LESSON 2: Whose Hand Covers the Most Space?

Important features of the unit
(And what do we do about parts?)

Overview & Big Idea: *This lesson is designed to help students begin to understand the utility of measure as they explore qualities of different prospective units of measure*, such as beans or square grid paper. Students "rank order" their hands according to the amount of space they cover. This activity is an opportunity for students to employ their previous understanding of additive congruence by superimposing pairs of hands to find which one covers the most space. This method will prove difficult as students struggle to order the hands of the whole class and discover that all hands are not scaled versions of one another. Students revisit measurement principles as they struggle with the difference between filling space with "resemblance units" and measuring space with a "privileged unit." Students are also asked to consider parts of units as a means to account for all the space covered.

Materials: Students' hands, construction paper, black felt-tip markers, scissors, a variety of materials such as beans, spaghetti, buttons, plastic discs, area tiles, grid paper (1" square and, 1cm square), color pencils, chart paper, marker.

LESSON 2: PART 1
COMPARING HANDS & MEASURING HAND REPRESENTATIONS

Qualities of a suitable unit of area measure

Activity Structure: *Whole Group Discussion & Measurement Exploration*. Ask students to work as a class to rank order their hands according to area and record their findings on a class chart. Students will most likely decide that they can determine a rank order by physically superimposing one hand to another. Allow them to explore this idea. Use the discussion prompts to begin the activity. After students have initial results (and frustrations) gather the class together for a discussion of their findings, including ideas about "*what worked and what didn't work.*" Use these ideas to revise the method and move toward measuring *representations* of each student's hand.

Teacher Role: After giving students initial instructions, rove the room and observe students' methods of comparison. Note student difficulties and discoveries to prompt a later discussion. Help students note parts of the fingers or hand that do not precisely match up and challenge them to think about a comparison when the two hands are not exactly the same shape.

Initial Discussion Prompts- Today I want you to think about putting your hands in order from biggest to smallest.

- *What do we mean by biggest?*
- *What part of the hand are we thinking about?*
- *Whose hand do you think covers the least amount of space? Explain.*
- *Whose hand do you think covers the most amount of space? Explain.*
- *How might we put all the hands in order?*
- *How will we record our order?*

Post-Exploration Prompts-

- *What did you find out?*
- *Were you able to put your hands in order?*
- *What problems did you have?*
- *What did you discover?*

Ask students to consider a different way for finding out the amount of space their hands cover. Use questioning to work toward a measurement strategy. Shift the discussion from the comparison of hands to the measurement of hands by asking students to think about past activities they have done where they have had to find out how big or small something is in order to compare the items (draw on class experiences from linear measure). Have materials available to promote the idea of representing the hand and using units to measure it- construction paper, scissors, and various items for measuring space (*see materials list*) including grid paper.

Discussion Prompts-

- *What problems did we have when comparing our hands? (I.e., parts didn't match, didn't know how to order the middle-sized hands)*
- *I've collected some materials on the table. Could we use any of these things to find out how much space our hands cover so we can put them in order? (Discuss the materials students see as viable choices)*

After students establish some ideas for measuring their hands (tools they could use, how they might use them, how they might record their measurements), invite students to trace their hand (decide on left or right, open or closed), cut out handprint, and use any of the available materials they think would work to complete the measurement task. Give students approximately 30 minutes to complete the task. Let them know they will be asked to share their findings with the whole group.

Teacher Role: Rove the room to support student reasoning about unit choice and discoveries they make while measuring (*What works? What doesn't? Explain. Do students consider any of the measurement principles learned during linear measure unit?*). Record student thinking and use that thinking during a follow-up discussion (Lesson 2, Part 2).

Teacher Support of Student Thinking: As students use various materials to "measure" the area of their hands, help them reason about the relationship between their unit choice and the necessary qualities of a suitable unit of area measure.

