Unit 1

This unit is designed to assess how students are thinking about measurement before beginning instruction. We aim to assess students' thinking about a few "big ideas" about measurement: the nature of units (attribute-unit relation, iteration, identity, tiling), partial units, scale-origin (zero-point, for instance, where 1 or 2 could stand in for 0), and the role and nature of tools commonly employed to measure common attributes, such as length. Students' written work and classroom conversation provide benchmark assessment. The *Measure Construct Map* describes how these big ideas typically develop, although individual students will not all fit this sequence.

Briefly, the big ideas assessed are:

Iteration. A unit is repeatedly applied to obtain a measure. For example, a measure of length is obtained by moving (translating) a unit-distance a finite number of times. The measure is the number of iterations of the unit (e.g., a 9 inch length is measured by translating an inch from the beginning to the end of the length 9 times).

Identity. The units must be identical in order for the iteration to yield a single measure.

Tiling. The units are translated "end-to-end" with no gaps. Units tile the line, plane, volume etc.

Attribute-Unit relation. An object can have multiple attributes, and some units may be more suitable than others for measuring different attributes or even different magnitudes of the same attribute (e.g., a longer distance might be measured more readily with a longer unit, an area is measured with a 2-D unit).

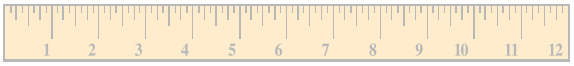
Partial units. Units can be partitioned to result in partial units (e.g., ½ inch.)

Zero-point. Any point on the scale can serve as the origin (any number can stand in for zero).

What is Measure?

Materials

Overhead or document projector
Transparencies or papers
Chart paper
Markers
Question clusters (in lesson)
Three different objects or groups of objects to measure: for example, dirt, rocks, water bottle



Task

Students consider the meaning of measurement as they answer a series of questions.

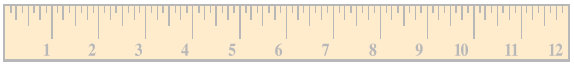
Teacher Role

The teacher:

- Facilitates whole-group discussion
- Asks clarifying questions
- Records student thinking on chart paper

In this lesson. . .

- Activity structure
- Question clusters
- Potential elaborations & homework
- Teacher reflection questions
- Appendix: Interview protocol for individual student follow-up



Activity Structure

Whole-group discussion

Ask each question in the question clusters below. The first question cluster aims to uncover student ideas about what measurement means to them and to probe whether or not students think about multiple measurable attributes for the same object (i.e., a water bottle, a pile of sand, etc.) The second question cluster aims to reveal students' ideas about methods and processes of measurement—how does measurement work? Do students mention units and if so, how do they think of them? The third question cluster clarifies how students think about tools that one can use for measurement. Are tools and measures fused or do students think that the same measurement can be obtained with different tools.

Compare and contrast ideas. Do not correct student thinking. Instead, ask for clarification, because the goal is to get a sense of how students think about measurement and to encourage explanation of ideas.

Cluster 1: What is measurement?

What does it mean to measure something?

Name 3 different things about this pile of dirt (or water bottle) that you could measure. Why are they different?

The intention is to find out if students can "see" multiple attributes of the same object. Often, there is a focus only on measures of length, such as height.

Students may respond with "keep track of the numbers," or "use a ruler," "inches or centimeters," etc. A follow-up probe might be: "How does a ruler work? What did the people who made the ruler know?" What about the ruler makes it a good way to measure? What do inches or centimeters help us know about the water bottle?

Cluster 2: Enacting measure

How do you measure how long something is?
What are some important things to keep in mind?

How do you measure how heavy something is?
What are some important things to keep in mind?

How do you measure how fast something is changing (like when something is growing or when you are traveling in a car)?
What are some important things to keep in mind?

Cluster 3: Tool use

How do tools help you measure? Give three examples.

The intention is to follow-up on the enacting measure question cluster to determine whether or not children's responses suggest anything more than reading numbers on a scale. Do they have any ideas about how measuring tools enact measure?

Potential Elaborations

Although the entire lesson is intended as an assessment of the collective, teachers have elaborated on the assessment to get a better fix on individual reasoning. Here we note some of the tasks and situations that help teachers understand how individual students are thinking.

