# LESSON 1: Which Covers the Most Space?

*Comparing Rectangles*

**Overview & Big Idea:** The lesson is intended to provoke spatial structuring of a 2-dimensional space as students compare the space covered (area) by 3 different looking rectangles. By folding and re-arranging pieces, students can establish that the three figures are in fact additively congruent—meaning that the pieces of one rectangle can be used to completely cover a second rectangle. (The assumption is that students understand that congruent figures cover the same amount of space.) Unit of measure emerges as a privileged partition (usually a square or a rectangle) that allows a student to more efficiently compare the space enclosed by each figure, simply by counting. *Rather than giving students a unit to use to cover and count, this problem asks that the student invent the unit, because this invention requires the student to structure the space by partitioning it.* Because area measure is a ratio of the space enclosed by a plane figure and the unit, the experience of inventing the unit is a practical introduction to this relationship. The lesson concludes with the problem of drawing different space figures using the same collection of units. This further elaborates the notion that despite appearances, figures can have the same area measure. Furthermore, the length around each figure (the perimeter) is found. The aim is to help students differentiate length from area measure. Extensions of the last part of the lesson might include investigation of the configuration that results in the least perimeter and the greatest. This extension investigation requires that students first consider rules for joining the units. For instance, must the units share a side?

**Materials:** Three rectangles constructed from large, unmarked chart paper with (horizontal x vertical) dimensions of: 12 x 1, 4 x 3, and 2 x 6 labeled A, B, and C (See Figure 1). Rectangles must be constructed so there are no folds or tape lines. Do not laminate or use paper with lines, grids, or other markings. If working with a whole group, make large rectangles with the dimensions scaled as: A = 48" x 4"; B = 16" x 12"; C = 8" x 24".

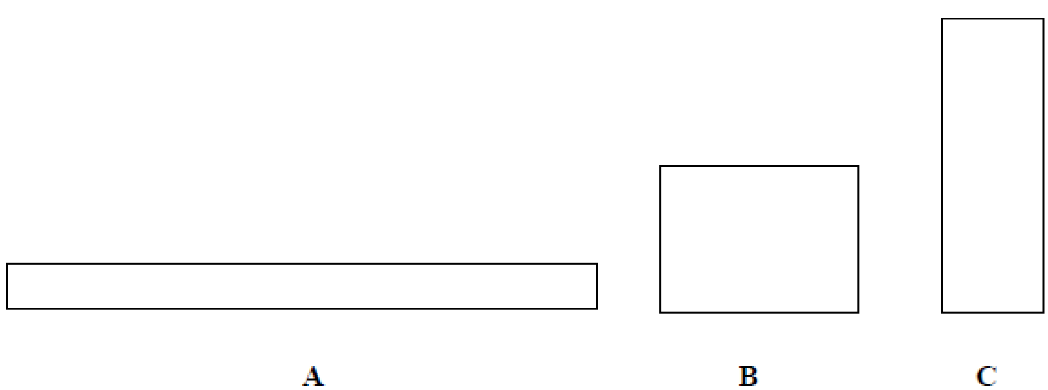


*Figure 1. Rectangles A, B, and C*

**Activity Structure:** *Whole Group Discussion.* Present the rectangles to the class, showing them one at a time. If possible, attach them to the board with magnets or tape so children can see all three at once and make comparisons across the rectangles. Tell the students you are making a quilt (or pirate flag, or picnic blanket, or rugs, or buying wrapping paper, etc.) and need to buy a piece that will cover the most amount of space because you want a large quilt (or the most wrapping paper for the same amount of money, etc.). Explain that the three pieces of paper represent the 3 different sizes the cloth is sold in. Ask student to help you figure out which piece you should buy that will get you the most cloth (see questions in next section).

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**Teacher Role:** Teacher presents the three rectangles and tells the students the story about buying cloth. Teacher facilitates whole group discussion, asks clarifying questions about student statements, asks for student reasoning or definitions, and juxtaposes ideas to promote mathematical argument around the structure and measurement of the rectangles. Students are permitted to physically fold the pieces, but they cannot use rulers or other tools. The aim is to support strategies of additive congruence---meaning that students split the area into parts and re-arrange these parts to establish the relative amounts of space covered by each rectangle. (See Units of Measure, below, for typical strategies involving matching parts).

