

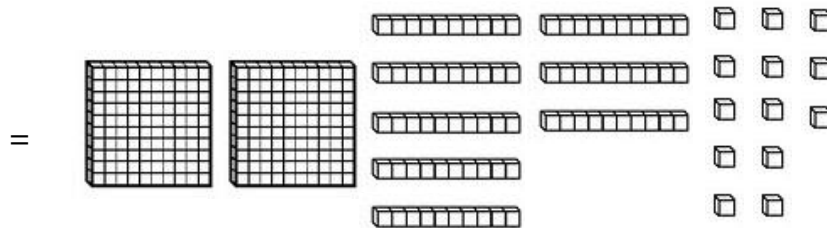
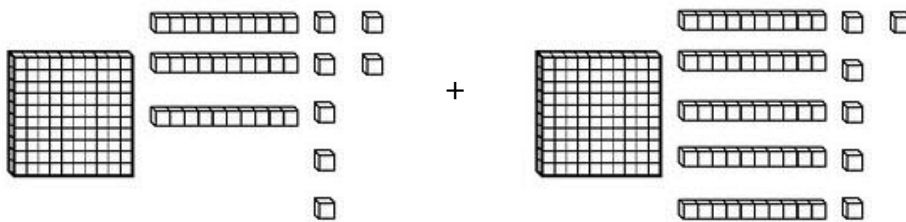
Direct Modeling by 1's–

- Draw to represent problem.
 $12 + 4 =$

**Direct Modeling by 10's–****By Tens**

- Draw to represent problem.

$$137 + 156 =$$

**Counting On****137 + 156**

137, 138, 139, 140, 141.....has to be by ones for +/- . Kids will abandon because it takes so much time.

Relational Thinking –

- **Combining like Place Value Units. Tens and Ones**

$$137 + 156 =$$

$$100 + 100 = 200$$

$$30 + 50 = 80$$

$$7 + 6 = 13$$

$$200 + 80 = 280$$

$$280 + 10 + 3 = 293$$

Relational Thinking –

Incrementing/uses relational thinking

Incrementing

- Break apart one number into tens and ones.

$$137 + 156 =$$

$$137 + (150 + 6) =$$

$$(137 + 150) + 6 =$$

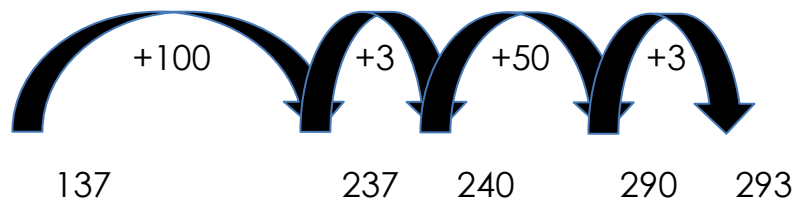
$$287 + 6 =$$

$$293$$

$$\begin{array}{r} 137 \\ + 156 \\ \hline 287 \text{ (137 + 150)} \\ + \quad 6 \\ \hline 293 \end{array}$$

- Use an open number line to add in chunks.

$$137 + 156 =$$



Relational Thinking –**Friendly Numbers/Compensating**

- Breaking apart or using a larger a number to make a friendly number.

$$\begin{aligned}
 137 + 156 &= \\
 (133 + 4) + 156 &= \\
 133 + (4 + 156) &= \\
 133 + 160 &= \\
 293
 \end{aligned}$$

$$\begin{aligned}
 137 + 156 &= \\
 \underline{- 4} \quad \underline{+ 4} & \\
 133 + 160 &= 293
 \end{aligned}$$

Relational Thinking –**Formal Algorithm**

- Break apart both numbers into place values (expanded form).

