Unit 5

Thinking About Scale

Overview

Students compare their personal tape measures to consider issues of scale, including the origin of scale and how to construct a scale so that markings on the scale are aligned with distance traveled. After considering their constructions, they compare four different exemplar-solutions to scaling with an eye toward deciding which is better for determining ¾ and 1 ¾ measured lengths quickly and easily.

Materials and Preparation

- Students' personal tape measures constructed during Unit 4
- Contrasting tape measures located in the Unit 5 Appendix: prepare as either an overhead, individual student copies, or both

Part One: Introducing the Unit

1 – Whole-group discussion about "lessons learned" → **2 –** Partners and groups consider ways of labeling tape measures

1 Whole-Class Discussion

Introduce the task by asking students to summarize their journal reflections or by choosing the journals of 3 different students for a discussion of "lessons learned" by constructing the second tape measure. This should be relatively free-form and make contact with today's activity.

This is an opportunity to revisit the importance of identical units, the "no gaps" principle, the origin of measurement as zero, the need to iterate units when one "runs out," and the utility of partial units when the measure is not a whole number.

Part One: Introducing the Unit

2 Small Group or Partner Work

Display all tape measures so they are visible to the class (or select a few with contrasting ways of labeling the scale). Provide 10 minutes of "think time" and have the members of the class work in pairs or table groups to consider the following:

- How do tape measures help a person measure length?
- Are these differences in how the tape measures are labeled? *(See Appendix for more information.)*
- How easy would it be to find a length that was 3 ¾ personal units using the tape measures? How about ¼ personal unit?
- Is it possible to begin a measurement some place other than the beginning of the tape? How would that work?

Teacher Note

Some teachers assign each small group one question to ensure that all are considered. The aim of the questions is to uncover how students are thinking about (a) labeling units, and (b) about the zero-point or origin of the scale.

Part Two: Comparing Different Scales of Measurement

1 – Individual work: students compare tape measures → **2 –** Whole-class discussion: students share which tape they preferred → **3 –** Individual work: Using Tape C to address zero-point → **4 –** Whole-class discussion: share solutions

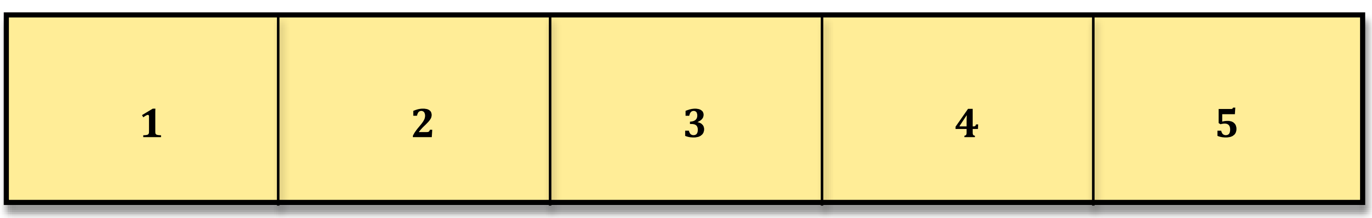
1 Individual Work

Students compare four different 5-unit tape measures. Each tape measure uses a different system for labeling units (master copies of each tape are in the Appendix). Students compare the tape measures (called A, B, C, D) and choose which they believe is best, and write about why they think so in their journal.

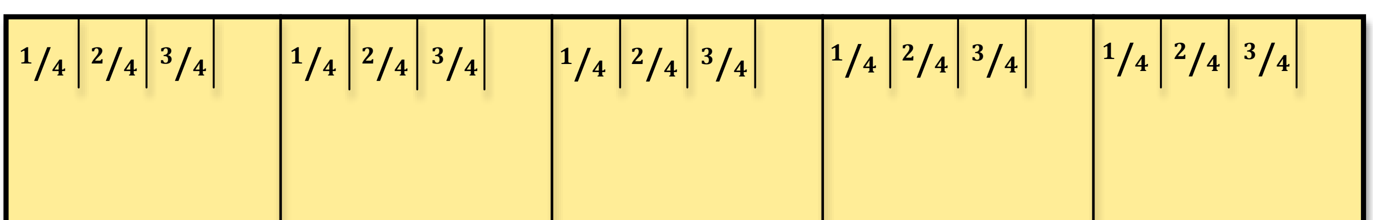
Teacher Note. . .

