Making Sense of Our Travels

Unit 4

Mathematical Concepts

- Fractions represent partial units.
- 1/b represents 1 copy of a unit partitioned into b congruent parts.
- a/b represents a copies of a unit partitioned into b congruent parts.
- a/b can be interpreted as traveling from zero to the location a/b.
- a/b can also be interpreted as iterating a length of 1/b, a times.
- 2-splits of units can be composed (e.g., ½ of 1u results in ½ u and ½ of ½ u results in 1/4u).
- Equivalent fractions mark the same distance traveled or location if the unit is the same. Hence, ½ unit = 2/4 unit = 4/8 unit.
- Relational thinking is assisted by the language of “times as long.”
- Understanding the number-line.

Unit Overview

This unit revisits concepts of measurement, but in this lesson the measurement units are not just tied to feet. Students are introduced to fractions as partial-units. The goal for students is to measure distances on a street and later transferred to a number line. The long side of the rectangle is what is used to measure length, but its 2D structure helps students better visualize the results of folds. Classroom discussion focuses on splitting units, so that lengths that are not multiples of whole numbers can be measured. Fractions, a/b, are quantities representing *a* copies of *b* congruent partitions of the unit. The symbolization a/b corresponds to partitioning the unit into *b* congruent partitions by folding, and then traveling (walking) *a* of these partitions, starting at the zero. Hence, ¼ unit represents traveling from the origin, 0, to the end of the first of 4 equal partitions of the unit. Similarly, 3/4 unit represents traveling from the origin to the end of the 3rd of 4 congruent partitions, and 5/4 unit represents traveling from the origin to the end of 5 of these congruent partitions, each of which is ¼ unit long. Traveling is complemented by iterating; a/b unit is *a* iterations of 1/bth partition of the unit. Hence, 2/4 unit is 2 iterations of ¼th unit.

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Materials & Preparation

Read

- ☐ **Unit 4**
  Start by reading the unit to learn the content and become familiar with the activities.
- ☐ **Mathematical Background and Sample Student Thinking**
  Reread the Mathematical Background and Student Thinking boxes to anticipate the kinds of ideas and discussions you will likely see during instruction.
- ☐ **Measurement Construct Map**
  Read the construct map and look at the multimedia map to help you recognize the mathematical elements in student thinking, and to order these elements in terms of their level of sophistication.

Gather

- ☐ Student math journals
- ☐ Teacher journal for note-taking
- ☐ Paper unit strips (enough for each student to have 2 – 3 strips and at least 10 extras to use for classroom demonstration) These strips need to be the same length as the paper strips used in the town street below.
- ☐ Tape
- ☐ Town Main Street map with unit strips with landmarks to describe location. Gather different clip art (or student drawings) with different items (houses, shops, school, playgrounds, trees, flowers etc). It might be best to laminate the town and the pictures so that they can be used repeatedly to explore the ideas of distances from a point of origin and making sense of units and partitions and to record students thinking. Leave space to add other items for
- ☐ Laminated sentence strips that can be used for a number line or butcher paper (still best to laminate so you can write on and use it repeatedly with different conversations). This is an extension that will relate directly to 3rd grade CCSS Fractions. Have extra sentence strips for folding units and iteration available. Three or four sentence strips should be long enough. You may want to tape it to butcher paper so that you do not have to start over in labeling the number line if you lesson goes over a class period.
- ☐ Some toy or item to use in traveling
- ☐ Writing assessment task instructions (Appendix)

