RESEARCH IN REVIEW

Young Children's Intuition for Solving Problems in Mathematics

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How do children become competent mathematics problem solvers? Researchers and educators have been interested in this question for many years. Mathematics is a science of patterns and relationships, and young children have far more ability to see those patterns than we may think. Furthermore, their natural ability to see quantitative patterns allows them to develop problem-solving strategies. This ability, because it arises naturally from real-world experiences, will be referred to by the authors as *intuition*.

Intuition for problem solving becomes more sophisticated as children get older. Infants as young as four to six months of age can distinguish differences in small quantities (Starkey & Cooper 1980). By the time they are three, most children have developed a nonverbal sense of number (Baroody 2000). It is not clear whether this sense is due to their recently acquired ability to manipulate mental images to differentiate quantity, their increasing ability to estimate quantity, or some other process. The ability to verbally count is usually present by about age four, when children become able to use counting to compare the sizes of sets up to 10 or sometimes higher (Clements 2004). Because four-year-olds use counting, they can tell which of two sets is larger, even if the sets have different types of objects (Baroody 2000). Note that at age four children's intuition about number (that is, their ability to understand the quantity of objects in sets of similar or dissimilar objects) is based on seeing and often touching actual objects, or at least pictures of objects, in a meaningful, real-life context.

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This review looks at how children in preschool through second grade (about ages three to eight years) intuitively solve the mathematical problems posed by adults. Understanding how children's intuition works makes it much easier to guide them to more formal conceptions of number and thus greater ability to think quantitatively. We begin the review by examining research that shows that young children—usually by the age of four—can solve *word problems*, or *verbal problems*, two terms we will use interchangeably. We follow with suggestions on how to encourage children to use and expand their skills for solving such problems.

Solving word problems

Much of our understanding of children's intuitive problem-solving ability comes from studies on how primary grade children solve word problems prior to formal instruction. One of the early studies in this area (Carpenter, Hiebert, & Moser 1981) looks at first grade children's strategies for solving various types of addition and subtraction problems in which numbers between 2 and 10 were added or subtracted. The children could have the problems read aloud as many times as they wished, and counters were available for children who wanted to use them.

For *join problems* (such as "Mary had 3 pennies. Her father gave her 8 more pennies. How many pennies did Mary have altogether?") and *part-part-whole problems* (such as "Some children were ice-skating. Five were girls and 7 were boys. How many children were skating altogether?"), 88 percent of the firstgraders used correct strategies and 80 percent correctly solved the problems. Although the most common strategy was making a set or sets with the counters and then counting the entire set, many children counted on from one of the numbers, and still others used a fact strategy, such as "I know 5 and 5 are 10, so 5 and 7 must be 2 more, which is 12." The children did not do quite as well on subtraction problems, but more than three-fourths of them used correct strategies for the problems (Carpenter, Hiebert, & Moser 1981).

Knowing that young children have natural intuitions about number and the ability to solve verbal problems, educators have developed curricula that encourage children to use their own strategies. One very successful program is Cognitively Guided Instruction (CGI) (Carpenter, Fennema, & Franke 1996; Carpenter et al. 1999). CGI helps teachers understand and encourage the use of children's intuitive strategies. Although CGI initially targeted first grade teachers, it has clear implications for and has been used in kindergarten through third grade.

Rather than using number sentences (for example, 2 + 4 = ?) for the children to solve, CGI presents children with word problems verbally and in written form and asks them to find and then explain their own ways of solving those problems. Over time, children naturally begin to write number sentences to solve their problems, but teachers do not introduce formal ways of writing mathematics and of solving problems until children are comfortable with their own strategies. A common theme of CGI and similar programs is encouraging children to share their ideas with each other so they come to understand the various approaches peers use to solve problems.