The tapes are designed to support students' consideration of the role and functions of numeric symbols in systems of measure.

Tape A has 5 equal partitions but labels each partition in the interior. This makes it difficult to know what is meant by "1" or "2." Moreover, the units are not split, making measurements of lengths difficult if the object does not correspond exactly to whole numbers of units.

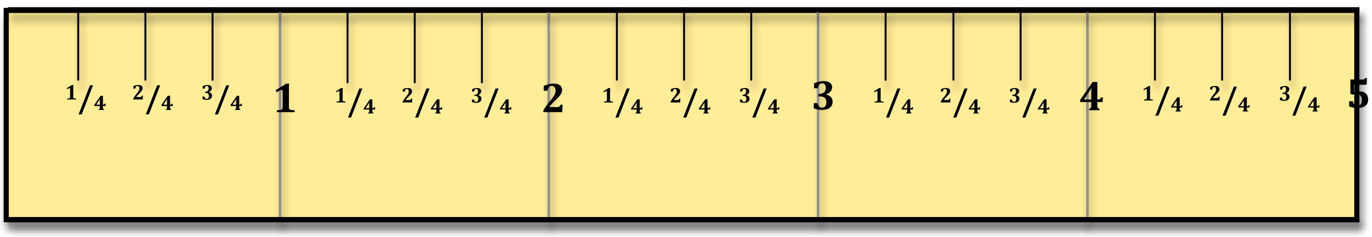


Tape B has 5 equal partitions, and each partition is further split into fourths. But the labels are ambiguous, so it is difficult to know the length of "3/4" or any other split-unit. The whole number units are not labeled.

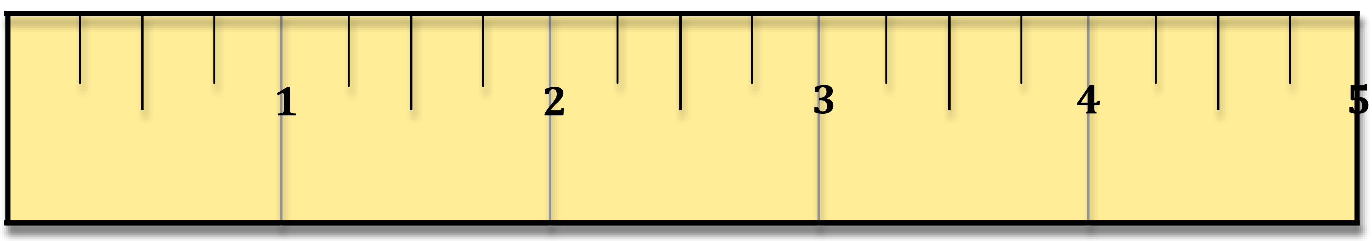


Part Two: Comparing Different Scales of Measurement

Tape C labels whole units and split units so that the label corresponds to a distance traveled.



Tape D labels whole units only, splits the units, and signifies ½ and ¼ partitions differently. It relies on user knowledge of the meaning of the split unit. This tape is closest to those that we use conventionally.



2 Whole-Class Discussion

After students write in their journals, the teachers asks selected students to share which tape they preferred and their reasons for their choice. Questions that will promote discussion are:

Which is best for finding a distance traveled (indicate with hands) that measures 3/4 unit? Why?

The ruler with the interior label makes this distance ambiguous.

Which is best for finding a distance traveled (a length that is as long as) of 1 ¾ units? Why?

Either the explicitly labeled tape measure or the implicitly labeled tape measure will do.

What do you think the people who made tape D had in mind? What do you think these people assumed you already know about measurement?

This ruler is closest to standard practice—but it relies on the users knowing the "rules."