Enactment of Measurement

Ask students to measure something, such as the length of a partner's arm span, with a variety of tools (including non-standard tools, such as empty plastic water bottles). Which aspect of the bottle might be used to measure the length of a person's arm-span? Looking at how students approach these problems often gives insight into their thinking. For example, some think that only rulers can be used. Some acknowledge unconventional units can be used but only if they are first measured with a ruler. Others say that they don't need multiple units: they don't know what to do when they "run out" of units. To determine whether or not a particular enactment works, students compare the arm-spans of two different people. Does the measure work?

Unit Mixture

Pose a problem where students use longer and shorter paper clips to measure a distance, but there are not enough of either type to span the distance. Do students mix units of measure? If so, what do they call the measure? What do they do when they "run out" of units?

Homework

Teachers can ask students to measure something interesting at home and make notes in their math journals (write about how they measured and the result of the measurement). Or, ask students to select an object, such as a car, and measure three different attributes of it.

Teacher Reflection and Preparation

Choose three students that represent three different levels of understanding. What conceptions or misconceptions did your advanced, proficient, and emerging-understanding students have about measurement? Use the reflection clusters on the following page to estimate about what percentage of students fell into each category. Feel free to create new categories – categories overlap, so the percentages do not need to sum to 100.

Cluster One
What is measure?

Initial responses to the meaning of measurement:

_____ **Procedural Focus**
Measurement was a procedure, such as using a ruler or reading values on a ruler.

_____ **Attributes**
Suggestions about multiple attributes that could be measured.

_____ **Other**
Please specify:

Students readily generated multiple attributes of the object that could be measured?

_____ **Yes,** but mostly about length.

_____ **Yes,** varied—length, volume, weight, compactness, etc.

_____ **No,** students focused on one attribute, such as height.

Cluster Two
Enactments of measure

When responding to questions about things to keep in mind, or about how to measure different attributes, students:

_____ Suggested different methods or units for measures of length, volume, weight, etc.

_____ Spontaneously mentioned qualities of units (give examples – perhaps students mentioned that units all had to be the same size, perhaps that you put one right after the other with no spaces, etc.)

When measuring:

_____ Reused unit (knew what to do when ran out of units)

_____ Mixed units of measure

_____ Left gaps or spaces when iterating

_____ Suggested that non-conventional units could not be used for measurement

_____ Treated partial units differently than whole units when appropriate

Cluster Three
Using tools

When asked about measurement tools, students:

_____ Mention ruler

_____ Mention other conventional tools (e.g., scales, measuring cups, stop watch)

_____ Mention unconventional tools

Additional Notes:

Digging Deeper: Individual Student Assessment

To focus on linear measure, some teachers have interviewed a few children individually (an emergent, proficient, and advanced mathematics student), especially when they were concerned or uncertain about how the children were thinking.

Materials

1 book (approximately 9-inches tall)

A measurement tool kit, consisting of:

- 8 paperclips (5 large, 3 small)
- 2 popsicle sticks
- 2 pencils (1 unsharpened, 1 approximately 3.5" long)
- 1 broken ruler (break or cut a ruler so that measure does not start at zero)

Instructions

Follow the initial assessment protocol, but feel free to ask follow-up questions that help clarify the student's thinking. Follow-up questions are listed in measurement principle categories and are to be asked in relation to the actions and tools a student uses when measuring the book. Potential follow-up probes are listed in the table below. Within each category there is a specific order the questions need to be asked depending on student response, but there is no specific categorical order. Ask categorical questions in relation to comments the student makes during the interview. When adding questions to the interview that are not listed, try to craft broad, open-ended questions first, then narrow the question content to focus on specific aspects of the child's thinking or action, or the concept you are trying to get at. Record as much of the student's responses as possible in order to have a record of the child's thinking for later analysis. After you have finished conducting the individual assessment, take a few minutes to look over your notes and write a short summary of the student's thinking and understanding of the measurement principles on the interview form before starting the next interview.

The interview script, follow-up probes, and sheet on which to record student responses follow in the appendix to this lesson.

Appendix

Length Measure Interview Protocol

Measurement Principles and Follow-up Probes

Length Measure Assessment

Length Measure Interview Protocol (for following-up with individual children)

Brackets and *italics* indicate directions for the interviewer.

Say: **I need some help with measuring today. I bought a new bookcase and want to make sure my books will fit on it before I move the bookshelf into the classroom. This is one of the books I'd like you to help me measure.** *[Present book to student.}*

This is my measurement tool kit. *[Present tool kit to student.]*

Let's look at what is in the tool kit. What do you see? *[Let student explore tools in kit and ask him/her to name each item.]*

You can use any of the tools in this box to measure. I'd like you to measure the height of this book. *[Point to the spine of the book to show the length that is to be measured.]*

What did you find out? How tall is the book? *[Listen to the student's response, then ask for a unit label if it wasn't provided (e.g., "8 what?")]*

Move on to follow-up probes.