- *Why did you choose that unit?(See Figure 5 for example of student unit choice)*
- *Did you measure all of the space?*
- *What did you do about the curved parts?*
- *What did you do when a unit hung over the edge of your hand/finger?*
- *What did you call this space that is only covered by part of a unit?*
- *How did you keep track of all the parts?*

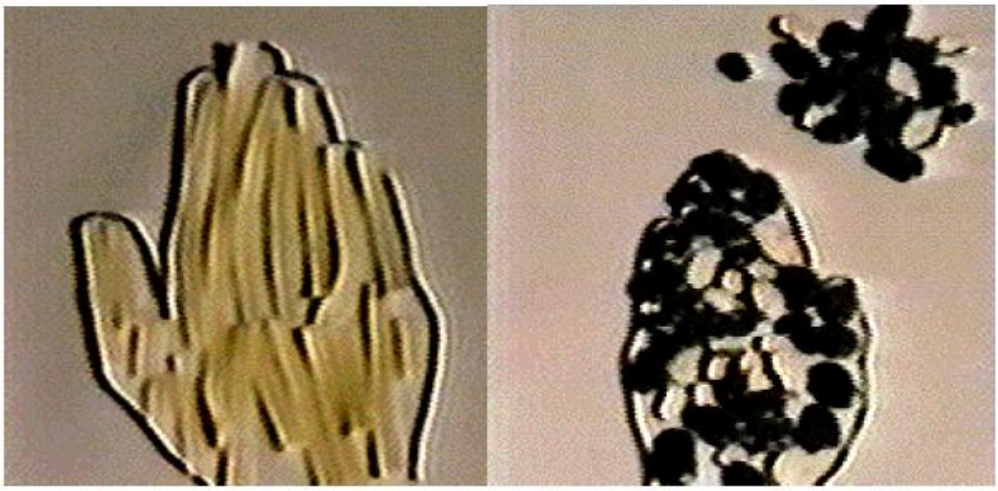

Figure 5. Handprint covered with different units: a) spaghetti, b) beans.

Individual Student Writing (Reflection): As students finish measuring their hands, have them post their hands and the measurements along the chalkboard. Remind students to label their measurements with the unit name. Have students spend about 15 minutes writing a reflection summary about the measurement activity. Ask students to write about the way they measured their hands and draw a diagram to go along with their writing (thinking trail). Check to make sure students explain why they chose the units they did to measure their hands and if they ran into any problems during this activity. Also ask students to take a few minutes to look over the class hand measurements and write down anything they notice.

Teacher Reflection & Preparation: Read the written reflections of the focus students (emergent, advancing, proficient) and take notes on the trends in thinking/reasoning. Take notes on the distribution of units used to measure the hands to get a grasp of potential student reasoning and comments that will arise in lesson 3. *Prepare grid paper (1" square, 1cm square), black felt-tip markers and color pencils for Lesson 2, Part 2.*

LESSON 2: PART 2- MEASURING WITH SQUARE UNITS

Privileging the Square and Partitioning Units

Activity Structure: *Whole Group Review*- Ask students to take a "Gallery Tour" of the hand measurements that are posted along the chalkboard and decide if they are ready to order the hands. Promote a discussion by again asking students, "*Are we ready to put the hands in order? Explain.*" This question provides the opportunity for students to address measurement principles such as identical unit, and tiling while reasoning about the ordering task.

Option 1 (*Reasoning from Activity*)- If the students all feel they are ready to order the hands, proceed by having students choose the first 5 hands that they think are the biggest and put them in order. If students have used different units, this task will become difficult early on. Question students about the reasons behind task difficulty. Make a list of problems they encounter and use these questions to promote a discussion about measurement principles/rules. *It is often helpful to have two different students measure the same handprint with the same units, such as beans or spaghetti. Students can consider why they did not obtain the same measure.*

Option 2 (*Reasoning from Representations*)- If immediately after the gallery tour students express that ordering the hands will be problematic, take 10- 15 minutes to talk about their observations with the class. Record student observations on chart paper. When all comments have been recorded, discuss student ideas that address the use of identical units to measure individual hands, the need to tile so all space is measured, and the difficulty of comparing measurements when students used different units from each other.

