Using Units to Measure Distances (Grades 1-2)

Unit

3

Mathematical Concepts

- A unit of length is itself a length.
- A length measure results from accumulating iterations of a unit length.
- The length of a unit and the resulting measure are inversely related.
- Partial units are obtained by equi-partitioning of a unit length.
- A measure is the distance traveled from zero. (One unit is only one unit after you have traveled from the beginning of the unit to the end of the unit.)

Unit Overview

Students build a connection between using a body part (the foot) as a unit of measure and a paper strip unit to measure a length. After discussion about which aspect of the foot the paper strip represents (its length from heel-to-toe), students measure pre-selected lengths with the paper strip units. The pre-selected lengths should be:

- Measurable with available paper-strip units with a whole-number result
- Measurable with a whole-number result but without enough paper strips to tile the length (to think about iterating the unit)
- Measurable by splitting one paper strip into 2 equal partitions (a length measured as ½ unit)

As a formative assessment, students predict the measure of the same lengths with a unit paper strip that is 1/4 times as long as the teacher's foot (this relationship is not revealed). Students estimate and then check their estimates. Class discussion focuses on the inverse relation between the measure of length and the length of the unit: Smaller unit lengths result in greater measures. Class discussion should also include a focus on the properties of units, and the partial unit of ½.

Contents

Materials & Preparation — Using Units to Measure Distances Unit 3

Read

- ☐ **Unit 3**
  Start by reading the unit to learn the content and become familiar with the activities.
- ☐ **Sample Student Thinking**
  Reread the Student Thinking boxes to anticipate the kinds of ideas and discussions you will likely see during instruction.
- ☐ **Measurement Construct Map**
  Read the construct map and look at the multimedia map to help you recognize the mathematical elements in student thinking, and to order these elements in terms of their level of sophistication.

Gather

- ☐ Student math journals
- ☐ Teacher journal for note-taking
- ☐ Post-it notes for marking units
- ☐ Markers
- ☐ Rubber band (one for demonstration, *see page 5*)
- ☐ String (for demonstration, *see page 5*)
- ☐ Construction paper to copy the teacher's footstep cutouts (about 10 sheets), or a few sheets if using paper strips to represent the foot (*see page 7*)

Prepare

- ☐ Space for students to measure a distance heel-to-toe (*see page 3)*
- ☐ Trace and cut out copy of teacher's foot.
- ☐ Construction paper strips longer than the teacher's foot (*see page 5*)
- ☐ If using paper strips, 10 strips (*see page 5*)
- ☐ At least 3 objects for students to measure (*see page 7, paragraph c)*

Using Units to Measure Distances (Grades 1-2)

Mathematical Background

Partial Units
Units can be partitioned to result in partial units (e.g., ½ inch). For example, 1/n unit is obtained by partitioning a unit length, u, into n congruent parts and representing one of these partitions as 1/n. Iterating the 1/n unit n times restores the original unit. There is a significant difference in student reasoning between a student can anticipate this relation and one who must resort to literal enactment to establish it.

Iteration
A unit of length is itself a length. Units can be iterated and the space left by iterating a unit has already been counted and represents the unit of length. A length measure results from accumulating iterations of a unit length.

Inverse Relation between Unit Length and Measured Length
For the same length, measuring with shorter units of measure results in a greater numeric measure when compared to the measurement obtained with longer units of measure. For example, a two foot length is measured as 2 feet or as 24 inches, because a foot is twelve times as long as an inch.

Walking Feet as a Means for Measurement

Students use movement to measure the length of the classroom with their feet and consider the relationship between their actions and resulting measurements. This builds from the work and the conversations the students had in Unit 2.

Individual/Partners

1. **Introduce the task by asking:**

   Q: If we all walk heel-to-toe like this (demonstrate) from here to there, will we all get the same measurement? Students will likely tell you no because everyone does not have the same sized feet. If they do not, you can get two students (with different sized feet and ask if they will both get the same measurement).
   Q: Why do you think so?

2. **What should we keep in mind when we walk?**

   a. Record student recommendations on chart paper.
   b. Have student pairs walk the length of the classroom.

   Note. The teacher should observe while students are walking. Ask students what they are keeping in mind about measurement as they are walking. Is their path straight? How can they tell? What about the foot is helpful for measuring length?

Whole Group

(This will likely be a review to bring to the front the importance of using some type of standard.)