$$\begin{aligned}
 137 + 156 &= \\
 (100 + 30 + 7) + (100 + 50 + 6) &= \\
 (100 + 100) + (30 + 50) + (7 + 6) &= \\
 200 + 80 + 13 &= \\
 293
 \end{aligned}$$

$$\begin{aligned}
 137 &= 100 + 30 + 7 \\
 + 156 &= 100 + 50 + 6 \\
 \hline
 200 + 80 + 13 &= 293
 \end{aligned}$$

$$\begin{array}{r}
 137 \\
 + 156 \\
 \hline
 200 \text{ (100 + 100)} \\
 80 \text{ (30 + 50)} \\
 + 13 \text{ (7 + 6)} \\
 \hline
 293
 \end{array}$$

Direct Modeling –

Base 1

- Draw to represent problem. $11 - 1 = 10$

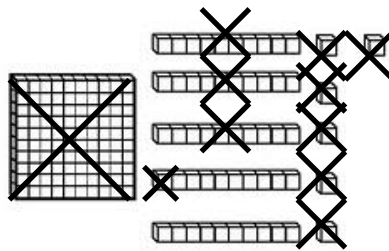


Direct Modeling –

Base 10

- Draw to represent problem.

$$156 - 137 =$$



Counting (would be by ones)

156, 155, 154, 153, 152, 151 22, 21, 20, 19

Most kids abandon this strategy because it takes too long

Relational Thinking –

Place Value Units

- Break apart one number into tens and ones.

$$156 - 137 =$$

$$100 - 100 = 0$$

$$50 - 30 = 20$$

$$6 - 7 = -1$$

$$20 - 1 = 19$$

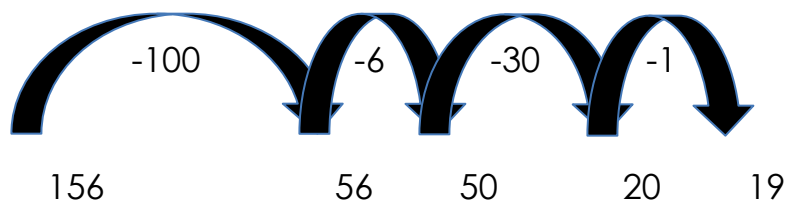
$$\begin{array}{r} 156 = 100 + 50 + 6 \\ - 137 = - 100 + 30 + 7 \\ \hline 0 + 20 - 1 = 19 \end{array}$$

Relational Thinking –

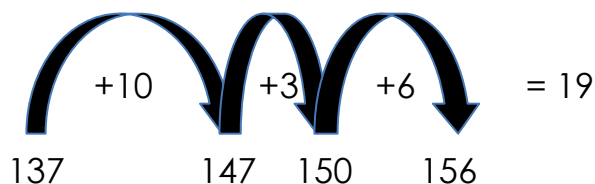
Incrementing

- Use an open number line to subtract or add in chunks.

$$156 - 137 =$$



$$156 - 137 =$$



$$156 - 137 =$$

$$156 - (130 + 7) =$$

$$(156 - 130) - 7 =$$

$$26 - 7 =$$

$$19$$

$$\begin{array}{r} 156 \\ - 137 \\ \hline 26 \quad (156 - 130) \\ - 7 \\ \hline 19 \end{array}$$

**Relational Thinking –
Compensating**

- Breaking apart or using a larger a number to make a friendly number.

$$\begin{aligned}
 156 - 137 &= \\
 (156 + 3) - (137 + 3) &= \\
 159 - 140 &= 19 \\
 100 - 100 &= 0 \\
 50 - 40 &= 10 \\
 9 - 0 &= 9 \\
 &= 19
 \end{aligned}$$

**Relational Thinking –
Formal Algorithm**

- Break apart both numbers into place values (expanded form).

$$\begin{aligned}
 156 - 137 &= \\
 (100 + 50 + 6) - (100 + 30 + 7) &= \\
 (100 - 100) + (50 - 30) + (6 - 7) &= \\
 0 + 20 - 1 &= \\
 19
 \end{aligned}$$

$$\begin{array}{r}
 156 \\
 - 137 \\
 \hline
 0 \text{ (100 - 100)} \\
 + 20 \text{ (50 - 30)} \\
 - 1 \text{ (6 - 7)} \\
 \hline
 19
 \end{array}$$

Modeling –**By Ones**

- Draw to represent problem.

$$25 \times 43 =$$

- (1)                                  

- Repeatedly adding groups of same numbers.