Part Two: Comparing Different Scales of Measurement

3 Individual Work

Students use tape C to answer the following questions (each addresses zero-point). *See Appendix for handout of questions.*

- What does this ruler consider 1 unit? How do you know?
- On tape C, if you started at 1 and traveled 1 unit, where would you wind up? How far have you traveled?
- On tape C, if you started at 2 and traveled 1 unit, where would you wind up? How far have you traveled?
- On tape C, if you started at ½ and traveled 1 unit, where would you wind up? How far have you traveled? How do you know?
- On tape C, if someone begins at 0 and travels 5/4 units on the ruler, where do they wind up? (Scaffold only if necessary by counting distances of ¼ at a time of by discussing what 5/4 means—composition of 4/4 and ¼. Make a mental note of more advanced counting strategies like jumping ahead by 4/4, or 1 unit, and then adding another ¼ unit, rather than counting by fourths to 5/4.)

4 Whole-Class Discussion

Students share their solutions. Stress the distance traveled as the measure of length (have students use their fingers and "travel"). The measure of the distance traveled is the difference between ending and starting points. Have students travel to 5/4 by asking them to travel ¼ at a time, and then travel another ¼ past their rulers. Subtraction is an operation we use to find differences. For example, a question to pose to students might be: If I start at ½ and travel until I reach 4, how far have I traveled? (4 – ½ - 3 ½)

Appendix

Teacher Note: Unit Labels

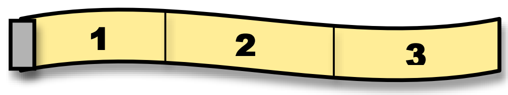
Tape Measures A, B, C, D

Handout, Tape C Questions

Teacher Note: Unit Labels

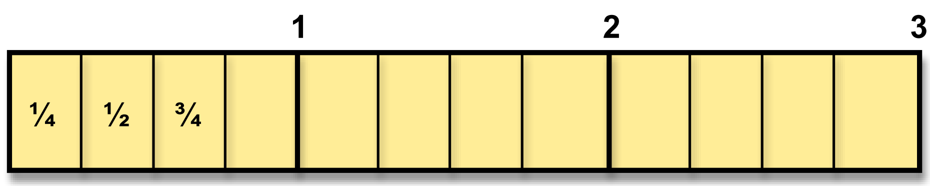
(See Part 1, Section 2 – page 2)

Many students take the view that the unit label should be in the interior of the partial units. This kind of labeling makes the indication of distance problematic. For example, in the tape measure below, what distance corresponds to 1? The marking does not indicate distance traveled unambiguously.



It is important that students consider the position of the label, partly because this is an important issue in mathematical symbolization (a marking should have a single referent, not multiple referents) and partly because it often indicates that the constructor of the tape measure is thinking about length measurement as counts of units rather than as a continuous distance traveled.

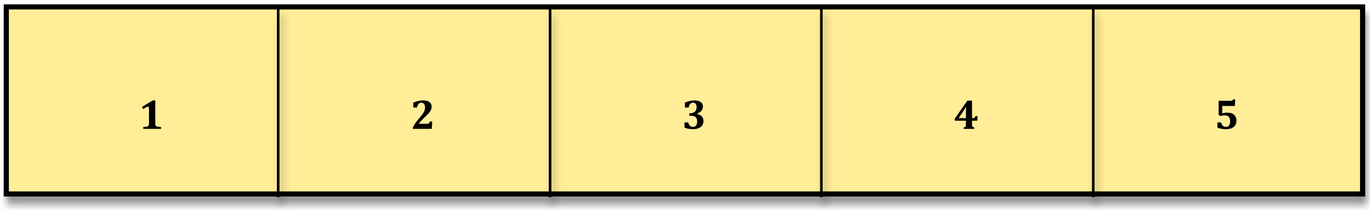
A similar issue arises when considering partial units. For example, if a unit is split into fourths and each is labeled in the interior region—a labeling of parts rather than of distance, where is a length corresponding to ½ unit? Is it at the end of the ¼ unit? In the middle between ¼ and ½? At the end of the ½ boundary?