A number of studies on the effectiveness of developmentally appropriate, intuition-based programs like CGI show that children in these programs gain a better conceptual understanding of mathematics without losing computational ability. One research team (Cobb et al. 1991), for example, compared the performance of second grade children in 10 classrooms in which teachers followed Vygotskian principles and "instruction was generally compatible with a socioconstructivist theory of knowledge" (p. 3) to children in 8 classrooms using traditional instruction. Using methods similar to CGI methods, researchers gave children in the socioconstructivist classrooms relatively challenging word problems, and the children were to work with classmates to develop their own methods of solving those problems. Levels of computational skill were similar in both types of classrooms, but children in the socioconstructivist classrooms felt less bound to traditional solution methods, had higher levels of conceptual understanding of mathematics, and believed more in the importance of collaboration in solving problems (Cobb et al. 1991).

Results of a four-year study of 21 primary grade CGI teachers and their classes were similar to those in the study of socioconstructivist classrooms. Fennema and colleagues (1996) found that CGI students' computational abilities improved at the rate expected from traditional instruction, yet their ability to solve problems and their understanding of underlying mathematics concepts improved significantly more than was typical with traditional instruction. Moreover, "students showed increasingly greater gains the longer they were in CGI classes" (p. 430).

One expectation of the CGI program is that teachers will gain a better understanding of how children intuitively solve problems and that this understanding can help them encourage their students to build problem-solving skills. In assessing instruction methods, the Fennema team (1996) found that of the 21 teachers, 18 fundamentally changed their views of what it means to teach mathematics. This change is important because "changes in instruction of the individual teachers were directly related to changes in their students' achievement" (p. 403). In short, the study shows that when primary-grade teachers believe that children can solve problems using their own rather than "textbook" strategies, and when they encourage students to build on their own intuitive strategies, problem-solving skills and understanding of mathematical processes improve without loss of computational proficiency.

Encouraging young children to use their intuition for number

While it may seem obvious, the key to getting children to use their intuition is giving them opportunities to use it. Hiebert and colleagues (1997) focus on several classroom features that help children understand mathematics. The first is the nature of the classroom task. While the research team focuses on tasks (here, verbal problems) for the primary grades, selection of appropriate tasks is just as important in the pre-K setting. Consider this exchange between teacher and child in a preschool classroom:

Educators have developed curricula that encourage children to use their own strategies.

> **Children who come to expect help with every problem lose faith in their intuition and never develop the confidence needed to tackle problems alone.**

During grocery store play, Myoung, the teacher, says to five-year-old Kailee, "Suppose you and Jane buy 12 bananas. You want to share them so that each of you has the same number of bananas. How many bananas do you get?" Kailee replies that she doesn't have enough fingers to answer the question. Myoung says that she can borrow his, but Kailee ignores the offer and says she will do it in another way. She draws a big rectangle-like shape and writes numbers in it—1, 2, 3, 4, 5, 6, 7 in the first row and 8, 9, 10, 11, 12 in the second row. Then she draws a line in the middle of the shape and counts the number of numerals on each side.

1 2 3 4	5 6 7
8 9 10	11 12

Kailee's Diagram

Kailee is clearly bothered by the difference between the sets, so Myoung offers her a stack of yellow counting tiles. Kailee counts out 12 tiles and then divides them into two groups by putting the first tile in pile A, the second in pile B, the third in pile A, the fourth in pile B, and so on. When all the tiles are used, she checks the height of both piles and notes that they have the same number. Then she counts the number of tiles in one pile. "Six bananas!" she declares.

When the answer was not immediately obvious to Kailee, she did not give up. The best way for teachers to know if a task is at the right level for a child is to try different tasks and adjust the difficulty up or down, depending on the child's response. In the language of CGI (see "CGI Multiplication and Division Problem Types, Examples, and Typical Strategies," p. 54), this was a *partitive division problem.* Partitioning a set into equal subsets by placing counters into piles one at a time is a strategy children often use. However, as will be discussed later, the real message of this example is that children should be allowed to figure out their own methods for solving problems.