*Discussion Questions*:

Which piece of cloth covers the most space?

Why do you think so?

How can we think about comparing the space without cutting it?

*Teacher Note: The area of the rectangles is the same.*

*Follow-up Question:*

How can 3 rectangles that look so different cover the same amount of space?

Small-Group Work (*Optional*)
Using the similar rectangles provided (see the attached materials), students work in small groups to compare the space covered.

**Teacher Support of Student Thinking:** Using the student responses as a starting point, elicit and elaborate the following ideas:

<u>Area as an amount of space covered</u>

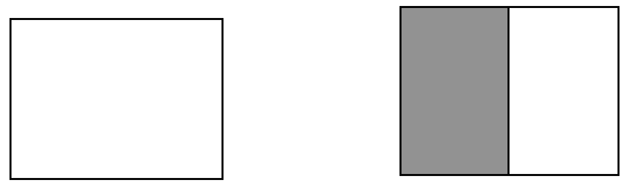
- Students may think about area as a singular dimension, such as the tallest or longest.
- Promote the idea that although one rectangle is long, another is wide and that both length and width need to be considered together.
- Help students to compare and contrast the attributes of each rectangle by focusing on the length of sides.
- Then ask students to think about the space that is covered by the entire rectangle.

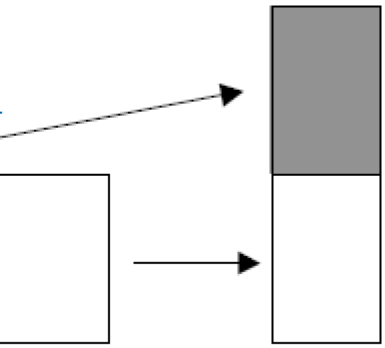
<u>Units of measure</u>

Students may suggest a number of different methods for comparing the area of the rectangles.

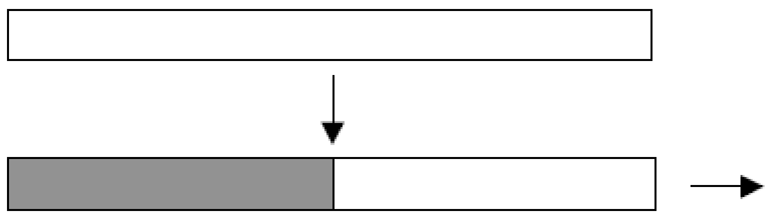
- Students may fold subsections of one rectangle and use congruence to compare that section to a section of another rectangle (equality of subsections) (see *Figure 2*).

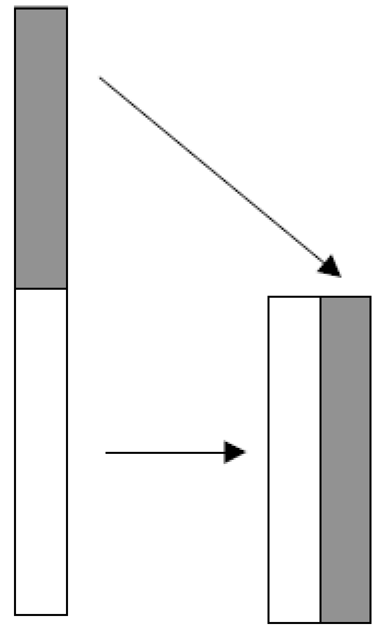
***Additive Congruence***
***B** rearranged to **C***
½ B = ½ C
2 (1/2B) = 2(1/2C)
B = C
B = 4 x 3, so the half spit is 2 x 3. When ½ B (or 2 x 3) is slid up and over the other half of B, the space is rearranged to rectangle C, 2 x 6.





