From Here to There (Grade 1-2)

Unit

2

Mathematical Concepts

- Length measure blends two ways of thinking. The first relies on the principles of a ruler. Measure results from repeated iteration of a unit length. The second relies on motion. The measure of a length represents a distance traveled.
- Measure of a length is obtained by iterating a unit and accumulating the number of iterations of that unit without gaps or overlaps.
- The length of a unit and the resulting measure are inversely related.
- The whole distance must be measured.
- There is a need for a standard unit.

Unit Overview

This unit is designed to give students a common measurement experience, finding the distance between two landmarks, in order to support further discussion about the principles of measurement. The landmarks also highlight length as distance traveled. The landmarks are set far enough apart so that choice of a measuring unit has visible consequences. Now that we have an initial assessment of students' prior knowledge about measurement, our goal is to give them some experience measuring that we can use to facilitate their development of big ideas, or principles of measurement, such as the nature of a unit or the need to iterate a unit. This activity is another opportunity to see how students invent measurement procedures, think about unit, and how they talk about these inventions as they discuss things they considered when they measured. We can use what we have learned from the previous assessment to guide our observation of students' activity and to inform instruction.

The results of the measurements give students the opportunity to think further about the relation between the unit measure (the accumulation of units) and the relative length of the unit (longer feet result in smaller quantities). This continues to create the need for standardization—agreeing about sharable units and methods of measure.

Contents

Materials & Preparation

Read

- ☐ **Unit 2**
  Start by reading the unit to learn the content and become familiar with the activities.
- ☐ **Mathematical Background**
  Reread the mathematical background carefully to help you think about the important mathematical characteristics within the unit.
- ☐ **Sample Student Thinking**
  Reread the Student Thinking boxes to anticipate the kinds of ideas and discussions you will likely see during instruction.
- ☐ **Measurement Construct Map**
  Read the construct map and look at the multimedia map to help you recognize the mathematical elements in student thinking, and to order these elements in terms of their level of sophistication.

Gather

- ☐ 2 flags or cones (or find a location with consistent distances for the students to measure)
- ☐ Teacher math journal (for note taking)
- ☐ Comparing Measurement Methods (page 10)
- ☐ Student math journals
- ☐ Chart paper and markers

Prepare

The first part of this lesson requires a large open space. Identify a large enough space (playground, field, gym, hallway) in which students can work to measure the distance between cones, flags or landmarks.

It helps students to think about a purpose for measuring and the importance of getting a specific, repeated measure. It is a critical step to help them make meaning of the need for rules of measure in order to get a consistent measurement otherwise, any measure will do. (i.e. We need to replace a piece of carpet or a pane of glass in our window, or make a sandbox the specified length.) Think about a context related to your school that students could connect to.

Mathematical Background

The big ideas of measurement emphasized in this unit are related to the nature of a unit.

Attribute-unit relation
An object can have multiple attributes, and some units may be more suitable than others for measuring different attributes or even different magnitudes of the same attribute (e.g., a longer distance might be measured more readily with a longer unit, or an area is measured with a 2D unit).

Iteration
A unit is repeatedly applied to obtain a measure. For example, a measure of length is obtained by moving (translating) a unit-distance a finite number of times. The measure is the number of iterations of the unit (e.g., a 9-inch length is measured by translating an inch from the beginning to the end of the length 9 times).

Identity
The units must be identical in order for the iteration to yield a single measure.

Tiling
The units are translated "end-to-end" with no gaps. Units tile the line, plane, volume, etc.

Inverse Relation between Unit Length and Measured Length
For the same length, measuring with shorter units of measure results in a greater numeric measure when compared to the measurement obtained with longer units of measure. For example, a two foot length is measured as 2 feet or as 24 inches, because a foot is twelve times as long as an inch.

Straight
When is a line straight? Often, when we measure a distance between two points, we imagine a line. A traditional way of thinking about a straight line is as the shortest distance between two points. But a way that is more consistent with bodily experience is to consider as straight a line formed by moving without any change in direction. For instance, walking at a constant heading towing a piece of chalk ideally creates a straight line.

Measuring the Distance Between Two Landmarks

Students use their bodies (such as footsteps) or other tools that they invent (e.g., some think to use notebooks, others pencils) to measure the distance between two landmarks.

Partners

1. **Take students to a large open space such as a playground or a field.**

    a. Set up two flags (or cones) at opposite ends of the space, making sure they are set far enough apart to make measurement challenging, so that different choices of units have consequences that are visible. (This distance can be any location as long as students have a set distance to measure. It is a good idea to have the students spread out so that you get a variety of strategies.)

    b. Ask students to work with their partner to find out how far it is from one flag to the next (or from one landmark to another). Mention they can try out anything they like or use any tool they like, except for rulers. Some students will walk heel-to-toe, others will stride, hop, etc. Some may use non-standard tools, such as pencils, clipboards, and lanyards. It is important to not allow students to use rulers or other standardized measuring tools.

    c. Take notes about what your students do as they measure in your math journal. Look for examples of students applying or not applying the big ideas of linear measurement from page 3.

Individual

2. **Ask each student to record his or her measurement in his or her math journal.**

    a. Ask students to record in their math journals how they measured the distance. Ask them to include a drawing.

    Note. Some students may draw a picture of themselves enacting a measure, others may choose to be more schematic and just show the unit used.