Teachers often ask how much help they should give when a child appears frustrated. In this situation Myoung suggested that Kailee use tiles to model the problem, but once Kailee gets comfortable using counters, such a prompt may be unnecessary. There is no rule for when to provide scaffolding during problem-solving activities and when to let a child struggle. Children who come to expect help with every problem lose faith in their intuition and never develop the confidence needed to tackle problems alone. More often than not, adults give too much help; yet, there certainly is a point at which too much struggling can diminish enthusiasm for solving problems.

One of the issues to address when selecting problems is that appropriate task complexity varies considerably among children of the same age. When working with a group of children, teachers should look for tasks that can be adapted to make them easier or harder. For example, if another child has trouble with the same banana problem that Kailee had, one option would be to restate the problem with 4 or 6 bananas rather than 12. Or for a child who quickly solves the problem, the teacher might ask him to share the 12 bananas among 3 or 4 children or ask how many bananas Ben would have if Ben had 2 more than he.

As has been stressed, to be meaningful to children, math problems need to be expressed in some sort of concrete context. In the problem that follows, children use counters (for example, cubes or tiles), and while the counters do not stand for other objects such as bananas, they are real objects and thus more meaningful than symbols. In this situation, the cubes are the "real world" context that makes the problem concrete for children.

Vicki gives each child in her preschool classroom a stick made of 10 interlocking cubes stacked together. She asks them to hold the cube sticks behind their backs and break them into two parts, then bring only one part forward. Vicki then asks the children, "How many cubes are left in the stick behind your back?" Some children try to count the cubes hidden behind their back by fingering them, but others try to figure out the answer based on the number of cubes in the part brought forward.

Andrew, counting 7 cubes in his one hand, announces he has 3 behind his back. Seeing 7, he knows that 3 more makes 10. Jenna looks at the cubes in her left hand and counts, "One, 2, 3, 4, 5, 6," pauses, puts the stack from her left hand on the table and says, "7 [unfolding one finger], 8 [unfolding two fingers], 9 [three fingers], 10 [four fingers]." Looking up, she says, "I have 4."

To extend this problem the teacher could have children work in pairs to figure out how many cubes their partner has behind his or her back, or she could change the total number of cubes to create a new problem. Sharing reasoning helps children see other ways of thinking about a problem. It also affirms for children that the strategies that make sense to them (their intuitive strategies) are appropriate.

Identifying problems by type

Although researchers differ somewhat on the names for various problem types (Fuson 1992), it is useful for teachers to consider problem classification when giving verbal problems to children. Carpenter and colleagues (1999) identify three types of *join* and *separate* problems, two types of *part-part-whole* problems, three types of *compare* problems, a single type of *multiplication* problem, and two types of *division* problems. (Join, separate, part-part-whole, and compare problem types are summarized in "CGI Addition and Subtraction Problem Types, Examples, and Typical Strategies"; multiplication and division problem types in "CGI Multiplication and Division Problem Types, Examples, and Typical Strategies," p. 54.)