3. **Compare measurements and discuss possible reasons for the different measures, looking for relationships between the method, the unit, and the resulting measure. Ask:**

   Q: What did you get as measures?
   Makes differences among measures visible

Q: We all walked end-to-end and we still don't have the same measure. Why would that be?
Entrée for conversation about the nature of the unit: why do smaller feet result in larger measures?

Q: What did we use to measure?
Invites conversation about the nature of the unit

Q: How did we use our feet to measure? What is important about how we used our feet?
Invites conversation about how students are thinking about iterating a unit

Q: If we wanted to all get the same measurement, what should we do?
This is an opportunity to talk about the value of everyone using the same length as a unit.

Units Represent Feet

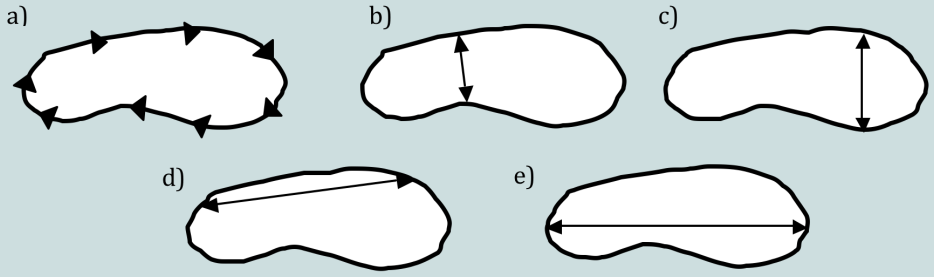
1. Introduce the teacher's foot unit.

Explain that you made a copy of your foot by tracing it, then cutting it out. Ask:

Q: What about my foot is important for measuring?

Student Thinking

It is important to prompt consideration of which attributes of a foot are being represented by the paper strip or string. For example, feet have curvature along the outline and arches. Does the paper strip represent the length from heel to big toe, from heel to little toe? Width? We want to be specific about what the representation (the cut-out of the foot or the paper strip) is representing and why the representation is useful for measurement of a length.



Above: Possible choices that students might make about what to pay attention to on the foot: a) perimeter, b) & c) width, d) & e) length.

Although some students may not choose (e), don't dismiss the other choices. Instead, try to enact their consequences for the measure of the distance from one end of the room to the other—they each could be used but would require a lot of work to use in that way. For example, you might try enacting choice (c) by sidestepping from one end of the room to the other. Another possibility is to cut out strips of paper and string to correspond to choices (a) – (e) and let children experiment by placing them on the footprint. Which do the best job of representing the distance between heel and toe?

2. Prompt a conversation about the important attributes of the foot cutout. Ask:

Q: What could we substitute for my foot? What would do the same job?
[Show a construction paper strip that is longer than the teacher's foot cutouts.]

Q: Could we use a (rectangular) strip of paper instead?

Q: What would we have to do to it so it stands in for my foot? Why?

Q: Does it matter how fat or skinny the strip is? Why?

Note. **Establishing equivalence between a paper strip representation of the foot and the foot is critical.** Units of length measure function like line segments, so it is important that students understand the paper strip stands in for this imagined relation between feet and strips. The aim is to foster representational competence: students need to develop awareness of the purposes and eventually, limitations, of different systems of representation. The strips can represent area as well as length. Hence, it is important to emphasize that length measure involves travel along the edge of the strip. Enacting this aspect of the representation is very helpful. Have students close their eyes and "travel" along the edge of the strip.

Q: If we start at one end and travel to the end of the strip, how far have we traveled? (Have students close their eyes and pretend that they are traveling from the start of the unit to the end of the unit.)

Q: Why wouldn't this piece of string work as well? (Be sure to kink the string a few times.)

Note. Make a decision about substituting a paper strip for the foot as a unit of measure. If the class seems to understand how a strip can stand in for a foot, then give each person 5 strips, each of which is as long as the teacher's foot.

Using Units to Measure Distances

Students measure different distances with paper-strip units.

Pairs

Be sure to have several different distances to measure. To reveal student thinking about iteration, tiling and partial units, have students measure distances that are:

- 4 units long (with enough units to measure to reveal ideas about tiling unit lengths)
- 8 units long (without enough units to measure, re-using units for iteration)
- ½ unit long (partial unit distance)
- 2 ½ units long (this is a more difficult problem in that it requires reconciling the number of units and the distance traveled)

Whole Group

4-unit length: Based on your observations, have one or more pairs demonstrate how they measured the 4-unit length. Ask students to consult the earlier chart about important ideas to keep in mind when measuring and to talk about which of these ideas they are seeing at work. You could also demonstrate a measurement of the 4-unit lengths using 5 overlapping units and ask which important idea you did not keep in mind. You could demonstrate the measurement by using 3 units with gaps, again asking children what important ideas about measurement that you had forgotten. Continue the conversation by using 6 units in a saw-shape, asking if the units are being used to measure a straight distance.