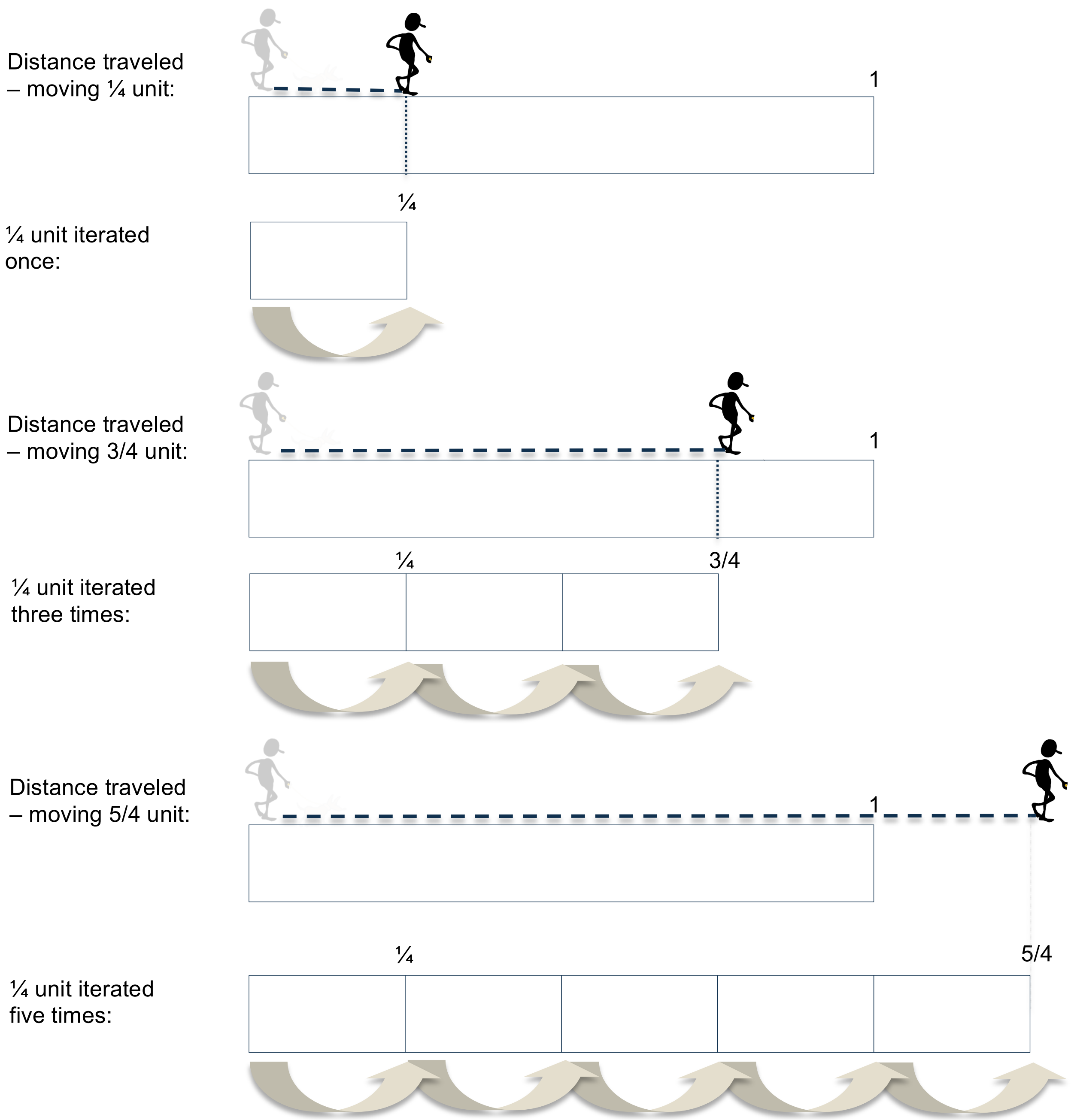
Materials & Preparation

Prepare

- ☐ Paper strips for each student for testing ideas
- ☐ Make Main Street map
- ☐ Laminate sentence strips for number line

Mathematical Background

There are two metaphors used to help students understand fractions. One relies on unit iteration where, for example, 3/4 means 3 iterations of ¼ unit. This sense of fraction relies on a copy of a partitioned unit, as in 3 copies (iterations) of ¼ unit results in 3/4 unit. The second metaphor is distance traveled where, for example, 3/4 means starting at 0 and moving 3/4 unit. Both of these metaphors, one static and the other dynamic, help students form images of fractions that will later help them locate fractions on the number-line, because the number-line is an idealized ruler.



Introducing the Unit

Each student will get 3 paper strip units; the goal is to create parts of units so that the measurement will be more accurate. Students discuss possible reasons for partitioning. What does partitioning help accomplish?