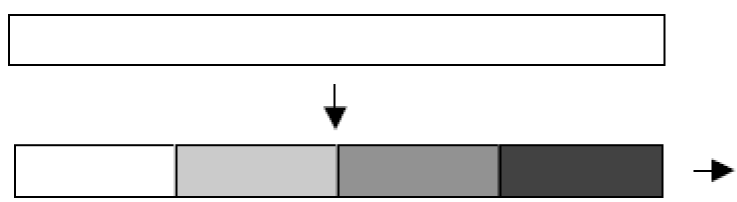
***A** rearranged to **C***

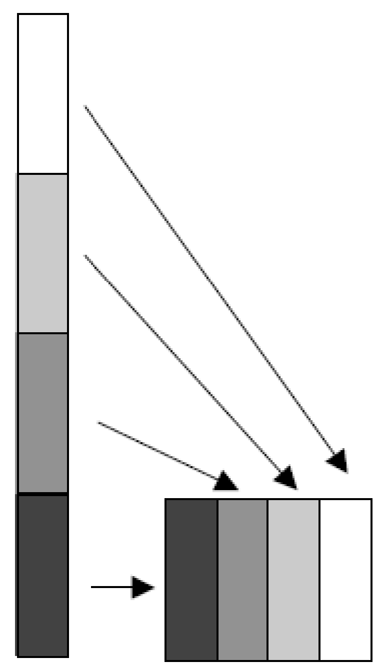




½ A = ½ C
2 (1/2A) = 2(1/2C)
A = C
A = 12 x 1, so the half split is 6 x 1.
When A is rotated ¼ turn right and ½ A is slid over and down, the space is rearranged to rectangle C, 2 x 6.

***A** rearranged to **B***





¼ A = ¼ B
4(1/4A) = 4(1/4B)
A = B
A = 12 x 1, so ¼ A = 3 x 1.
When A is rotated ¼ turn right, and each fourth is slid so the long side is aligned with the next fourth, A is rearranged to rectangle B, 4 x 3.

*Figure 2. Additive Congruence*

- Students may match unequal subsections to parts on each of the other rectangles until all the space is accounted for. They may need support to account for all the parts as they superimpose them on each other. (See *Figure 3*)

**Additive Congruence Using Unequal Parts**

*The shaded areas indicate one example of the progressive matching of sections in rectangles B and C.*