CGI Addition and Subtraction Problem Types, Examples, and Typical Strategies

Problem Type	Example	Typical Strategy*
Join result unknown	Bob has 7 apples. Jane gives him 5 more. How many apples does Bob have now?	Children count out a set of 7, add 5 more to that, and then count the total.
change unknown	Bob has 7 apples. Jane gives him some more. Now he has 12 apples. How many apples did Jane give to Bob?	Children count out a set of 7 and add counters to that until there is a total of 12 counters. Then children count the number of counters added to the initial set.
beginning unknown	Bob has some apples. Jane gives him 5 more. Now he has 12 apples. How many apples did Bob have to start with?	Children start with a pile of counters and add 5 to that pile. Then they count the new pile. If the total is not 12, they adjust the pile by either adding more counters or taking some away until they have 12. Then they take away 5 and find that 7 remain.
Separate result unknown	Katie has 12 apples. She gives Nick 5 apples. How many apples does Katie have left?	Children count out a set of 12 counters and remove 5 from the set. Then children count the remaining counters.
change unknown	Katie had 12 apples. She gives some to Nick. Now she has 7 apples left. How many apples did she give to Nick?	Children count out a set of 12 counters. Then they remove counters from the set until the number of remaining counters is 7. They count the number of counters removed.
beginning unknown	Katie has some apples. She gives Nick 5 apples. Now she has 7 apples left. How many apples did Katie have to start with?	Children start with a pile of counters. They remove 5 counters from the pile and count the remaining counters. If there are not 7 remaining, they adjust the remaining counters by adding more or taking some away until there are 7 counters. They then add back the 5 counters and count the whole pile to get 12.
Part-part-whole whole unknown	Enrique has 5 nickels and 7 pennies. How many coins does he have?	Children make two sets of counters, one containing 5 and the other containing 7. Then they count the total.
part unknown	Enrique has 12 coins. 5 are nickels and the rest are pennies. How many pennies does he have?	Children count out a set of 12 counters. Then they remove 5 counters from the set and count the remaining counters.
Compare difference unknown	Chris has 5 marbles. Ellen has 12 marbles. How many more marbles does Ellen have than Chris?	Children make two sets of counters, one containing 5 and the other containing 7. Then they match counters from both sets in a 1-to-1 manner until one set is used up. They count the number of unmatched counters in the larger set.
larger set unknown	Chris has 5 marbles. Ellen has 7 more than Chris. How many marbles does Ellen have?	Children count out a set of 5 counters, add 7 more to that, then count the total.
smaller set unknown	Ellen has 12 marbles. She has 7 more than Chris. How many marbles does Chris have?	Children make a row of 12 counters. Then they construct another row of counters just below the first, adding counters until the difference in number between the rows becomes 7. They then count the number of counters in the second row.

*These are typical strategies for relatively novice problem solvers, although other strategies are possible and frequently seen. Beginning unknown problems are challenging because children are not sure how to model the initial set. Children often try a variety of trial-and-error-based strategies to solve this type of problem, although the strategy described is common. Part-part-whole and compare problems tend to be more difficult than join and separate problems because there is no explicit action described in the problems.

Adapted from Carpenter et al. 1999, p. 12.

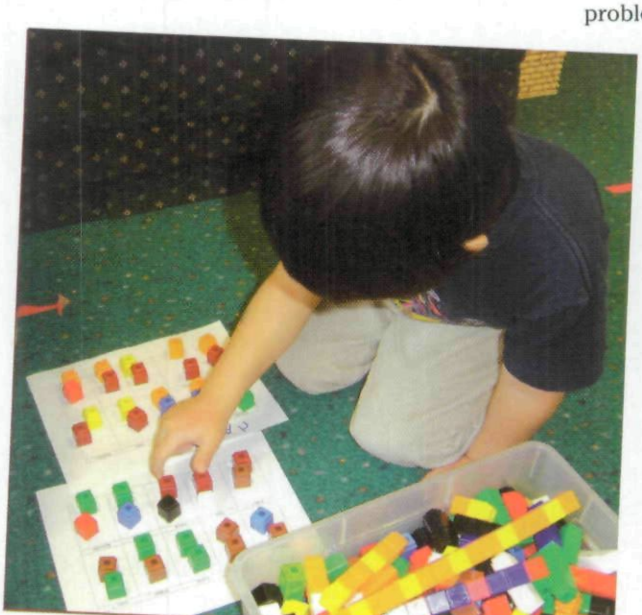
CGI Multiplication and Division Problem Types, Examples, and Typical Strategies

Problem Type	Example	Typical Strategy
Multiplication	Nichole has 4 boxes of beads. Each box has 5 beads. How many beads does she have in all?	Count out a group of 5, then another set of 5, a third set, and the fourth set. Then children count the total in all sets.
Measurement division	Nichole has 20 beads. She wants enough boxes to put 5 beads in each. How many boxes does she need?	Count out 20 counters. Then children make a pile of 5 counters, another pile of 5, and so forth until all the counters are used up. Then they count the number of piles.
Partitive division	Nichole has 20 beads. She puts the beads in 4 boxes, with the same number of beads in each box. How many beads are in each box?	Count out 20 counters and then put one counter in each of 4 separate piles. Children continue by putting a second counter in each pile and then a third, until all counters are used. Then they count the number of counters in each pile.