Measurement Principles & Follow-up Probes					
Units (Tools used?)	Zero Point (Count at start? *Any # can stand in as zero?)	Tiling: Space Filling (Gaps/overlaps?)	Iteration (Repeated unit use?)	Fractional Parts (Partitions unit? Name of partition? Count with partition?)	Label (Inches or tool name?)
When the student finishes choosing and arranging units ***ASK:*** **Why did you choose that object (those objects) for measuring the book?** ***ASK:*** **Is that the only tool you could use to measure the book?** *RECORD ANSWERS.*	*Point to the start if the spine of the book and* ***ASK:*** **How long is the book right here?** *RECORD ANSWER.* *If student says* ***"ZERO"*** *move to the next principle.* *If student says* ***"1"*** *ask the following question:* *Point to the end of the first unit (1 unit) and* ***ASK:*** **How long is it here?** *RECORD ANSWER.* *If a student says,* ***"1"*** *for both the start and the end of the first unit,* ***ASK:*** **How can the book be "1" both here and here?** [*Point to the start of the first unit and then the end of the first unit*] *RECORD ANSWER.*	*Look at the way the student arranged the units. If there are spaces between units* ***ASK:*** **Did you measure all the space?** *If student says "YES," point to empty space between two units and* ***ASK:*** **What about this space? Do we need to measure this? Why?** *If the student says "NO" to the first question,* ***ASK:*** **Do we need to measure all the space? Why? Why not?** *If the student space fills with different objects, remove their tools and place 5-large paperclips along the object with space between and* ***ASK:*** **Is this a good way to measure?** *If student says "NO" say:* **Can you fix it for me?** *RECORD ANSWERS.* *Next arrange paperclips so the overlap and* ***ASK:*** **Is this a good way to measure? Why or why not**	*If student fills the space with different objects* ***ASK:*** **Why did you these objects to measure the space?** *RECORD ANSWER.* *Remove the student's units and lay out 5 large paperclips starting at zero point and leaving no space between. There will not be enough to measure the whole length,* ***ASK:*** **I want to use just <u>these</u> paperclips to measure the book, but I don't have quite enough. What can I do?** *RECORD ANSWER.*	*Look at the end of the book to see if the student used units that overhang the edge,* ***ASK:*** **How long was the book?** *If the student does not mention the fractional part take 2 and lay them along the spine of the book. The last straw will extend the spine of the book.* ***ASK:*** **How many straws long is the book?** *If the student answers in a whole number* ***ASK:*** **Why did you say [*student's answer*] straws long?** *RECORD ANSWER.* *Point to the part of the straw that is measuring the book,* ***ASK:*** **What about this?** **Is this a whole straw?** **What might you call this part?** **So how long is the book?** *RECORD ANSWER.*	*If student uses mixed objects to measure the book,* ***ASK:*** **How long is your book?** *RECORD ANSWER.* *If student does not label the length measure,* ***ASK:*** **[*Student's answer*] what?** *(I.e. 14 what?)* *RECORD ANSWER.* *If student uses just one type of object but says "inches",* ***ASK:*** **Why did you say inches?** **Could you show me where 1 inch is?** *RECORD ANSWER.* *If student shows you the end of one object,* ***ASK:*** **Why do you say that is an inch?** *RECORD ANSWER.* *If student insists that the objects are inches,* ***SAY:*** **I though you said you used [object] to measure. Why did you call it inches?**

Length Measure Assessment

Student & Date_____

Interviewer _____

Note student's actions in relation to each measurement principle. When finished write a short summary of student thinking on back.

Units (Tools used?)	Zero Point (Count at start? Any # can act as zero?)	Space Filling (Gaps/overlaps?)	Iteration (Repeated unit use?)	Fractional Parts (Partitions unit? Name of partition? Count with partition?)	Label (Inches or tool name?)

Clear the measurement tools away and close the measurement took kit. SAY: **If you needed to explain how to measure this book to a new student who was just joining the class, what would you tell him or her?** *Record Answer.*

Give student broken ruler. ASK: **Could you use this ruler to measure the book? Why or why not?** *Record Answe*