43
43 → 86
43 → 86 → 172
43 → 86 → 172 → 344
43 → 86 → 172 → 344 → 688
43 → 86 → 172 → 344 → 688 → 1032
43 → 86 → 172 → 344 → 688 → 1032 → 1075
43 ————— + 43
1075

Factor x Factor = Product**Efficient Multiplication****Relational Thinking –
Complex Doubling**

- Double groups of factor repeatedly.

$$25 \times 43 =$$

$$\begin{array}{r}
 43 \text{ (1)} \\
 + \underline{43 \text{ (1)}} \\
 86 \text{ (2)} \\
 + \underline{86 \text{ (2)}} \\
 172 \text{ (4)} \\
 + \underline{172 \text{ (4)}} \\
 344 \text{ (8)} \\
 + \underline{344 \text{ (8)}} \\
 688 \text{ (16)} \\
 + \underline{344 \text{ (8)}} \\
 1032 \text{ (24)} \\
 + \underline{43 \text{ (1)}} \\
 1075 \text{ (25)}
 \end{array}$$

**Relational Thinking –
Partitioning Multiplicatively
(Associative Property)**

- Break apart factor(s) into easier parts using *multiplication*.

$$25 \times 43 =$$

$$(5 \times 5) \times 43 =$$

$$5 \times (5 \times 43) =$$

$$5 \times 215 =$$

$$1075$$

**Relational Thinking –
Partitioning one of the factors
(Distributive Property)**

- Break apart factor into tens and ones.

$$25 \times 43 =$$

$$(10 \times 43) + (10 \times 43) + (5 \times 43) =$$

$$430 + 430 + 215 =$$

$$860 + 215 =$$

$$1075$$

$$\begin{array}{r}
 25 \\
 \times 43 \\
 \hline
 430 \text{ (10} \times 43\text{)} \\
 430 \text{ (10} \times 43\text{)} \\
 + \underline{215 \text{ (5} \times 43\text{)}} \\
 1075
 \end{array}$$

**Relational Thinking –
Partitioning Both Factors
(Distributive Property)**

- Break apart both numbers into place values (expanded form).

$$25 \times 43 =$$

(Adapted Area Model)

	40	3
20	800	60
5	200	15

$$1000 + 75 = 1075$$

$$\begin{array}{r}
 25 \\
 \times 43 \\
 \hline
 15 \text{ (3} \times 5\text{)} \\
 60 \text{ (3} \times 20\text{)} \\
 200 \text{ (40} \times 5\text{)} \\
 + \underline{800 \text{ (40} \times 20\text{)}} \\
 1075
 \end{array}$$

**Relational Thinking –
Partitioning Additively
(Distributive Property)**

- Break apart factor(s) into easier parts using *addition*.

$$\begin{aligned}
 25 \times 43 &= \\
 (25 \times 40) + (25 \times 3) &= \\
 (25 \times 4 \times 10) + (25 \times 3) &= \\
 (100 \times 10) + 75 &= \\
 1000 + 75 &= \\
 1075 &
 \end{aligned}$$

$$\begin{array}{r}
 25 \\
 \times 43 \\
 \hline
 1000 \quad (25 \times 40) = (25 \times 4 \times 10) \\
 + \underline{75} \quad (25 \times 3) \\
 \hline
 1075
 \end{array}$$

**Relational Thinking –
Compensation**

- Using a larger a number to make a friendly number.

$$\begin{aligned}
 25 \times 43 &= \\
 25 \times (50 - 7) &= \\
 (25 \times 50) - (25 \times 7) &= \\
 1250 - 175 &= 1075
 \end{aligned}$$