Adapted from Carpenter et al. 1999, p. 34.

Knowledge of problem types makes it easier for a teacher to avoid inadvertently confusing children. For example, in Kailee's partitive division problem, the teacher asked Kailee to divide a set into equal parts. But the teacher could have posed the problem as a *measurement division problem*: "Suppose you and Jane bought 12 bananas and you want to give 3 bananas to as many people as you can. How many people will get bananas?" In this case, rather than dividing a set into equal parts, the problem involves "measuring out" parts of one size into subsets and then determining how many subsets have been constructed (see "CGI Multiplication and Division Problem Types, Examples, and Typical Strategies"). Adults who are used to doing division often see no difference between partitive and measurement division. To young children, however, they are very different.

The most important reason that knowledge of problem types is useful is that conscious awareness of problem type gives important clues about the way in which children solve the problem. Intuitive strategies for solving verbal problems are usually based on the way the problems are worded (Carpenter, Hiebert, & Moser 1981; Carpenter, Fennema, & Franke 1996; Warfield 2001). In Kailee's banana problem (p. 52), where the task involved breaking a whole into two equal parts, the methods that Kailee tried involved breaking 12 into equal parts. Her first attempt involved writing numbers in a rectangle. This is an *equal parts strategy;* Kailee just could not figure out how to get the same number of numerals on each side of the rectangle. She solved the problem by using tiles to make equal stacks.



Had Kailee been asked to solve the problem about how many people would get 3 bananas, it is likely she would have started by making a pile of 3 tiles (representing bananas) for the first person, another pile of 3 tiles for the second person, and so forth until she ran out of tiles. She would then have counted the number of piles and discovered that 4 people could have 3 bananas (see *measurement division* in "CGI Multiplication and Division Problem Types, Examples, and Typical Strategies").

Too often, adults fail to see why children treat these two types of problems differently; we try to get children to use a partitive model for a measurement problem or vice versa. However, even when adults do not initially see problems the way young children do, we can understand children's intuitions by listening as they explain how they solve the problems. Consider another actual classroom example between the teacher and Chad:

Myoung places a toy horse and about 20 yellow counters in front of five-year-old Chad. "Could you help me solve this problem?" he asks Chad. "Horse has 9 treats . . ." Chad counts out 9 counters and arranges them in a horizontal line. "Now what?" he asks. Myoung responds, "She [Horse] has 4 more than her friend. How many does her friend have?" Chad makes another line of counters, above the first, continuing to add counters until the difference in number between the two lines is 4. "Horse has more than her friend," he says. Myoung says, "Right! So how many does her friend have?" Chad counts the counters in the top line and announces, "Five."

Mathematically, this problem involves subtraction. However, in contrast to a subtraction problem that involves traditional *take away* (like "Cathy had 9 cookies. She ate 5 of them. How many cookies does Cathy have left?") the wording of the horse/treat problem implies comparison of sets. Making two sets made sense to Chad because the problem indicated that he needed a set of treats for Horse and another set for her friend. Chad then did what the problem implied: figured out how many are available for the friend if 4 are left on the table.

When children's intuitions are respected and valued, and when they are encouraged to listen to other children explain how they answer questions, they naturally pick up more advanced ways of solving problems.

As Chad becomes more comfortable with numbers, he will no longer need to model two sets to solve a comparison problem like this. Stepping in and pushing Chad to make a set of 9 and take away 4 would undermine his intuition. And asking him to write a number sentence (9 – 4 = 5) to solve the problem probably is of no use. Listening to Chad and observing and encouraging his strategies is the best way to build his confidence in his problem-solving skills.