Potential Discussion Prompts:

- *What did you observe about the class measurements of your hands? (List ideas)*
- *What problems might we have ordering the hands using these tools?*

Shifting to a Privileged Measurement Unit- One of the ideas students should mention in their observation of the hand measurements is that individuals in the class chose different units of measure from each other resulting in a difficulty comparing areas. This is an important idea that can be used to promote the use of a particular unit of area measure- the square.

Ask students to consider alternative measurement tools. If no one suggests (or no one has used) grid paper, use the following prompt questions:

Present grid paper and ask (*promoting identical units*):

- *Could we use this as a tool?*
- *If we use the grid paper, what is the unit we are using to measure?*
- *Which of our measurement rules can we follow if we use the grid paper to measure the area of our hands?*
- *What problems might we encounter?*
- *Which size grid paper should we use? (Inch squares or cm squares) Explain.*
    - Inverse relation between unit size and count
    - Discussion about accuracy
- *How should we position our hands? (Fingers together or spread apart) Explain.*
    - *(Work toward agreement on the above 2 questions)*

After the class agrees upon the open or closed hands and the size of graph paper, have each student trace his/her hand on the paper and begin to measure the area by counting the number of square units the hand covers. Support students in reasoning about how to account for *partial units*. When the students finish measuring the area of their hands, have them post their hand tracings with their measurements. Use these measurements to order the hands. List all the hand measurements, ask students to arrange the data using the numerical measurements, and then check on the rank order by using the measurements to order the actual hand representations. Ordering according to the data first and then ordering the actual hand representations will provide an opportunity for students to compare the data to the actual object being measured and look for discrepancies between the two (a nice introduction to causes of measurement error and finding discrepancies in naming fractional parts).

Teacher support of student thinking: The introduction of grid paper may seem problematic to students who continue to reason that the unit must resemble the attribute they are measuring. For example, students who are thinking about a single dimension, such as length of hand may argue that they cannot use the square because it is not long like the hand, or that it is not curved like the fingers. Students may also reason that the curved shape of the fingers leaves parts of the square unit unfilled. This is an opportunity to look at the measurement rules that privileging the square will support and then problem solve about partial units as students begin to quantify square units.

Strategies for Accounting for Partial Units in Irregular Figures:

- *Fractional parts & naming fractions*
  - Find an example on a student page where the hand does not cover the space of a complete square.
  - Ask: *What did you call this? Explain. About how much of a square is covered? Why do you think that?*
  - Folding squares of paper may support students in estimating partitions and naming the part covered in relation to the whole square (1/2 is 1 part covered of a total of 2 parts).
- *Key coding fractional parts (consistency in naming)*
  - Students may decide it is necessary to create a "key" (such as the key on a map) of fractional parts to help them name rational values more consistently. If a student develops a key, have that student explain how she/he came up with the code and how it helps with naming parts.
  - If a student does not see a need for a fractional part key, promote the idea of consistency in naming rational values by asking:
    - *What are you calling ½? ¼?*
    - *How do we know your ½ is like my ½?*
    - *Is there something we could do to help us be more consistent in naming fractional parts?*
- *Compensation Strategies (adding fractions to make a whole)*

- Students may choose from several different strategies to account for all the fractional parts.
    - *Matching parts*- students may simply indicate parts of two different sections of the hand that "look like" they would add up to a whole square (*See Figure 6*).
    - *Adding like fractions*- students may add together fractional parts with the same denominator (i.e., count the number of ½ square units and add them together, then finding all the ¼ square units and adding them together)
    - *Adding fractions with different denominators*- students with a strong understanding of equal fractions (i.e., 2/4 = ½) and operating on fractions may choose to make an unordered list of fractions and add them together (i.e., ½ + ¾ → 1 ¼ + 2/8 → 1 ½)