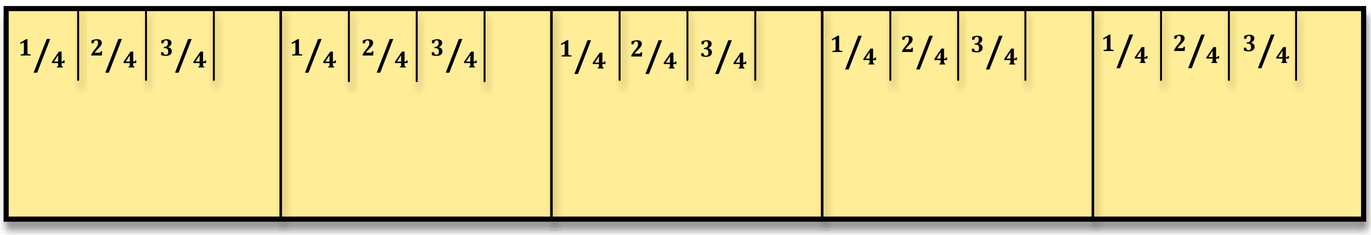
Some students associate the beginning of the measurement with the first unit counted. So measurement begins at 1 for these students. This is another manifestation of not attending to distance traveled when reasoning about length measure. Instead, the student focuses only on the number of units covering a length.

Many students do not understand that any point on the scale can serve as the zero-point. This means that if the measurement starts at 2 units and ends at six units, they respond that the measure is 6 units, instead of 4 units. Knowing and symbolizing the start of measure as zero is an important first step. Eventually, we want students to know that any number can "stand in" for zero, but when it does, the student needs to compensate (usually by subtracting the starting value from the ending value).

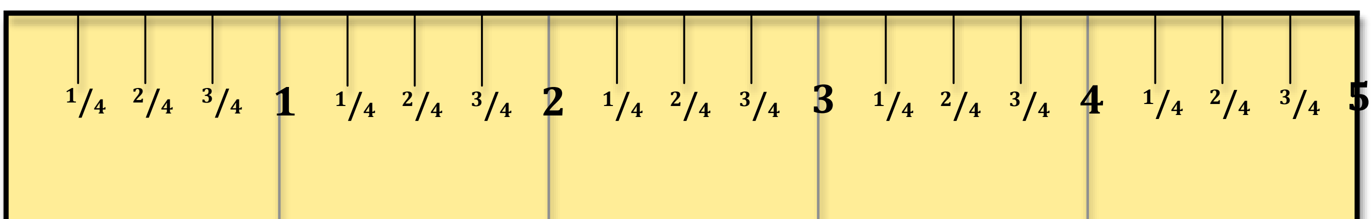
Ruler A



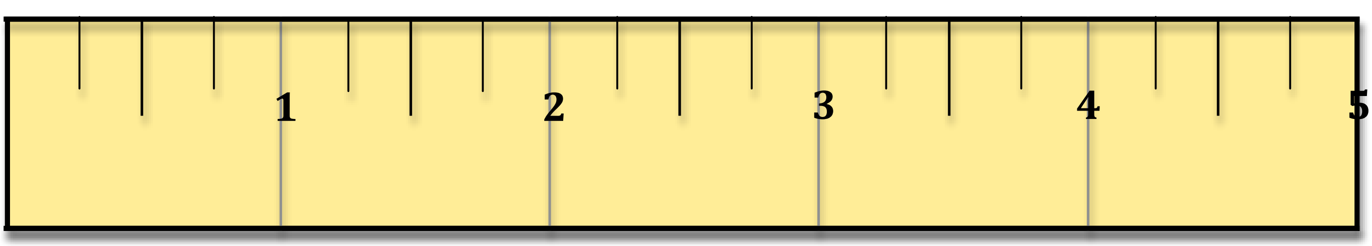
Ruler B



Ruler C



Ruler D



Name ____ Date________________________

Part Two – Individual Work

What does this ruler consider 1 unit? How do you know?

On tape C, if you started at 1 and traveled 1 unit, where would you wind up?

How far have you traveled?

On tape C, if you started at 2 and traveled 1 unit, where would you wind up?

How far have you traveled?

On tape C, if you started at ½ and traveled 1 unit, where would you wind up?

How far have you traveled?

How do you know?

On tape C, if someone begins at 0 and travels 5/4 units on the ruler, where do they wind up?