Whole Group

1. **Introduce paper strip units.**

   a. Facilitate a discussion about issues students had with their footstrip units focusing on issues with accuracy and problem solving around measurements that were partial unit lengths.

   b. Ground the need for partial units by reminding students that when they used the footstrip unit to measure, some students mentioned that some of the things you were measuring did not come to the end of a unit. (Have several students' footstrip unit handy for demonstration purposes.) Ask:

   Q: Who can remind us of that conversation?
   Q: What problems did you have?
   Q: What did you do when the object you were measuring did not end where a unit ended?
   Q: What did you have to think about?

2. **Demonstrate Partitioning.**

   a. Begin by asking: How could I split my unit so that I could measure exactly something that was one-half times as long as this unit? Most often, students will suggest folding it. If they do not, proceed to demonstrate.

   b. Ask: How should I fold my unit to make 2 parts, each exactly the same length? Then, demonstrate splitting the unit by folding into 2 congruent pieces. Ask:

   Q: How do I split a unit to make sure the parts are the same (identical, congruent) length?
   Q: What do I call each part?
   [One-half <unit name>, e.g., one-half Ricardo; **be sure to mention the unit!**]

Q: How do you know it's really a half?
Q: How far have I traveled along my unit? (Demonstrate moving half way across your unit.)
Q: What do I call the length from here *(start)* to here *(midpoint)*? [Move fingers along the paper strip. Students will likely say "one-half."]
Q: One half of what?
[One half of <unit name>]

3. Build Relational Thinking

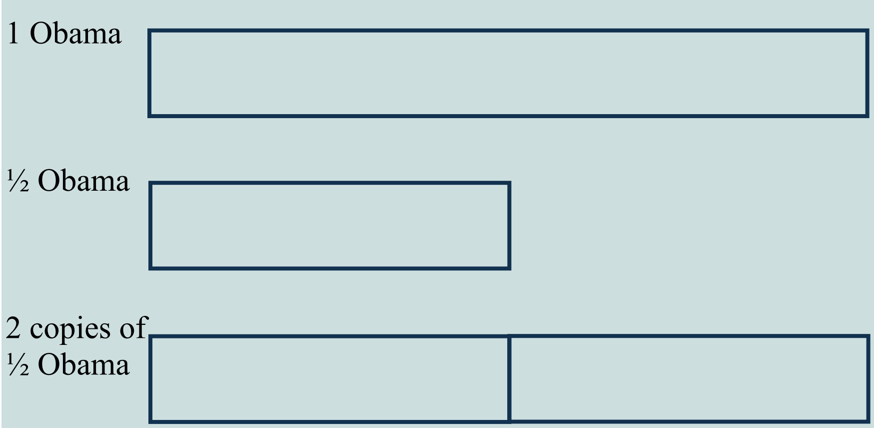
a. Ask:

Q: How many times would you have to copy ½ <unit name> to make 1 <unit name>? (2 times)
Q: 1 <unit name> is how many times as long as one-half <unit name> (2 x)

Student Thinking

The construction "times as long" may seem awkward, but it is a gateway to building reasoning about relations. This is often called algebraic reasoning and multiplicative reasoning. So, please stick with it. If your class seems to be grasping these ideas quickly, you might ask: How many times would I have to copy ½ <unit name> to make 2 <unit name>? *Keep the unit and the copy of the unit separated*, so that students do not think we need only make 1 copy of ½ unit to make 1.

For example:



4. **Write:**

   a. How many one-half <unit name> are in 1 <unit name>?

   b. 1 <unit name> is ? times as long as one-half <unit name>

      Alternate phrasing: One Nina is how many times as long as one-half Nina?"

   c. How many one-half <unit name> are there in 2 <unit name>?

