AREA MEASURE

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This is a series of lessons designed to support the growth of students' understanding of area measure.

The first lesson, *Comparing Rectangles,* is intended to provoke spatial structuring of a 2-dimensional space as students compare the space covered (area) by 3 different looking rectangles. By folding and re-arranging pieces, students discover that despite appearances, all three rectangles enclose the same space. Unit of measure emerges as a privileged partition—the same subdivision of space can be counted and used to compare the space enclosed by each figure. Either rectangular or square units typically emerge during this activity. After establishing the equivalence of the space enclosed by each figure (by equivalent counts of the same units), students are further challenged to construct many different space figures of equal area. The perimeters and area of each figure are recorded. The aim is to help children differentiate length and area measure.

The next lesson, *Whose Hand Covers the Most Space?* provokes consideration of the qualities of a suitable unit of area measure, and introduces the need to label and combine partitions of units. Students trace an outline of their hand, and they propose means to rank order the space covered by each handprint. Tools provided include beans, buttons (because their contour resembles that of the hand), spaghetti, rope and square grid paper. Students work in pairs to develop a strategy to determine the space covered (the area). They can use any of the tools that are provided. As students compare strategies, the teacher selects strategies for comparison that will make evident qualities such as identical units (are the beans all of the same size?) and space-filling (are there any "cracks?"). After students have a chance to consider the qualities of their choice of unit, the teacher either introduces square grid paper or asks students who have chosen square grid paper to demonstrate their solution. This sets the stage for using partial units, because the measure using squares of the grid is not a whole number.

The third lesson, *Classroom Space*, introduces the notion of comparing the areas of different enclosures (classrooms) by coordinating units of length measure to constitute units of area measure. The enclosures are rectangular or are formed by combinations of rectangular units. Students can use rulers to measure the length of the sides, and they are challenged to consider how knowing the length of each side can be used to create a unit of area measure. Students then partition a few of the enclosures and count the number of units formed by these partitions. The lesson culminates in the development of a formula for finding the area of any rectangle.

The fourth lesson, *Sweeping Space*, emphasizes area as constituted by motion. Given a fixed length, children sweep that length through another length (see Figure i below). The lesson poses problems with rectangles and concludes with a challenge involving the surface area of a cylinder.

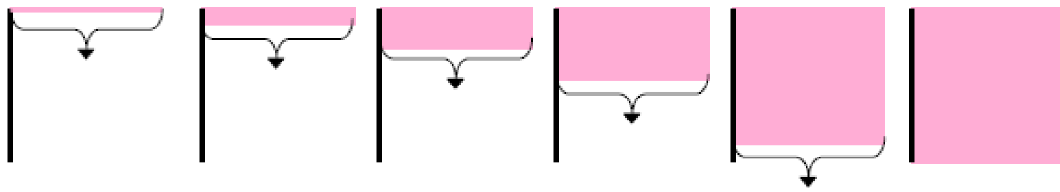

Figure i. Sweeping Area

Two lengths, with one pulled *along* the length of the other, construct a planar region. The area is measured by finding the ratio of a unit (or a unit sweep) to the total area swept.

The fifth lesson, *Parts and Wholes,* suggests that length need not be considered only as measured in whole numbers, so problems like those posed in the third and fourth lesson are revisited from the perspective of lengths involving fractional quantities.

The sixth lesson, *Finding and Comparing Areas of Triangles,* capitalizes on the formula for the area of the rectangle to develop the area of any triangle.

The seventh lesson, *Polygon Area,* employs the area for the formula of a triangle to find the area of other polygons, such as quadrilaterals and hexagons. Students are challenged to develop a general formula for area.

The eight lesson, *Area of a Circle*, considers a circle as an n-gon and develops the formula for the area of a circle as an extension of the general polygon area formula as the number of sides becomes infinite.