Enact traveling: Have students watch as your fingers travel along the paper strips from the starting point to the ending point of the distance. Then re-enact and ask them to use a finger to show 1 when your fingers have traveled ONE teacher-foot. How about TWO teacher-foot? THREE? FOUR?

Set up a contradiction by saying that someone else said that your fingers traveled one about here (gesture to ½ way along the first unit). Ask students to say what the person might have been thinking (it is the first unit) and why the distance traveled is not yet one.

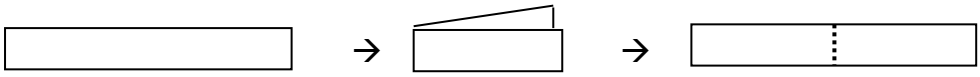
8-unit length: Enact running out of units when measuring the 8-unit length. Based on your observations, ask a pair of students to complete the measurement by re-using the units. Possible questions include:

Q: Can we reuse a unit?
Q: Why did you put your finger there (at the 5 unit mark)? What are you trying to keep track of?
Q: How much of the distance has already been measured?
Q: How many foot units is this item? How can you know for sure?

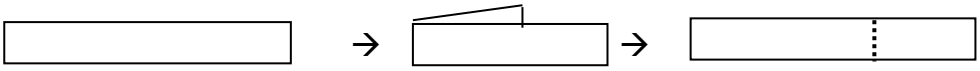
Enact traveling, as before. Be sure to engage students in the measure as a distance traveled, again using fingers to indicate ONE, TWO, etc.

½-unit length: Ask students if the distance is 1 foot, less than 1 foot or more than 1 foot. Align the unit with the distance to provide students with a visual referent.

a. Give each student a foot strip and ask them to fold it so that it is ½ times as long as the foot.



b. Most students will say that there are 2 pieces. Present them with an unequal partition:



c. Ask students to think about what is the same and what is different about the 2 splits. Tell them that one-half foot means that the foot is split into 2 parts that are exactly the same length. We can tell they are exactly the same length because we can put one on top of the other without any leftovers. They both cover one another exactly (they are congruent pieces).

d. Enact traveling. Ask students to unfold the ½ unit strip and to touch one end of the strip. Then ask them to close their eyes and move their finger until they have traveled ½ unit.

2 ½ unit length: If students appear to be comprehending the preceding measures, then ask them to demonstrate the difference between a measure of 3 foot and a measure of 2 ½ foot. Emphasize again the travel metaphor. Have students follow the teacher's fingers as she walks 1 unit, 2 unit, and then ½ more unit for a total of 2 ½ units.

Formative Assessment

The formative assessment, located on the next page, indicates student progress in:

a. Representation: What aspect of a foot is represented by a unit of length measure?

b. Partial unit: How can a partial unit, ½, be established by folding (partitioning) a unit length?

c. Conceptions of Units: How can a new unit of length measure be used to re-measure the same distance? (This affords an opportunity to view how children think about iteration and explain why re-measuring with a shorter length unit results in an increase in the measurement—the inverse relation between unit size and measure.

d. Coordinate partial and whole units to measure a length.

After children have completed the assessment, conduct a conversation about each question. The intention of the conversation should be to clarify (a) – (d) above. It is especially important that children understand how units are accumulated to result in a measure, with attention to what happens when we run out of units, the importance of no gaps between units, and the inverse relation between the length of the unit and the measure of the resulting distance. For partial units, it is critical that children understand that ½ refers to an equal partition of the unit length. This can be established by superimposing one part on the other—it fits perfectly. The other way to check is that if a copy of the unit is set aside, it takes 2 ½ units to travel the same distance.

Assessment — Using Distances to Measure Distances Unit 3

NAME_____

1. Here is a footprint. If we want to use it to measure how long something is, draw with a pencil the part of the foot that we should use to make a unit.

2. Here is a new unit for measuring length. Fold your paper strip so that is one half times as long. How can you tell that it is ½ times as long?

3. Use your new unit to measure some of the same distances that we measured before. What do you notice?

4. How tall is this building?