      2 <unit name> is ? times as long as one-half <unit name>

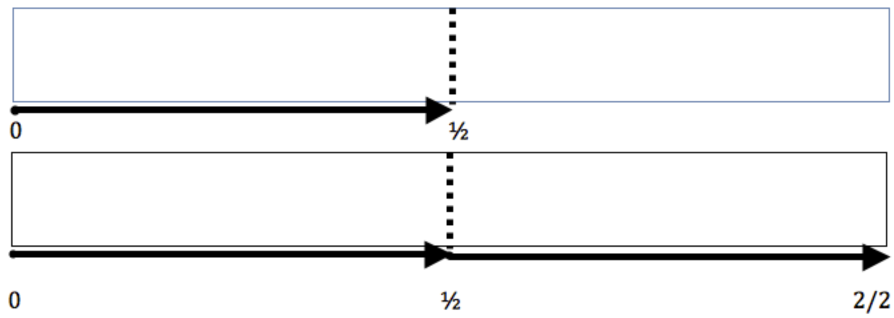
      Alternate phrasing: How many times do you have to copy ½ Nina to make a length that is 2 Nina units long?

   d. Have the students use of iteration to prove the relation (move the half-unit 2 or 4 times) working with a partner. Then have someone come up to the front of the group and prove it for the class. Discuss.

5. **Introduce students to writing fractions: how might we write one-half with numbers?**

   a. Here is how we write so that other people know what we mean: ½. The bottom number tells us how we have *split* our unit—how many congruent parts. *Congruence means that the parts match exactly, so when we put one length on top of another, the distance is exactly the same.* The top number tells us how many copies of these parts we have. So, if we have ½, it means that we have 1 copy of the unit that we have split into two congruent parts, so it should take 2 of them to make one unit.

   b. Review the previous demonstration by iterating ½ twice to make 1 unit. Then emphasize travel: So, when we travel ½, we start at zero and travel all the way to the end of the first part (see illustration on the following page). If we keep traveling, and travel all the way to the end of the second part, we write 2/2. This means that we have traveled 2 of the ½ units.



c. Ask:

Q: Thinking about this ½ unit, what does 2/2 unit mean? *(Notate their ideas as they describe their thinking. At this point, introduce mathematical notation of 1/2 (the bottom number or denominator is the number of splits and the top number or numerator is the number time the unit has been iterated). Begin to press on mathematical relationships and communication through more formal notation. For example: ½ + ½ = 2 x ½*

The goal is to notate what they say. If you have not yet introduced the multiplication symbol, it does not hurt to tell them that mathematicians also use this symbol 2 x ½ to represent the number of times a group is repeated. Make sure that you connect it to repeated adding. The goal is to make connections. Some students will get the notation and start using it. Students who are not ready will not use it. That is okay.

Q: What about 3/2 units? ½ + ½ + ½ = 3 x ½

Q: Working with a partner, travel from the beginning (0) to ½ unit. Now travel another ½ unit. Then travel another ½ unit. How far have you traveled? (½ + ½) + ½ = (2 x ½) + (1 x ½) = 3 x ½

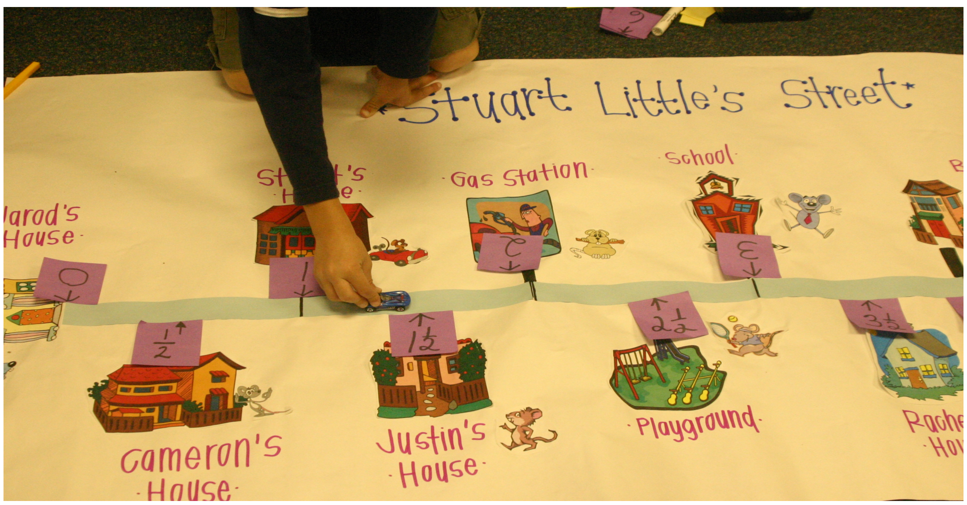
Q: What about if you traveled another ½ unit? How far would that be altogether?

