Unit 3

Representing Our Actions

Students build a connection between using a body part (the foot) as a unit of measure and using these units to create their own measurement tool. Students work in pairs to measure the length of their classroom in footsteps. Following discussion about which aspect of the foot allows it to serve as a unit of measure, students either use cut-outs of their feet or a paper strip that stands in for the student's foot to make a 5-foot tape measure by gluing units to paper tape. Using their measuring tape, students measure objects preselected to pose problems about leftover parts of units (partitioning—a gateway to fractions), and repeated use of a unit if the measuring tool is too short (iteration). The results of the measurements give students the opportunity to think about the relation between the unit measure (the accumulation of units) and the relative length of the unit (longer feet result in smaller quantities.) This creates the need for standardization--agreeing about sharable units and methods of measure. The unit concludes with an assessment that reveals student thinking but also provides an opportunity for further instruction, especially about the meaning and placement of numeric symbols on the measuring tape by the student-designers.

Materials

- Open space to measure (space large enough for students to get different measures, such as length of the room)
- Adding machine tape
- Glue/glue sticks
- Thin felt-tip markers
- Scissors
- Math journals
- Cut-out of teacher's foot
- Rubber band
- String
- Mal-ruler (sample included in lesson)
- "Where do the marks go?" ruler (see section on Assessing Thinking)

Planning Time

Part one: 1 class period
Part two: 2 class periods
Part three: 2 class periods

In this lesson. . .

Part One: Reviewing heel-to-toe walking as a means for measurement

Students use movement to measure the length of the classroom and to begin consideration of the relationship between their actions and resulting measurements.

Part Two: Constructing a footstrip tape measure and using it

Students begin to represent the action of measurement with foot units as they discuss measurement principles.

Part Three: Formative assessment

Principles of measurement are represented on the footstrip tape measures. Traveling—moving—along units of measure supports conceiving of length as a path which can be measured by iterating (moving) a unit.

Part One: Reviewing Pacing as a Means for Measurement

1 – Individual prediction and partner work → **2 –** Whole-group discussion → **3 –** Student journal writing (individual) → **4 –** Teacher reflection

Task: Students measure the length of the classroom with their feet and consider the relationship between their actions and resulting measurements.

1 Students work with a partner to measure the width of the classroom using their feet, walking heel-to-toe. Prior to measuring, ask students "If we all walk heel-to-toe like this (demonstrate), will we all get the same measurement? Why do you think so?" Ask students to record in their math notebook their prediction of how many of their feet it would take. After predictions are recorded, each student measures the width of the room with their partner, walking heel-to-toe, one partner at a time. Students record their measurements on post-it notes (one per student) and put them on the board. **Teacher role during partner work:** Teacher roves while students are walking, observes student activity, and asks students what they are keeping in mind about measurement as they are walking. Is their path straight? How can they tell? What about the foot is helpful for measuring length?

2 In whole-group discussion, compare measurements. Discuss possible reasons for the different measures, looking for relationships between the method, the unit, and the resulting measure.