**Relational Thinking –
Ratio**

- Adding and/or multiplying to build up to factor.

$$25 \times 43 =$$

1	43
2	86
3	129
5	215
10	430
20	860
25	1075

**Relational Thinking
Doubling and Halving (works best
when one or both factors are even)**

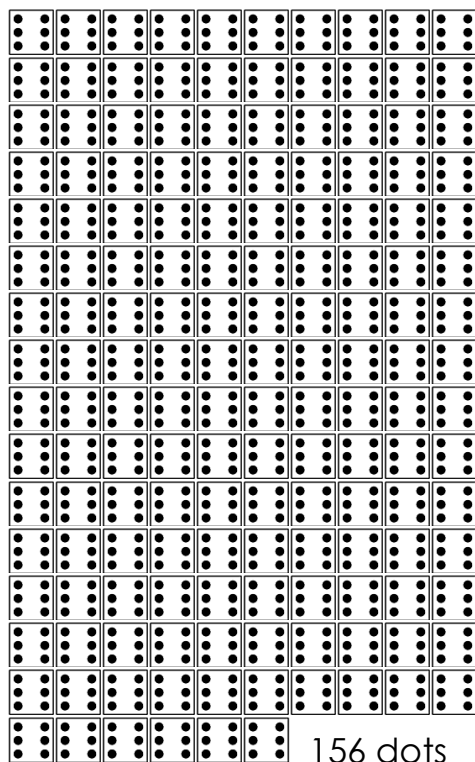
$$\begin{aligned}
 12 \times 36 &= 6 \times 72 \\
 6 \times 2 \times 36 &= 6 \times 72 \\
 (6 \times 2) \times 36 &= 6 \times (2 \times 36)
 \end{aligned}$$

Modeling –**By Ones or Tens and Ones**

- Draw to represent problem.

$$936 \div 6 =$$

6 in each group

**Counting –****Repeated Subtraction/Addition/Skip Counting**

- Repeatedly subtract or add divisor.

$$36 \div 9 =$$

$$36 - 9 = 27$$

$$27 - 9 = 18$$

$$18 - 9 = 9$$

$$9 - 9 = 0$$

4 nines to make 36

Counting –**Simple Doubling**

$$36 \div 9 =$$

$$0 + 9 = 9$$

$$9 + 9 = 18$$

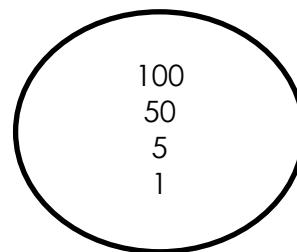
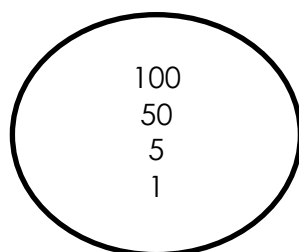
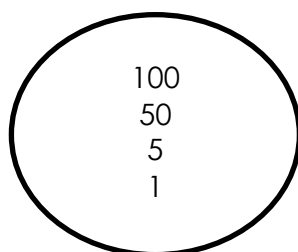
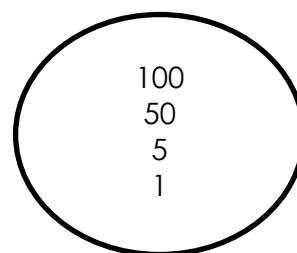
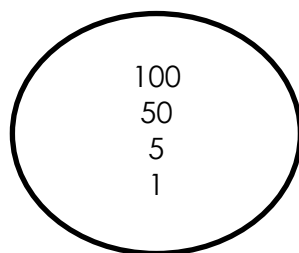
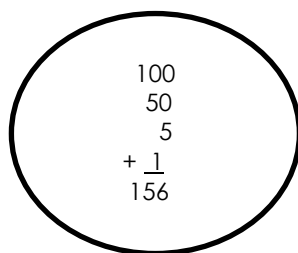
$$18 + 9 = 27$$

$$27 + 9 = 36$$

4 nines to make 36

$$936 \div 6 =$$

6 groups



Relational Thinking –**Building Up to Dividend**

- Add and/or multiply divisor to build up to dividend.

