unit Lengths** - Prepare paper strips that are 1 inch x 12 inches for the development of the unit of units (1 per student) and 2 inches long for the unit (2 per student).
- Prepare Unit Lengths** - Prepare paper strips that are 1 inch x 18 inches for the development of the unit of units (1 per student) and 1½ inches long for the unit (2 per student).
- Identify items that students can measure that can be measured accurately to the fourths. Identify items that can be measured to fourths with the following properties or make items (or paper strips) that can be measured using the student rulers.
  - Less than a 18-inches long that results in a measure that is a whole unit
  - Less than a 18-inches long that results in a measure that results in half and fourth partitions.

## Instruction

## Connections to Conventions Unit 5

### Introducing the Unit

Students participate in a whole-group discussion about important lessons learned, before considering problems in small groups or partners.

### Whole Group

1. **Introduce the task: summarizing journal reflections, introducing standard unit, 2-splits of the unit.**
  - a. Ask students to summarize their journal reflections or choose the journals of 3 different students for a discussion of “lessons learned” by comparing scales in the previous unit.
  - b. Revisit the importance of the origin of measurement as zero and the rationale for how units are labeled.
  - c. We have been using different units to measure lengths. If we all agreed to use the same unit of measure, would that make easier? (Standard units are agreed upon to make comparisons of lengths of different objects easier.)
  - d. Why do we split units? What does that help us do (increase accuracy of measurements)?
  - e. Why do we have measuring tools with units of units (for measuring larger objects or distances)?

Note: The goal here is to review the key ideas that the students have explored in the past 4 units to help them ultimately connect to standard conventional units.

- Zero point
- Partitions are needed for accuracy
- Iterations of units are needed when measuring larger objects or units

### Introducing the Unit

Exploring Compositions & Conventions

Making a 12 Unit Ruler

Comparison to U.S Customary

## Instruction

## Connections to Conventions Unit 5

## Exploring Compositions &amp; Conventions

## Individual/Student Teams

**1. Students make a 6-unit measuring tool.**

- a. Introduce the <name of unit> unit (2 inches long) and the <name of unit> unit (12 inches long). The students will make a measuring tool by using their <Jacob> units and iterating it to make one <Anthony> unit (ruler or measuring tool) and labeling it.
- b. Give each student a 2 <name of unit> units (2 inch strips) and one <name of unit> unit (12-inch strip). Then have the students make a measuring tool by iterating the <Jacob> unit across the paper strip (one <Anthony> unit) and label their rulers. (At this point, let them partition and label as much or as little as they choose. This will be part of the discussion later.)

**2. Students compare different 6-unit tape measures.**

- c. Select measuring tools that exemplify the ideas from ones described below in the Student Thinking box to share or is likely to reveal students' conceptions/misconceptions of the measuring tool. (This is going to be heavily dependent on what your students do and what ideas they still need to make sense of.) If no one makes the measuring tools that have qualities you need, make some so that you have examples that can bring out ideas that do not come naturally from theirs. Students are not likely to make a tool like D. It is a good one to share (but it is important for students to think this was created in a different class as opposed to offering it as a standard you want them to consider). Students compare the measuring tools, identify which would be problematic, and which are good examples of a tool and why. Have something that could be measured (at least to the half unit and  $\frac{3}{4}$  unit). Discuss.

Then have them choose which they believe is best tool, explain their thinking and write about why they think in their journal. If students make other errors that seem to be typical, include those as well in the set for discussion.

Introducing the Unit  
Exploring Compositions &  
Conventions  
Making a 12 Unit Ruler  
Comparison to U.S Customary

# Instruction

# Connections to Conventions Unit 5

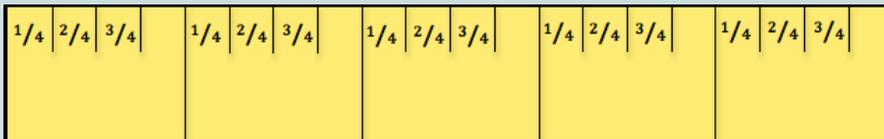
## Student Thinking

The tapes are designed to support students' consideration of the role and functions of numeric symbols in systems of measure and make connections from the act of measuring and how it connections with conventional tools.