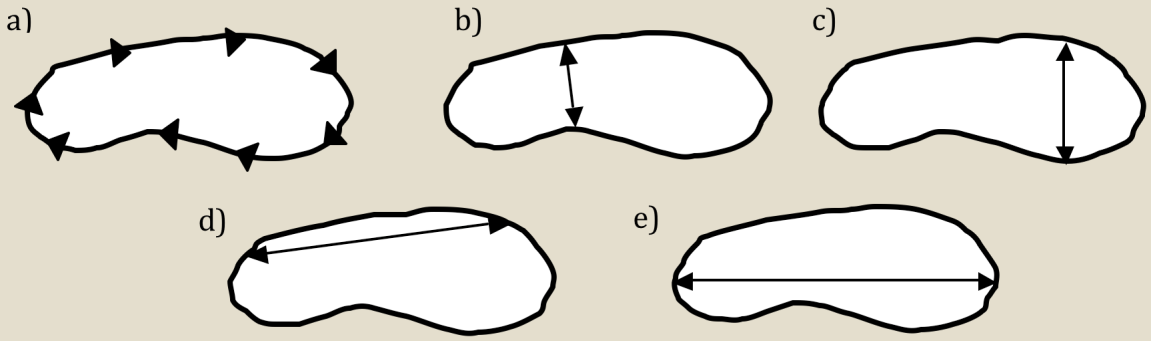
Discussion Questions

- **What did you get as measures?** (Makes differences among measures visible)
- **We all walked end-to-end and we still don't have the same measure. Why would that be?** (Entrée for conversation about the nature of the unit. . .why do smaller feet result in larger measures?)
- **What did we use to measure?** (Invites conversation about nature of unit)
- **How did we use our feet to measure? What is important about how we used our feet?** (Invites conversation about how students are thinking about iterating a unit)
- Introduce the teacher's foot unit, explaining that you made the copy of your foot by tracing it, then cutting it out. **What about the foot is important for measuring?** (Focuses attention on which attributes of the foot let it serve as a unit. *See teacher note on the following page for examples of student thinking.*

Part One: Reviewing Pacing as a Means for Measurement (continued)

Teacher Note

It is important to prompt consideration of which attributes of a foot are being represented by the paper strip or string. For example, feet have curvature-along the outline and also arches. Does the paper strip represent the length from heel to big toe, from heel to little toe? Width? We want to be specific about what the representation (the cut-out of the foot or the paper strip) is representing and why the representation is useful for measurement of a length.



Above: Figure 1. Possible choices that students might make about what to pay attention to the foot: a) perimeter, b) & c) width, d) & e) length.

Although some students may not choose (e), don't dismiss the other choices. Instead, try to enact their consequences for the measure of the distance from one end of the room to the other (They each could be used but would require a lot of work to use in that way. For example, you might try enacting choice c by side-stepping from one end of the room to the other.

Discussion Questions (continued)

Prompt a conversation about the important attributes of the cut out of the foot, by asking students if we can substitute other things for feet:

- What could we substitute for your foot? What would do the same job?
- Show a construction paper strip that is longer than the teacher's foot cut-outs. Could we use a (rectangular) strip of paper instead? What would we have to do to it so that it stands in for someone's foot? Why? Does it matter how fat or skinny the strip is? Why? **(Establishing an equivalence between a paper strip representation of the foot and the foot is critical.)**

Part One: Reviewing Pacing as a Means for Measurement (continued)

Teacher Note

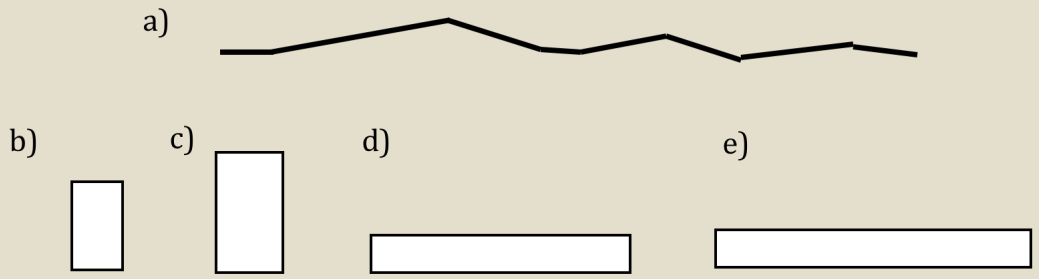
Units of length measure function like line segments, so it is important that students understand the paper strip stands in for this imagined relation between feet and strips. The aim is to foster representational competence: students need to develop awareness of the purposes and eventually, limitations, of different systems of representation. Hence, conversations about what the strip represents matters for the measurement of length is important. The strips can represent area as well as length. Hence, it is important to emphasize that length measure involves travel along the edge of the strip. Enacting this aspect of the representation is often very helpful. Have students close their eyes and "travel" along the edge of the strip.