More advanced strategies

The two CGI chart summaries include samples of strategies children are likely to use when first confronted with join, separate, part-part-whole, compare, and multiplication and division problems. When children's intuitions are respected and valued, and when they are encouraged to listen to other children explain how they answer questions, they naturally pick up more advanced ways of solving problems. Consider this scenario:

Myoung says to five-year-old Ashley, "If you bought 6 apples in the morning and bought 4 more apples in the afternoon, how many apples

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do you have altogether?" Ashley uses her fingers to count 6 and then 4 more and responds, "Ten." Myoung then asks, "What if you had 6 apples and bought 5 more?" When Ashley quickly answers, "Eleven," Myoung asks, "How do you know that?" Ashley explains, "Because the last one was 4 and 6. This time was 5 and 6. Four and 6 make 10, and one more is 11."

Another child might have taken the second question and repeated the process used for the first, but Ashley was comfortable enough with what she knew to build on her response to the first question. It is appropriate for a teacher to prompt children to see if they can come up with more advanced problem-solving strategies; but if they cannot, there is no need to push. The children will use more sophisticated strategies when they are ready.

Finally, consider the following class activity:

Vicki gives each child in her classroom a sheet of paper containing a list of everyone's names. She says, "There are 20 children in our class, right? I'd like to know how many noses there are in the room." Many children shout out "Twenty!" Vicki asks, "How do you know that? You did not go around counting everyone's nose—1, 2, 3 . . ." Jeff responds, "Because everybody has one nose," and Vicki says, "Good! Now, this is going to be the challenging part: How many ears are in our classroom?"

Some children immediately say 20, but five-year-old Ian disagrees, stating, "No, it is 20 plus 20." Vicki replies, "Yes, that's one way to get the answer." Having noticed that five-year-old Emma had touched each name twice when counting, Vicki says, "Emma, I saw you doing something there with your sheet of paper. What were you doing? Why don't you tell everyone?" Emma answers, "I was touching each name two times." Vicki says, "So you are doing 1, 1, 2, 2, 3, 3?" "No," Emma says, "It's 1, 2, [pause] 3, 4 [pause]."

The children continue working to get their own solutions, some using manipulatives such as color tiles and Unifix cubes and others using letters (like E for eyes) or dots. Later, Vicki asks an even more challenging question: "How many noses and ears are in the classroom?"

In this situation, the first task—finding the number of noses—was relatively straightforward, and it was clear from the multiple responses to the question about noses that most children understood the answer. Vicki moved quickly to the more challenging question of how many ears were in the classroom. Note that when children were presenting strategies for determining the number of ears, Vicki accepted their ideas, but the classroom culture was one in which the children were expected to continue working on the problem to get their own solutions. Too often, when a problem is solved by one child, the other children think it is time to move on. In this setting, the children believed they had to find answers that made sense to them. After working on the total noses and ears problem, they shared the multiple ways in which they had solved it.

Getting correct answers is important, but it is the process of getting those answers—including talking through procedures and coming to consensus on an answer—that is key in getting children to build and trust their intuitions. Building this type of classroom culture takes time and thought, but when children know they are expected to think and explain and when teachers listen to what children have to say, deep learning of mathematical ideas occurs.

Conclusion

Young children have a natural capacity for number and considerable intuition for solving problems. The vignettes in this article, all with children who can count small sets, demonstrate that to solve problems, children do not need to be able to count appreciably larger sets or to write number sentences like those used in primary grade textbooks. In fact, when allowed to use their own strategies to solve problems, many children come up with their own primitive forms of number sentences as an aid in solving problems (Hiebert et al. 1997).

When teachers know the categorizations of simple word problems, such as those used in Cognitively Guided Instruction, they can provide structure and variety in the problems they pose to children. More important, however, is building a classroom culture in which children are expected to share their thinking and encouraged to use their intuition. Early childhood educators are accustomed to inviting young children to share their ideas and thoughts as part of a literacy curriculum. We need to promote the same type of expression in number and problem solving. Knowledge of how children solve problems is helpful, and the easiest way to get that knowledge is to listen to the children. They are eager to share.

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