Note. Establish a relation between *(for example)* a measure of 4 units (also written as 4/1) as 4 iterations of a single unit and 3/2 as 3 iterations or copies of ½ unit. ½ means 1 copy of ½, 2/2 means 2 copies of ½, 3/2 means 3 copies of ½, 4/2 means 4 copies of ½. We want children to get comfortable thinking about measurement as iteration of a unit, but at the same time, we want children to start to think about "times as long" to consider relations between splits of units and the original unit. Language such as 3/2 unit is 3 times as long as ½ unit, and 3/2 unit is 1 ½ times as long as 1 unit helps students think about these relations.

Introducing the Unit
Main Street
Partitioning Problems
What Have We Learned?

Main Street – Using Context

Whole Group



6. Explore these ideas using the road map with (Jacobsville, Mousetown or whatever you choose to name your Main Street). Your class will use Main Street as a way to further explore the big ideas of the number line and measurement (zero point, distance from the point of origin, need for partitions (halves and fourths), comparing the relationships between whole units, half units and fourth units, iterating partitions, and identifying congruent distances in context.

You can begin by having them begin with one <name your unit> and establish the landmark at the end the street is at zero. The town starts at the (landmark). Then using a critter or your hand moving down the street, have them identify where one <name your unit> is, two <name your unit> is and mark them on the map and label it.

Extend your discussions by identifying halves on Main Street. Have the students describe different distances using half a unit.

Q: If I start (zero point landmark) and travel by half units to the landmark 2 units away (describe the landmark – do not quantify how far it is away), how many half units will you have traveled? How do you know? How else can we describe how far you have traveled? *Record the notation on the board they describe: "So you are saying your traveled four half units so ½ +½+ ½ + ½ = (or) 4 x ½.*

Q: How else could you describe the distance traveled? Are there other names that you could describe the units? Record these equivalences on the board as you work on Main Street. "*So you are saying that you traveled 2 whole units or 2/1. So are ½ +½+ ½ + ½ = 4 x ½ = 2/1?"*

Note: Repeat this discussion by having the students travel to different locations on the main street. (Continue to use mathematical notation to describe travels.) Make sure to include distances that:

- Are not always traveling from zero point but from other locations (points of origin).
- Are not whole units (i.e. 3 ½ units)
- Are not whole units and travel to another point that is not a whole unit (a half unit location to a different half unit location)
- Travel backwards up or down Main Street (does direction matter). If I am here and travel back, how far have I traveled?
- Travels from a half unit to a whole unit

Partitioning Problems

Introducing the Unit
Main Street
Partitioning Problems
What Have We Learned?

After students have played with exploring concepts on Main Street, move to using a number line to continue to explore the same ideas. You can have students make a number-line on their directly on desks by having them tape 3 – 4 paper strips on butcher paper. Also have one up on the board (using sentence strip units so partitions can be seen easily across the room).

Students work in pairs to solve two different partitioning problems involving one half and then one quarter.

Partner

7. Students work in pairs to solve problems using the number lines on their desk.

a. Present problem 1: How could I make ¼? Post the following questions:

Q: How could we fold so the result is ¼ of 1 unit?
Q: What would we need to think about if we wanted to find a length that is ¼ of 1 unit?
Q: How would we know it is ¼?

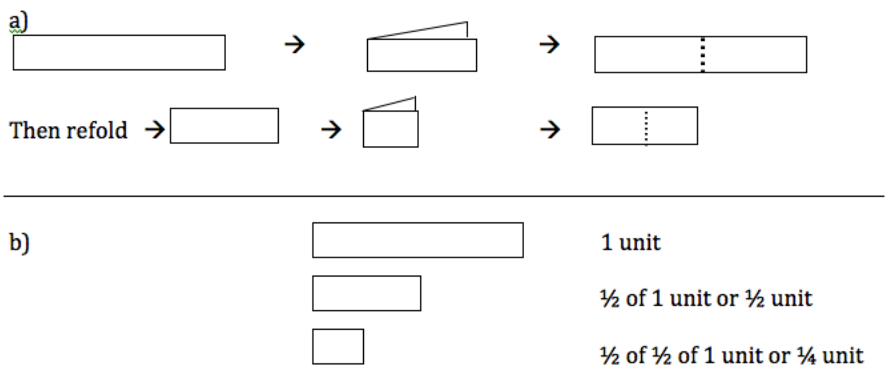
b. Students should discuss the questions with a partner and then use paper strips to test their ideas. Tell students they should be able to explain and demonstrate their thinking so they can convince their classmates they have found ¼ of 1 unit. Have students record their thinking into their math journal.