Discussion Questions (continued)

How about using:

- String?
- Thread?
- Rubber bands? (Stretch a rubber band if there is no objection. This helps focus attention on rigid units.)

Teacher Note



Above: Figure 2. Paper strip choices that correspond to attribute choices in Figure 1. Choice a) is made of string.

Make a decision about substituting a paper strip for the foot as a unit of measure. If the class seems to understand how a strip can stand in for a foot, then give each person 5 strips. Each student cuts each strip to represent her or his foot. If this substitution appears too difficult for your class, or if you prefer to bridge more explicitly between using one foot and using a representation of a foot, then let each student trace their foot on construction paper to make 5 paper cut-outs of their feet. (Or let some students use the cut-outs and other the lengths.) Ask students to work in partners to trace each others' foot (class must decide if shoe will be on or off), cut them out, and label them with the student's name.

Part One: Reviewing Pacing as a Means for Measurement (continued)

3 After the discussion is complete and students have traced and cut out their feet, have them copy the class measurements listed on the board and answer the following question (it helps students record questions in their journals): **Why didn't we all get the same measure?**

4 **Teacher reflection:** Write a short summary about your impressions of the discussion. Read and summarize your focus students' journal writing.

Prepare

- ☐ If students have not made sufficient copies of each foot, make 4 or 5 photocopies of each student's foot before the next part of the lesson. . .

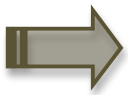
  - or -

  Prepare paper strips that will serve as substitutes.

- ☐ Prepare *Mal-Ruler* (See Part 2: Assessing Thinking, Points 1 & 2)

- ☐ Prepare *Where Do the Marks Go? Ruler* (See Part 2: Assessing Thinking, Points 1 & 2)

Part Two: Constructing and Using a Footstrip Tape Measure

1 – Creating and using tape measure (individual)  **2 –** Whole-group discussion

Task: Students begin to represent the action of measurement with foot units as they discuss measurement principles.

1 **Construct tape measure.** Each student cuts out the copies of his or her foot and constructs a ruler by gluing their foot units to adding machine tape. *If* students agreed in the last part of the lesson that rectangular construction paper strips could be used in place of their feet, each student constructs a ruler by gluing thin rectangular paper strips to machine tape.

Label tape measure. Students label the beginning of the tape measure and the units **however they like.** (Student choices often reveal thinking about counting rather than focusing on distance traveled. Hence, the labels for each unit are often in the center of the units, making it difficult to coordinate the numeric label with a measure. For example, if "2" is located in the center of a unit, if the endpoint of the length falls at the beginning of the unit, is that 2?

Use tape measure. Students measure objects in the room that will produce (a) approximately whole number units; (b) the need to use parts-of-units, and (c) the need to re-use the units, because the length is greater than that of the ruler. Choose (at least) 5 lengths for students to measure and record in their journal. Ask students to think about how they will organize their measurements so they can later compare their measures with a partner and report the measurements to the class. Let them come up with organization strategies on their own or with a partner.

Teacher Note

Rove the class while students are building their measuring tape. Observe how students (including your focus students) are constructing their ruler. Take notes that help you support structuring exampleas, counter-examples, and mathematical arguments about the construction of the ruler and using it to measure.

Part Two: Constructing and Using a Footstrip Tape Measure

2

To conduct a whole-class conversation about building the measuring tapes and using them to find the measures of lengths of the objects you selected, the following structure may be useful:

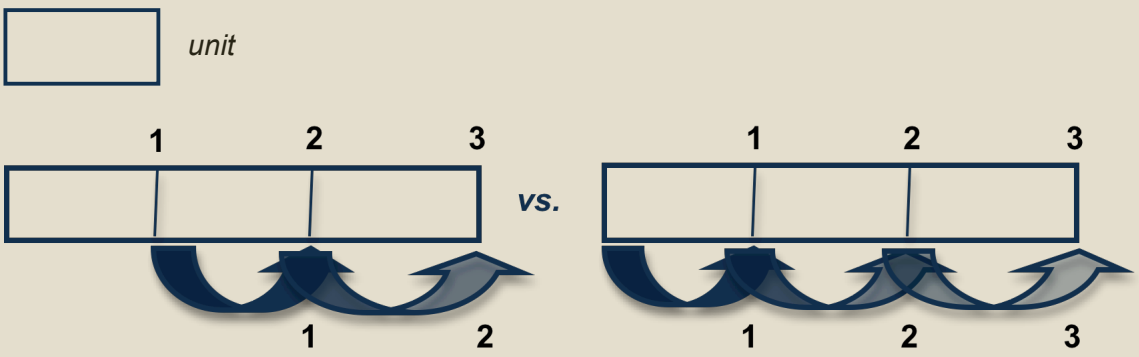
- Make a list of issues that children noticed or experienced; add your noticings to the list.
- Choose one issue that arises frequently, such as "where do you start?" or "what do you do when the object's length is longer than the tape measure?"
- Choose two children with different methods or solutions to demonstrate how they met that challenge. This promotes opportunity for mathematical argument.

If children continue to disagree, write down one or two different positions. For example, some students may think that measurement begins with 1, and others with zero. Then, a way to resolve this issue needs to be determined. For example, if a child believes that measurement starts with one, ask how far they have traveled at the starting point. Most children will say nothing (as in they've not traveled anywhere)—and we can then agree to signify this as zero. **The following issues should be raised, unless your observations of students suggest it will not be fruitful:**

- How did you place your units? Why did you do that?
  This often raises issues related to tiling or space-filling, gaps and overlaps on the ruler, even when there were no gaps or overlaps when they paced heel-to-toe.
- How did you measure? What was involved? Why did you do it that way?
  Try to promote an image of iteration as copying a unit n times. Be sure that you hold the unit apart from its copies. Enact at least 2 iterations of whole numbers of units so that everyone can see what you are doing.

Teacher Note

Sometimes, children think of iteration as starting at 1, **if the copy of the unit is not held apart from the unit being copied.** For example, a measure of 3 units is thought of as the original unit and two more, rather than as starting at the beginning of the length and iterating the unit three times. As a result, there is a breakdown between the number of iterations of the unit and the accumulated measure. From the child's perspective, the unit has been iterated twice but the measure is three.



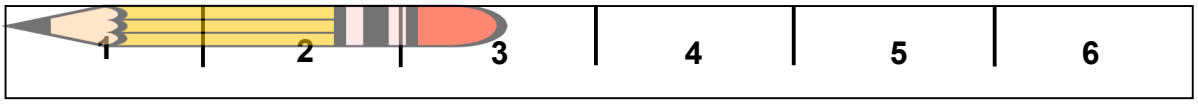
Part Two: Constructing and Using a Footstrip Tape Measure (continued)

Issues to raise (continued)

- What did you do if the length being measured was longer than your ruler? Why did you do that?
  This raises re-use or iteration of a unit.

- What did you do if the length being measured included a part of your foot-unit but not the whole footunit?
  This question helps create the need for partitioning units.

- How did you mark your ruler? Why did you mark it in that way?
  Students often make rulers in ways that show counts rather than distance traveled.



Above: Ask, **"Is this 3?"**

- Where did you begin to measure? What did you label that, and why?

- Let's pick one of these (choose an object). What were some measures of this length? Why were they different?

Teacher Note

Be sure to pick someone with a smaller foot and someone with a larger foot to make the comparison clearer. If you can't locate a foot big enough to reveal a difference, introduce a giant foot (or your foot) and then use that as the comparison. This question draws attention to the inverse relation between the length of the unit and the measure of the resulting length. One needs to iterate a smaller foot more than a longer foot to travel the same distance. It may help if the teacher can discuss *and* demonstrate using a bigger-foot tape measure and smaller one, getting a couple of children to actually measure the object, and then put up the measures beside those foot rulers, so they can really see and connect the size of the unit with the measurement result. The comparison also highlights the benefits of a standard unit.