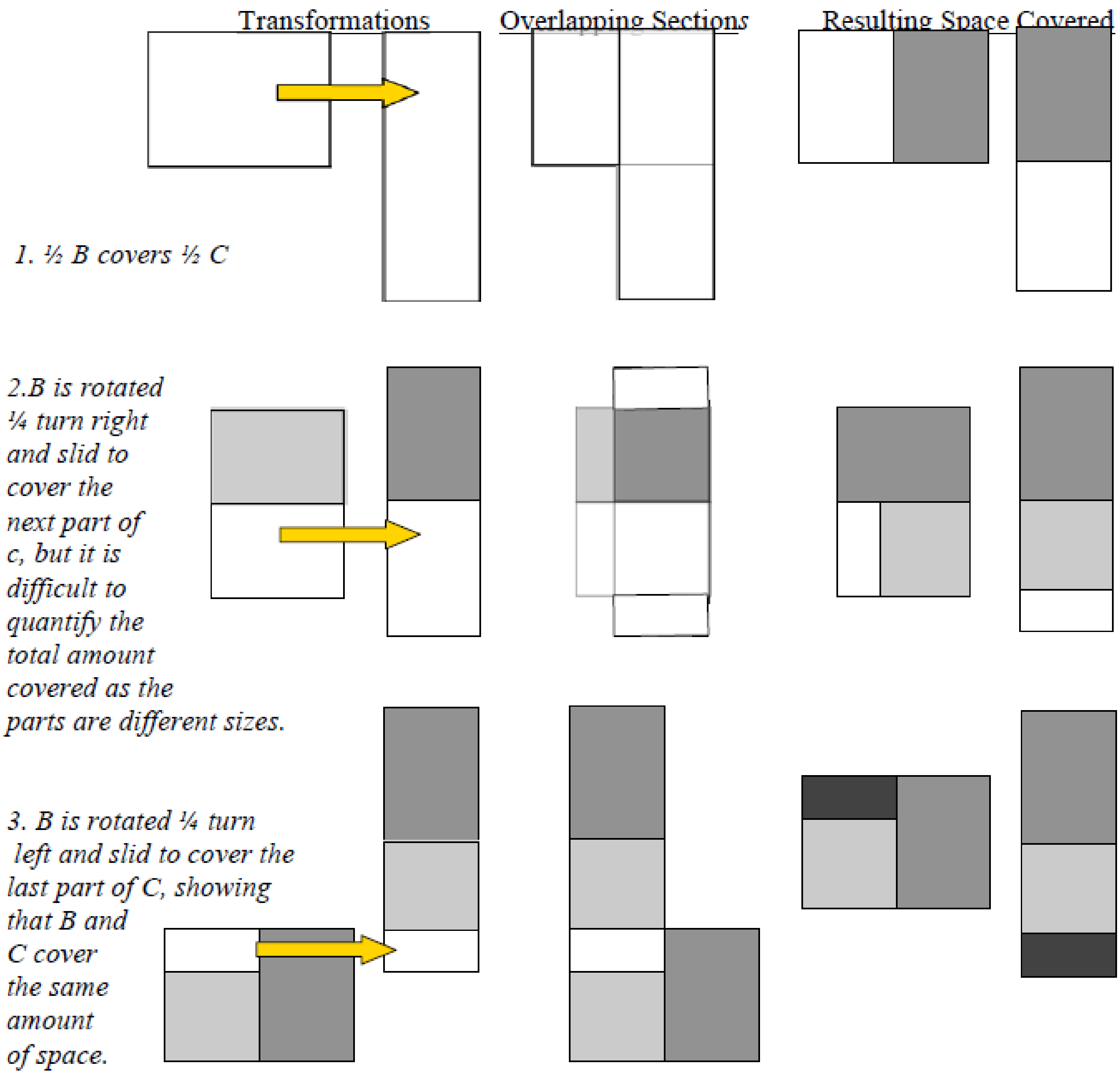



*Figure 3. Additive Congruence of Unequal Parts*

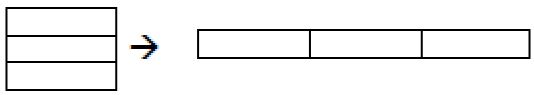
- As students reconfiguring the space by folding and comparing one piece to another, units may emerge. Folds may create an array of identical squares that can be used as units to find the measure of any space (Units may result in squares or rectangles). Support action of unit creation and use it to discuss "privileging a partition." (See *Figure 4*).

**Measurement Congruence**

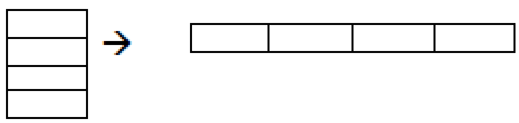
*(Structuring a Square Measurement Unit from Rectangular Folds)*

Students may:

Fold thirds of B and use the 1/3 of B for measuring, for example, A.



Then rotate and fold fourths of B and used ¼ for measuring, for example, A.



How can it be that when you split ***rectangle B*** into thirds it covered ***rectangle A*** and when you split ***rectangle B*** into fourths it also covers ***rectangle A***?

Unfold rectangle B and it is composed of 12 squares

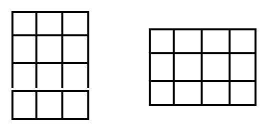


*Figure 4. Measurement Congruence*

- Allow students to explore these various methods (without cutting or physically disassembling the rectangles- and have them share and compare strategies.
- After students have shared their strategies, ask them to consider which methods would work all of the time or with any rectangle.

**Exploration & Extension (Optional)**:

*Partner Exploration*- Give pairs of students small sets of the three rectangles (same dimensions, but scaled down) and allow them to discuss their ideas for finding which rectangle covers the most space. Reconvene as a class and share strategies.

*Extension Activity*- After students create a unit of measure and agree to privilege that unit, give everyone 12 identical units (if squares, or 6 if rectangles). Ask students to find at least 5 different ways the same amount of space can be covered. For each configuration of units, students should record the perimeter. Which configuration has the least perimeter? The most? Why? Record findings in math journal.

**Blackline Masters for Rectangles A, B, & C**
**The first rectangle doesn't fit the page**