c. Observe as partners work, and support thinking about strategies. Take notes about student difficulties. Try to get each pair of students to see how they are thinking about fractional parts and labeling those parts, without giving direct instructions.

Note. This problem may or may not be challenging because it asks students to compose splits of a unit: ½ of ½ (of 1 unit). If the students discovered this relationship in earlier units, it will be a matter of re-discussing it. Let students work in pairs to answer the first problem, then return to the whole group. Compare solution strategies (see discussion questions on the next page). One common strategy is to fold the paper in half twice. Be sure to allow children to perform this folding. After the first fold, ask children to name the part of the unit. Review the notion that the unit has been split into two congruent pieces. Then, refold the unit, and ask children to fold the ½ by ½ again. Is it often helpful to tape these successive actions on the unit strip to the board, so that students can see 1 unit (it is always important that students see 1 unit), ½ unit, and ¼ unit. Ask student to unfold their strips and count the number of partitions. Ask how they might say and write the distance traveled between the beginning and end of one of the 4 partitions (¼ or one-fourth). Some students may not be challenged by this problem or may finish it quickly. If so, ask them to try to create 1/8 unit. Depending on the grade level, you can decide if

Introducing the Unit
Main Street
Partitioning Problems
What Have We Learned?

your students are ready to explore eighths or not. If you do, follow the same discussions as you did with halves and fourths.



Whole Group

2. Ask students to display their results for finding ¼ unit.

a. Post the strips that students offer, and ask:

Q: Why do you think this shows ¼ of 1 unit?
Q: How far is it from here (begin at 0) to here (first fold line should be ¼)?
Q: How do you know for sure?
Q: How could we test it?
Q: How did you create the ¼ unit?
Q: How many copies of ¼ unit are needed to make 1 unit?
Q: How can you tell? (Connect iterating ¼ unit 4 times with 4 copies of ¼ unit.)

Note. Students may have used two consecutive vertical folds to find ¼, or they may have made 1 vertical split and 1 horizontal split. If this happens, be prepared to compare and discuss their 2 different methods of folding and the difference in results. Make sure to ask which ¼ unit would best serve measuring *distance traveled*. The vertical and horizontal split will be a good way of measuring area. Students may also present a unit that has been split twice, but they may open it up. This will call for a discussion about how much of the unit is showing and what we could call it (4/4 or 1 unit split into 4 equal lengths). Then compare it to the unit folded to show ¼. Ask: How would we write ¼? Why would we write ¼ that way? What does each part of the symbol mean? Be sure to symbolize this as ½ of ½ of 1 <unit name> (as well as ¼) to emphasize that the symbolism captures what is essential about the activity—even though the result is a different length for differing personal units, the result is the same: 1 is now measured in 4 of these new sub-units. Emphasize again that the part-of-the-unit traveled is one-fourth of the length of the unit and that the unit is four times as long as the part-unit. If students are finding this well within their grasp, ask them to compare 2/4 unit to 1 unit: 1 unit is 2 times as long as 2/4 unit.