Part Three: Formative Assessment

1 – Assessment tasks → **2 –** Student writing (individual) → **3 –** Teacher reflection

Assessing student thinking during instruction forms a solid foundation for changing, elaborating or simply continuing a lesson, as needed. Here we note some of the tasks and situations developed by teachers to assess student reasoning and to support its development. *These may be used as whole group assessment questions in a second part of the discussion. Make sure to take notes on individual focus student responses as representatives of the 3 levels of students. You may choose any of the following (or invent a new one).*

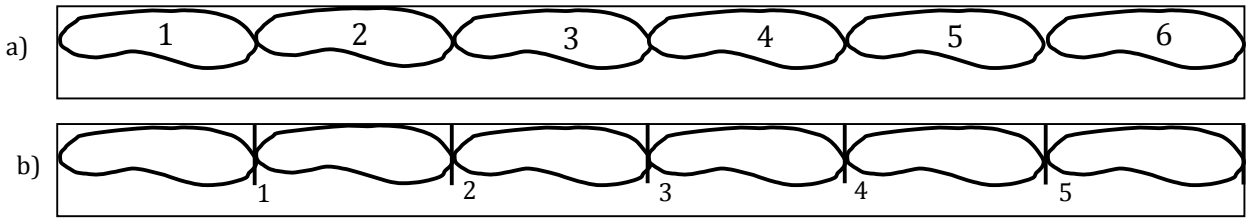
Mal-ruler

Create a mal-foot-strip ruler with units that are not identical and with uneven spaces between units. Measure several objects with the ruler and report the results to the class. Ask students to comment on what they see as potential difficulties with the mal-ruler. Ask students to propose remedies and rationales for these remedies. Be careful not to simply accept convention as a rationale (e.g., do not simply accept that "there need to be no spaces." Ask why. It is important that students understand function, not simply form.)

Figure 3. Mal-ruler

Where do the marks go?

Create two foot-strip rulers with labels for each unit in the middle of the unit *(a)* and at the end of the unit *(b)*.



Part Three: Formative Assessment (continued)

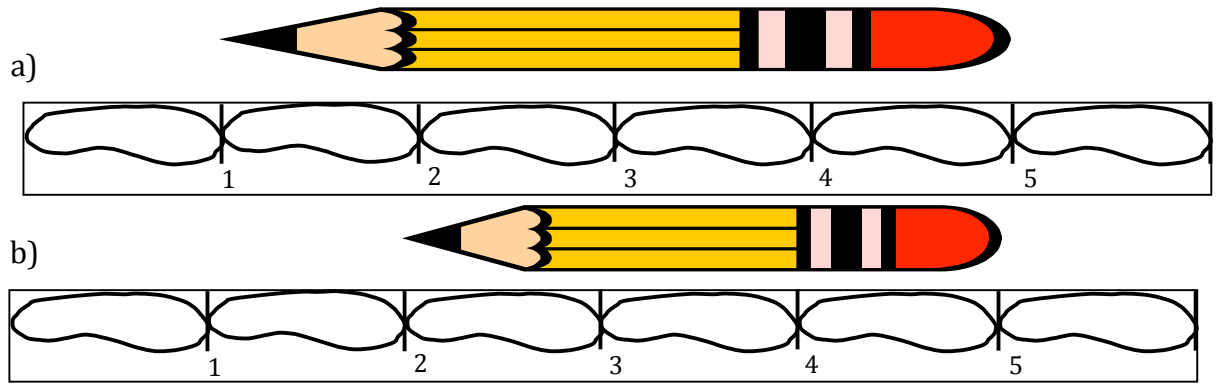
Where do the marks go? *(continued)*

Ask students to compare the placement of the labels and ask them to speculate why rulers are conventionally labeled at the end of the unit (to signify the distance or length of each unit). The consequences of different choices of marks can be made explicit by having students *move their finger* from the start to the end of the unit. Have students compare that motion to the motion made by starting at the origin (zero) or at the end of the unit and then traveling until they see the mark somewhere within the next unit. If students have addressed the issue of fractional parts of units, ask them to label those parts as well.