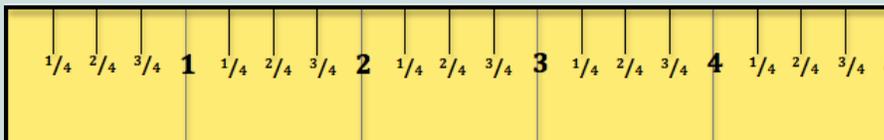
**Tape A** has 5 equal partitions but labels each partition in the interior. This makes it difficult to know what is meant by "1" or "2." Moreover, the units are not split, making measurements of lengths difficult if the object does not correspond exactly to whole numbers of units.



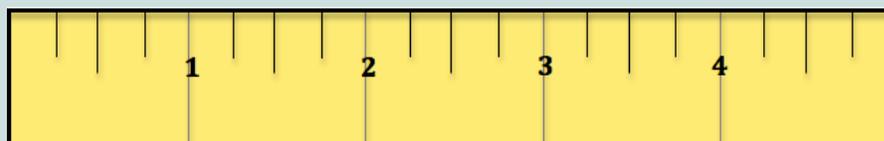
**Tape B** has 5 equal partitions, and each partition is further split into fourths. But the labels are ambiguous, so it is difficult to know the length of "3/4" or any other split-unit. The whole number units are not labeled.



**Tape C** labels whole units and split units so that the label corresponds to a distance traveled.



**Tape D** labels whole units only, splits the units, and signifies 1/2 and 1/4 partitions differently. It relies on user knowledge of the meaning of the split unit. This tape is closest to those that we use conventionally.

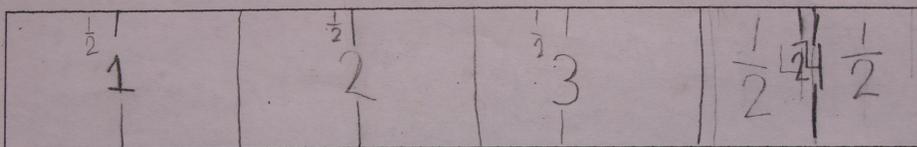


Introducing the Unit  
 Exploring Compositions &  
 Conventions  
 Making a 12 Unit Ruler  
 Comparison to U.S Customary

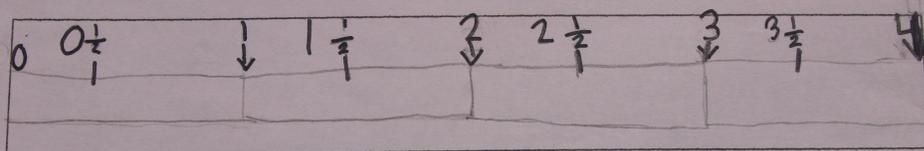
## Instruction

## Connections to Conventions Unit 5

**Tape E** labels the unit in as a count without regard to zero point but labels half units using zero point (distance from the beginning of the unit). This causes difficulty as the ruler mixes conventions (area and linear model) and makes interpretation and use of the ruler difficult.



**Tape F** labels the ruler thinking about the distance from zero but forgets about the importance that halves are equi-distance measures.



Introducing the Unit  
Exploring Compositions &  
Conventions  
Making a 12 Unit Ruler  
Comparison to U.S Customary

### Whole Group

3. **Students identify the measuring tools they prefer and which tend to be problematic.** Have the students identify which rulers they think work and which do not. They should justify their thinking about what is productive and what is not. This will provide some formative assessment in terms of what students understand about measure and how that thinking relates to a measuring tool.
  - a. Questions to promote discussion include:
    - Q: Which is best for finding a distance traveled (indicate with hands) that measures  $\frac{3}{4}$  unit? Why?

## Instruction

## Connections to Conventions Unit 5

*Tape A, with the interior label (and no partitions marked), makes this distance ambiguous. Tape E is labeled only to  $\frac{1}{2}$ . Tape E has issues of the half marked as a distance from zero but the unit labeled in the space (area model). They are also not labeled to fourths.*

Q: What is problematic about tape F? *Tape F is labeled with zero point in mind. The units are equal but the partitions are not equal.*

Q: If you used Tape E & F to measure, what might be confusing? What would you do with these measuring tools to make it so that they worked better?