$$936 \div 6 =$$

1	6
2	12
3	18
5	30
6	36
10	60
100	600
50	300
150	900
156	936

Relational Thinking –**Partitioning the Dividend with Tens and Ones**

- Break apart dividend into tens and ones

$$936 \div 6 =$$

$$(900 \div 6) + (30 \div 6) + (6 \div 6) =$$

$$150 + 5 + 1 =$$

$$156$$

$$936 \div 6 =$$

$$900 \div 6 = 150$$

$$30 \div 6 = 5$$

$$6 \div 6 = \underline{1}$$

$$156$$

Relational Thinking –**Complex Doubling**

- Double groups of divisor repeatedly.

$$936 \div 6 =$$

$$\begin{array}{r}
 6 \text{ (1)} \\
 + \underline{6 \text{ (1)}} \\
 12 \text{ (2)} \\
 + \underline{12 \text{ (2)}} \\
 24 \text{ (4)} \\
 + \underline{24 \text{ (4)}} \\
 48 \text{ (8)} \\
 + \underline{48 \text{ (8)}} \\
 96 \text{ (16)} \\
 + \underline{96 \text{ (16)}} \\
 192 \text{ (32)} \\
 + \underline{192 \text{ (32)}} \\
 384 \text{ (64)} \\
 + \underline{384 \text{ (64)}} \\
 768 \text{ (128)} \\
 + \underline{96 \text{ (16)}} \\
 864 \text{ (144)} \\
 + \underline{48 \text{ (8)}} \\
 912 \text{ (152)} \\
 + \underline{24 \text{ (4)}} \\
 936 \text{ (156)}
 \end{array}$$

Relational Thinking –**Partitioning the Divisor**

- Break apart numbers into easier parts. (right distributive)
- Can only break dividend apart additively

$$936 \div 6 =$$

$$936 \div (3 \times 2) =$$

$$(936 \div 3) \div 2 =$$

$$312 \div 2 =$$

$$156$$

$$936 \div 6 =$$

$\begin{array}{r} 12 \\ 3 \overline{) 936} \\ \underline{- 900} \\ 36 \\ \underline{- 36} \\ 0 \end{array}$	$\begin{array}{r} 6 \\ 2 \overline{) 312} \\ \underline{- 300} \\ 12 \\ \underline{- 12} \\ 0 \end{array}$
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Relational Thinking –**Friendly Numbers/Compensating**

- Breaking apart or using a larger a number to make a friendly number.

$$936 \div 6 =$$

$$(1200 - 300 + 36) \div 6 =$$

$$(1200 \div 6) - (300 \div 6) + (36 \div 6) =$$

$$200 - 50 + 6 =$$

$$150 + 6 =$$

$$156$$

Relational Thinking –**Partitioning the Dividend**

- Rely on known facts and multiples of 10.

$$936 \div 6 =$$

$\begin{array}{r} 6 \overline{) 936} \\ \underline{- 600} \\ 336 \\ \underline{- 300} \\ 36 \\ \underline{- 30} \\ 6 \\ \underline{- 6} \\ 0 \end{array}$	$\begin{array}{l} 100 \times 6 \\ 50 \times 6 \\ 5 \times 6 \\ 1 \times 6 \end{array}$
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$$936 \div 6 =$$

(Adapted Area Model)

	6
100	600
50	300
+ 6	36
156	936

Relational Thinking –**Ratio**

- Dividing the divisor and dividend by a fraction equivalent to 1

$$936 \div 6 =$$

$$\frac{936}{6} \div \frac{3}{3} = \frac{312}{2} \div \frac{2}{2} = \frac{156}{1} = 156$$