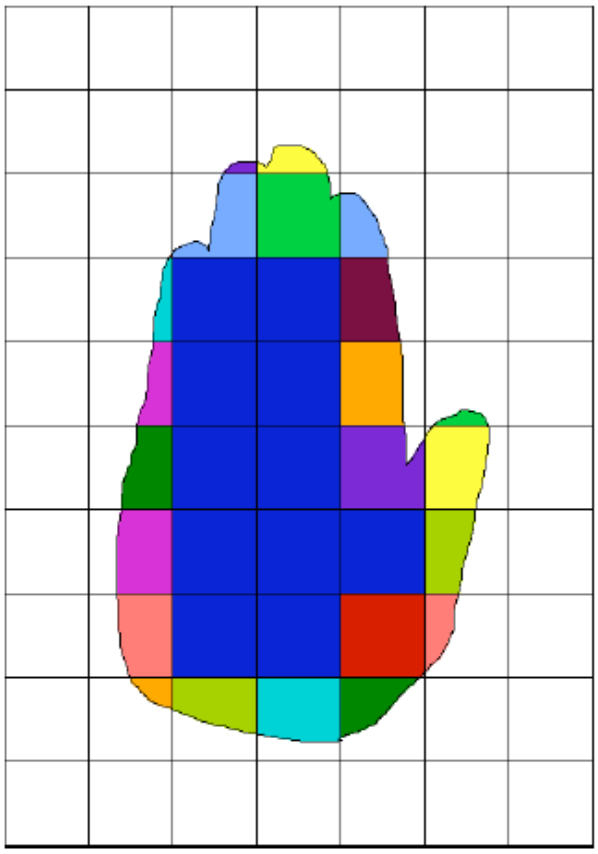

Figure 6. Students using matching parts to make "whole squares."

Extension Activity: Which covers more space- an open hand (fingers spread) or a closed hand (fingers together). How do you know? How could you convince someone of your thinking?

Extension Activity: Discuss the function of leaves on a tree, how they absorb sunlight. To collect sunlight, the leaf must be designed to cover as much area as is possible without becoming too heavy to be supported by the stem or branch. If the children collect some leaves of irregular

shape, such as ivy or sycamore, we can discuss the amount of leaf (or area of leaf) exposed to the sun and air. Rank order the area covered by different leaves or find the area covered by one leaf and then determine the total area covered by all of the leaves of the plant/tree (*Figure 7*). You might wish to switch to cm squares.

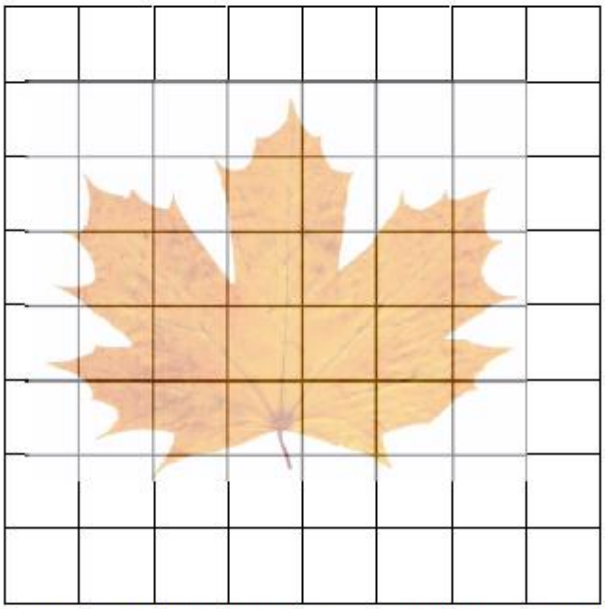

Figure 7. Revisiting Area measurement with an irregular shape