Teacher Reflection and Planning

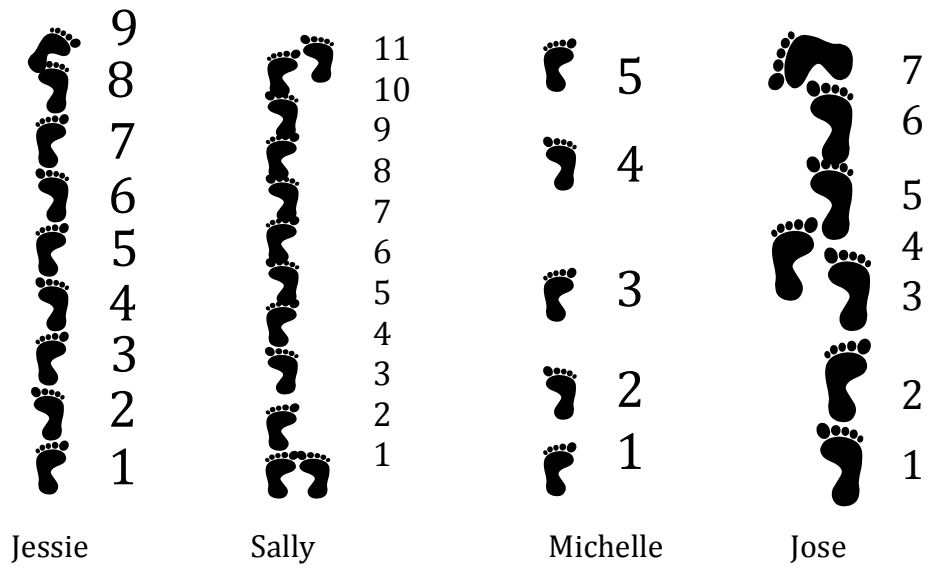
3. **Read student summaries and your observation notes about students' measurement methods.** (Select three methods to use in the classroom discussion and record on Comparing Measurement Methods worksheet.)

Note: Think about the students' actions and comments during the activity to help you plan a conversation. (see the Comparing Measurement Methods WS). Questions to guide your planning could include:

What big idea about measurement do I want to start with (suitable units, identity, iteration, tiling, inverse relation between unit length and measured length, the meaning of straight)?

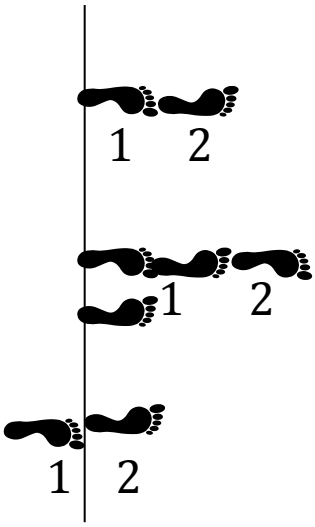
What did students do that tells you about how they were thinking about identical units and or tiling? (Did they leave gaps? Did their strategies indicate that they value identical sized units? How did they count the last unit if it was a partial unit?

Student Thinking Some students will think of using their feet. They often do so in ways that reveal their thinking.



Jessie Sally Michelle Jose

Measuring the full length or distance*



Are they counting all the space? Where do they begin their count? Before the measure? Skip the first space? Or do they start where the first footstep begins?

How are they describing the end point? Are they rounding up or partitioning the last unit? (There will be lots of different responses. Keep track of what they are but more emphasis will be placed on this big idea in unit 3.)

- *"Using Children's Understanding of Linear Measure to Inform Instruction" Teachers engaged in Research: Inquiry into Mathematics Practice Grades K-2* (D. S. Mewborn, series ed.). Reston, VA: National Council of Teachers of Mathematics.

Things We Think About When We Measure

Students reason about measurement principles based on their experience with measuring the playground.

Whole Group

1. Elicit Student Ideas about Measuring the Distance.

a. To open up the discussion, ask the class: "What did we think about when we measured?" First, listen to the directions in which students take the conversation and then consider questions you prepared to support further discussion.

b. Conversation starters include:

Q: What did we do?
Q: What were your measures? (Record on board or chart paper.)
Q: Why did we all get different measures?
Q: What were some important things you did when you measured?
Q: Would someone else get the same measurement if they did what you did?
Q: Someone said that they measured the straight distance. What did they mean by straight?

c. Using the planning sheet, ask three pairs to present their methods. As each pair presents their method, ask other students to point out one positive feature of the method and one thing that might be improved so that someone else could use the method. For example, when using their bodies, it might be important for students to tell other students to walk heel-to-toe and to keep moving without changing direction—this will keep the path straight. (It is sometimes helpful for the teacher to enact a method that changes direction, so that this tacit assumption of "straight" can be made explicit.)

d. Following presentation, compare and contrast methods and try to highlight important ideas, such as iteration, tiling, accumulating units by counting the number of iterations, identical units, and the suitability of the unit for the task. Ask:

Q: Could other people do it easily?
Q: What if we used a pencil to measure—would it work? Would you want to be the one using it?
Q: What happens when we measure the distance with a small unit, like a paper clip?

2. Summary

Use chart paper to summarize what students have learned about what they need to keep in mind when they measure. Be sure to label big ideas, such as "no gaps" and "keep using the same unit"

3. Formative Assessment

The teacher demonstrates using pencils to measure the distance between 2 points that are visible to everyone in the class. She asks children to tell her whenever they disagree with how she is measuring or to write about each mistake that she makes in their journals. Mistakes that a teacher could enact (depending on how the previous conversation went) include leaving gaps between the pencils, using pencils of different lengths, and stopping the measurement when she runs out of pencils.

Comparing Measurement Methods

Summary of what students did:

Learning Goals:

Method/Students: ____________	**Method/Students:** ____________	**Method/Students:** ____________
Ideas to Focus on	Ideas to Focus on	Ideas to Focus on
____________________	____________________	____________________
____________________	____________________	____________________
____________________	____________________	____________________

Connections 1/2:

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Connections 2/3:

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Connections 1/3:

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