Q: Which is best for finding a distance traveled (a length that is as long as) of  $1\frac{3}{4}$  units? Why?  
*Either the explicitly labeled tape measure (C) or the implicitly labeled tape measure (D) will do.*

Q: What do you think the people who made tape D had in mind? What do you think these people assumed you already know about measurement?  
*This ruler is closest to standard practice—but it relies on the users knowing the “rules.”*

4. After the class has had their conversations and stated their views, have two items available that students can measure that results in a measure that is related to first a half and then a fourth. This will help to see if they can prove how the rulers of their choice are useful for measuring and which are not.

### Making a 12 Unit Ruler

#### Individual/Student Teams

5. Introduce the <unit name> unit ( $1\frac{1}{2}$  inches long) and the <unit name> unit (18 inches long). Have the students make a measuring tool by using the Alyssa unit ( $1\frac{1}{2}$  inches long) and Clint unit (18 inches long) units. We will then have them use them to measure selected items.

Introducing the Unit  
Exploring Compositions &  
Conventions  
**Making a 12 Unit Ruler**  
Comparison to U.S Customary

## Instruction

## Connections to Conventions Unit 5

- d. Today, we are all going to use this unit (one Alyssa unit) to make a measuring tool (one Clint unit). Then we will use them to measure items in the class.
- e. Give each student 2 Alyssa units (1 ½ inch strips) and one Clint unit (18-inch strip). Then have the students make a measuring tool by iterating their Alyssa unit across the Clint paper strip) and label their measuring tool. (At this point, let them partition and label as much or as little as they choose. This will be part of the discussion later. Hopefully after the previous conversation, most students' rulers will be done correctly but if not, this is one more time to discuss and consider the important ideas behind the construction of a measuring tool. This will be both a formative assessment (how much did they make sense of the ruler and what are they still struggling with) and the foundation for the classroom discussion.
- f. Have the students measure the set of items in the room and record their answers.

## Whole Group

### Student Thinking

As students construct their measuring tool, probe their understanding of how to label the partial units.

What do students call each partition? We want to ensure that students understand that each partition is called  $1/b$ , where  $b$  corresponds to the number of congruent partitions of the unit.

Where do they write the numeric label? This is often particularly revealing. Students who do not label the unit at its endpoint may not have a firm grasp on the measure as a distance traveled.

After students have constructed their measuring tool, be sure they try to use them to measure a length that is *less than* the length of the unit, and a length that is *greater than* 12 of the units.

3. Use the students' measures and thinking to press on further on the big ideas about measuring tools, understanding partial units and zero point and conventions for labeling measuring tools.

Introducing the Unit  
Exploring Compositions &  
Conventions  
**Making a 12 Unit Ruler**  
Comparison to U.S Customary

## Instruction

## Connections to Conventions Unit 5

a. Conduct a whole group conversation to explore the issues related to measuring tools and measuring. Choose student responses to the questions addressed individually and with partners.