Support thinking by asking: *How far have you traveled? What would you call that? How might I label that on my ruler?* (**Teacher Note:** The use of motion helps students reason about length measure as unit-distance traveled, rather than as discrete counts of motion.)

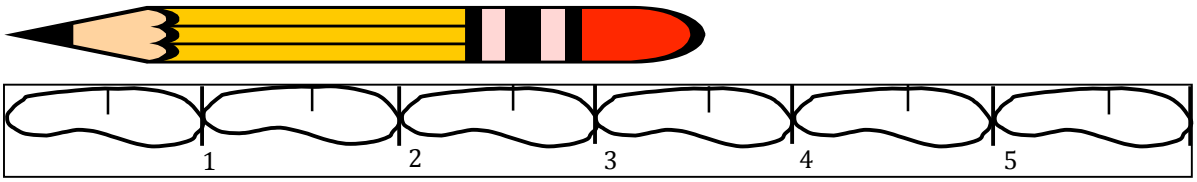
Zero point

Have students use their ruler to measure an object beginning at one on the ruler and ending at 4 or 5. Do they compensate for the shift in origin or do they simply read off whatever number they see (zero-point?) If they do, what happens when you start at 2 and go to 4 or 5?



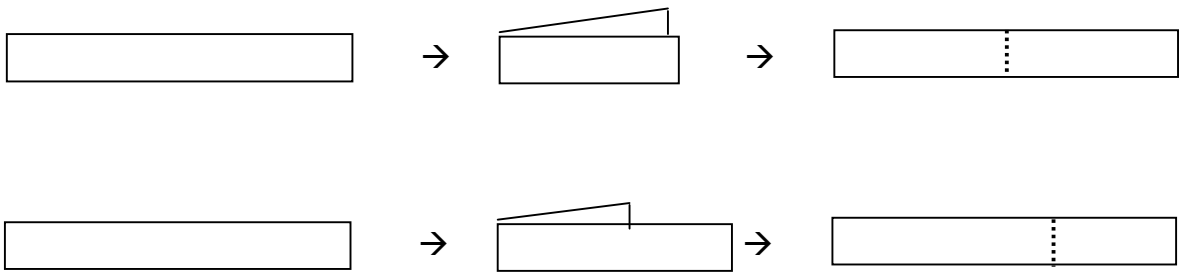
Part Three: Formative Assessment (continued)

Give students a footstrip ruler with ½ unit markings. Have students use their ruler to measure something that ends at one of the ½ unit marking. What do they call the measure? Some students who are thinking about units as labels and not lengths may treat 3 ½ as 4 ½ because they think: "1, 2, 3, 4" because it is the fourth count, and ½ because it is half of the fourth unit. *(See figure below.)*



Parts of units

Take one of the rectangular strips of paper representing a foot (do not do this unless students understand how a strip represents a person's foot) and ask students to comment on two different kinds of folds. One fold splits the unit into 2 congruent lengths. The other split does not. Ask what each split could be named. Ask students to compare each kind of split, noticing what is the same and what is different. Ask if students can see any advantage to congruent partitions. This will be the focus of the next unit, so you might wish to delay this portion of the assessment.



Part Three: Formative Assessment (continued)

2 Ask students to spend 15-20 minutes writing a response to the question below in their math journal:

Write a letter to your friend describing all the steps you took to build your ruler. Make sure you tell your friend the most important things you need to remember when you build a ruler, and explain why those are important.

3 Read each written assessment and analyze student understanding of measurement in relation to the progress map. Are students making progress? Take notes on the understanding and misunderstandings of your focus students. Use this summary to prepare for Unit 4 demonstration and discussion.