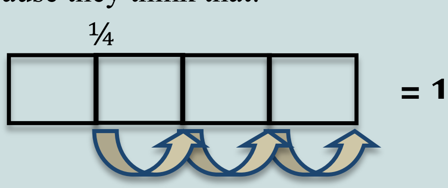
Partner

3. Students work in pairs to solve problem 2 to build relational thinking. Ask:

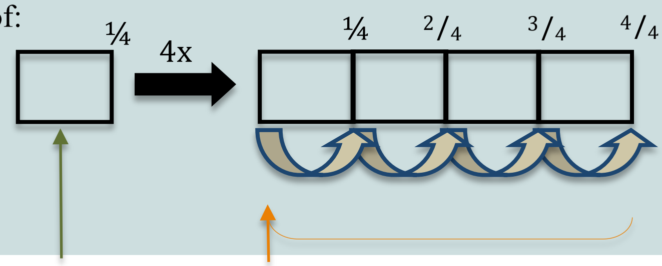
Q: How many copies of ¼ <unit name> do we need to make 1 <unit name>?
Q: 4 of ¼ <unit name> is 1 <unit name>.
Q: 1 <unit name> is ______ times as long as ¼ <unit name>.

Student Thinking

Another way to emphasize the reciprocal relationship between the part-unit and the original unit-length is to ask students to consider how they might recreate the unit-length using the partitioned unit. For example, 1 unit can be recreated by iterating the ½ unit twice; 1 unit can also be recreated by iterating the ¼ unit 4 times. It is important that children see that there are 4 iterations of ¼ to recreate 1 unit. Some may say that it only takes 3 iterations, because they think that:



instead of:



Keep **the unit** and **its copies** literally separate.

We are aiming for the time when students will anticipate that 4 iterations of ¼u *must* result in 1u, 8 iterations of 1/8u *must* result in 1u, and more generally b iterations of 1/b u must result in 1 u. You can help students make this transition by asking them to consider what changes and what stays the same when they re-create the unit starting with one-half, one-fourth, one-eighth of the unit.

Introducing the Unit
Main Street
Partitioning Problems
What Have We Learned?

Whole Group

4. To build equivalence, post the strips that students offer.

a. Ask:

Q: Travel ¼ unit. Now travel another ¼ unit. How far have you traveled?
Q: Travel ½ unit. How far have you traveled?
Q: How is 2/4 unit the same as ½ unit? How is it different?

b. Explain that when we travel the same distance from the beginning (we land at the same place), we call these distances equivalent. So, 1 copy of the unit split into 2 partitions (½) means the same thing as 2 copies of the unit split into 4 partitions (2/4).

Note. For the same unit lengths, equivalence means the same distance traveled, although the split-units might be different. That is, ½ <unit name> vs. traveling 2 of ¼ <unit name>. Clearly, ½ and ¼ are not identical because they represent different partitions of the unit. This lack of identity is confusing to some children, who think that "equal" is a synonym for "exactly the same." This is another opportunity to build relational thinking.

What Have We Learned?

Individual

1. Introduce student writing assessment task.

a. Ask students to spend approximately 15 minutes writing a reflection in their math journals telling how their thinking changed from measuring with foot-strip units and traveling on Main Street or the number line.

b. Tell students you will choose several journal entries to be read to the whole group during the next measurement lesson. Allow students to write unprompted, as this is a nice opportunity to see the topic on which the students focus.

c. Display the instructions on chart paper (or use the page provided in the Appendix). Instructions are as follows:

Write complete sentences telling what you know about measurement. After you finish your reflection, go back and read your previous measurement entries, then make any necessary additions to your reflection. Some questions to consider are:

Q: How is your thinking the same?
Q: How has your thinking about measurement changed from your previous journal entries?
Q: What questions do you still have about measurement?
Q: Include in your reflection: Where do you start measuring? What do you call it?

d. Read through the entries before the next class and choose 3 to share. Write a summary of your three focus students, using the following questions:

Q: How are your students thinking about fractional lengths?
Q: What surprised you?
Q: What, if anything, did they find difficult?
Q: Do students understand the advantages of using a standard unit

Introducing the Unit
Partitioning Problems
Constructing the Tape Measure
What Have We Learned?

Written Response

a. Ask students to spend 15-20 minutes writing a response to the question below in their math journal.

   Draw a picture of Main Street or a number line. Label it to show what you understand about what is important about measurement. Explain your thinking.

b. Read each written assessment and analyze student understanding of measurement in relation to the progress map. Are students making progress? Take notes on the understanding and misunderstandings of your focus students. Use this summary to prepare for Unit 5 demonstration and discussion.