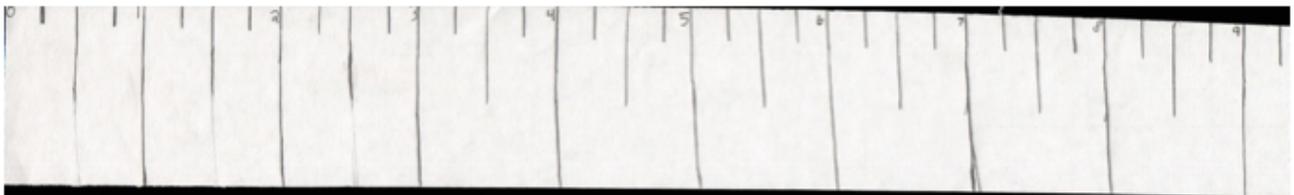
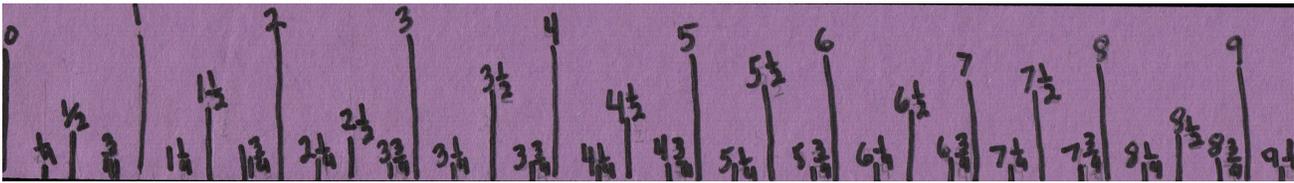
- 1) Begin by having the students share measures that resulted in whole Alyssa units and less than a Clint unit. Look for:
  - a) How did the students label their rulers? Did they label the whole units, if so where did they label it?
  - b) How did they describe their measure? Were their measures influenced by students understanding of the big ideas of measure? If not, what big ideas are they struggling with? What is your evidence?
  - c) Does their thinking reflect understanding about zero point?
  - d) Ask the students to describe and justify their thinking?
    - i. Why did they label it at the end of the unit or the middle of the unit? Does it matter? Why does it matter?
    - ii. Have them compare and contrast the different measures for the same items. Ask them why there are different measures? Are these measures equal to each other or not? Ask them if they can justify their thinking.
- 2) Next have them look at measures that less than a Clint unit but results in partitions of the Alyssa unit.
  - a. Again, how did the students label their rulers? Did they just label the whole Alyssa or did they partition them? Did they label the partial units? If so, how did they label? Did they label the partitions reflect understanding of zero point (i.e.  $\frac{3}{4}$  is  $\frac{3}{4}$  from zero or did they label all partitions as  $\frac{1}{4}$ ,  $\frac{1}{4}$ ,  $\frac{1}{4}$  ... regardless of the distance from zero).
  - b. This is another place for students to talk about zero point and distance from zero. It will also give you a chance to talk about equivalence if students quantify their measures differently.
  - c. Place items on the ruler but do not start at zero (different point of origin).

Did they label their ruler continuously  $1\frac{1}{2}$ ,  $1\frac{3}{4}$ ,  $2\frac{1}{4}$ ,  $2\frac{1}{2}$ ,  $2\frac{3}{4}$  or by non-continuous partitions  $\frac{1}{4}$ ,  $\frac{1}{2}$ ,  $\frac{3}{4}$ . Did they let the lines show the different partitions? Which is the easiest to read? What if you do not start measuring at zero? Which tool is easiest to measure with?

Introducing the Unit  
Exploring Compositions &  
Conventions  
Making a 12 Unit Ruler  
Comparison to U.S Customary

## Instruction

## Connections to Conventions Unit 5



- d. Ask the students to describe and justify their thinking.
  - i. Have them compare and contrast the different measures for the same items. Ask them why there are different measures? Are these measures equal to each other or not? Ask them if they can justify their thinking.

### Comparisons to U.S. Customary

#### Individual/Student Teams

6. Pass out standard U.S. rulers (ruler should only have fourths labeled) out to each of the students. Have the students examine this tool and ask them to compare and contrast this measuring tool to the ones that they invented. How are they alike or different? (same markings but enlarged)
  - a. Try to pull out that both measuring tools consists of 12 units.

Introducing the Unit  
 Exploring Compositions &  
 Conventions  
 Making a 12 Unit Ruler  
 Comparison to U.S Customary

## Instruction

## Connections to Conventions Unit 5

- b. Each unit is partitioned into fourths similar to the way some of the measuring tools were in the class.
  - c. Some students labeled their rulers (partitions) where on this measuring tool, the partitions are marked but not labeled.
7. Explain to the class that this is the measuring tool that we use in the United States. It is called a ruler that is a foot long (about the same as our Clint unit). The ruler (we call a foot) is comprised of smaller units (similar to our Alyssa units) and are called inches .
8. If you are using a word wall, this would be a good place to write the terms inches and feet and help them to connect convention with their inventions.
  - a. Suggest to students that they have both rulers on their desk as they write the reflection. Have them include diagrams if it helps the students clarify their thinking and to support the writing skills of your grade level.
  - b. Give the students some paper strips or items to measure using the U.S Customary ruler. Using the students' measures, engage them in discussions about how they are using the ruler and how the use is like their student made rulers.