Addend + Addend = Sum

Efficient Addition

Relational Thinking –
Tens and Ones

- Break apart one number into tens and ones.

137 + 156 =
137 + (150 + 6) =
(137 + 150) + 6 =
287 + 6 =
293

137
+ 156
287 (137 + 150)
+ 6
293

Relational Thinking –
Friendly Numbers/Compensating

- Breaking apart or using a larger a number to make a friendly number.

137 + 156 =
(133 + 4) + 156 =
133 + (4 + 156) =
133 + 160 =
293

137 + 156 =
- 4 + 4
133 + 160 = 293

Relational Thinking –
Formal Algorithm

- Break apart both numbers into place values (expanded form).

137 + 156 =
(100 + 30 + 7) + (100 + 50 + 6) =
(100 + 100) + (30 + 50) + (7 + 6) =
200 + 80 + 13 =
293

137 = 100 + 30 + 7
+ 156 = 100 + 50 + 6
200 + 80 + 13 = 293

137
+ 156
200 (100 + 100)
80 (30 + 50)
+ 13 (7 + 6)
293

Addend + Addend = Sum

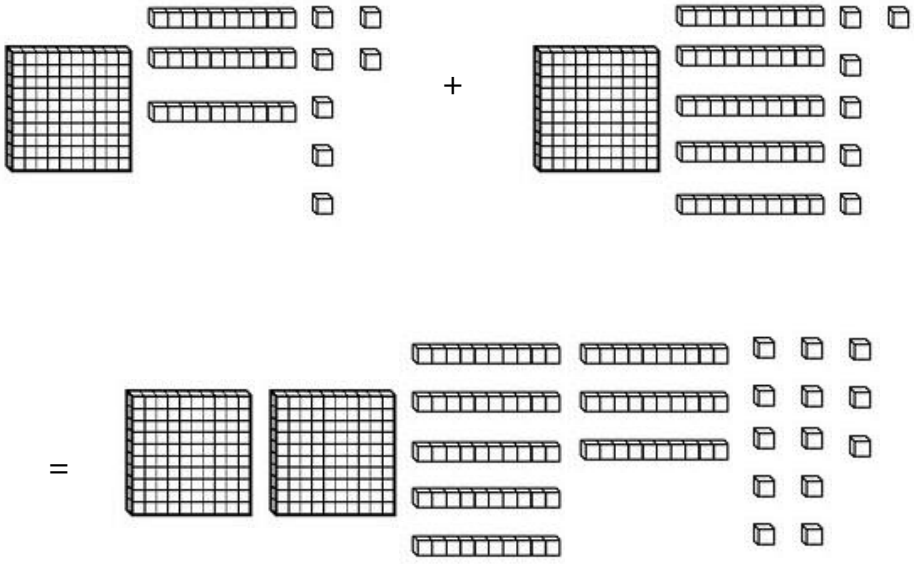
Addition

Modeling –

By Tens

- Draw to represent problem.

137 + 156 =

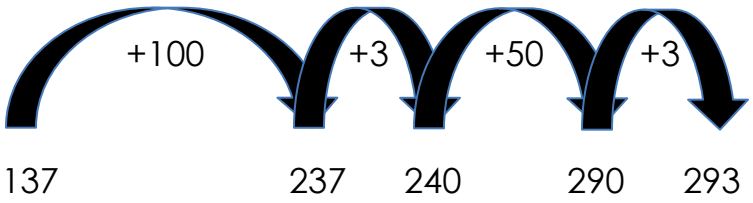


Counting –

Incrementing Up

- Use an open number line to add in chunks.

137 + 156 =



Minuend + Subtrahend = Difference

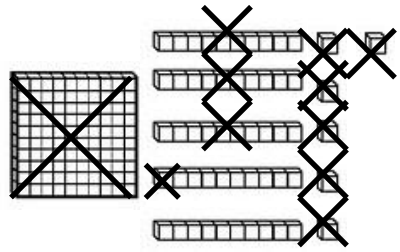
Subtraction

Modeling –

Base 10

- Draw to represent problem.

156 – 137 =

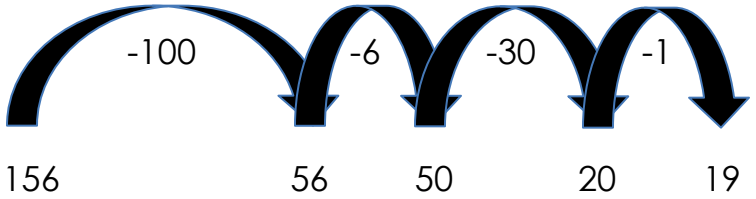


Counting –

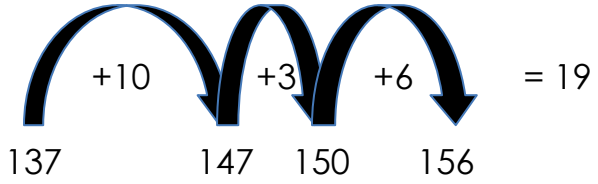
Incrementing

- Use an open number line to subtract or add in chunks.

156 – 137 =



156 – 137 =



Minuend + Subtrahend = Difference

Efficient Subtraction

Relational Thinking –
Tens and Ones

- Break apart one number into tens and ones.

156 – 137 =
156 – (130 + 7) =
(156 – 130) – 7 =
26 – 7 =
19

156
– 137
26 (156 – 130)
– 7
19

Relational Thinking –
Friendly Numbers/ Compensating

- Breaking apart or using a larger a number to make a friendly number.

156 – 137 =
156 – (135 + 2) =
156 – 135 – 2 =
21 – 2 =
19

156 – 137 =
+ 4 + 4
160 – 141 = 19

Relational Thinking –
Formal Algorithm

- Break apart both numbers into place values (expanded form).

156 – 137 =
(100 + 50 + 6) – (100 + 30 + 7) =
(100 – 100) + (50 – 30) + (6 – 7) =
0 + 20 –1 =
19

156 = 100 + 50 + 6
– 137 = 100 + 30 + 7
0 + 20 – 1 = 19

156
– 137
0 (100 – 100)
+ 20 (150 – 130)
– 1 (6 – 7)
19

Factor x Factor = Product

Efficient Multiplication

Relational Thinking –
Building Up Factor

- Adding and/or multiplying to build up to factor.

25 x 43 =

1	43
2	86
3	129
5	215
10	430
20	860
25	1075

Relational Thinking –
Complex Doubling

- Double groups of factor repeatedly.

25 x 43 =

43 (1)
+ 43 (1)
86 (2)
+ 86 (2)
172 (4)
+ 172 (4)
344 (8)
+ 344 (8)
688 (16)
+ 344 (8)
1032 (24)
+ 43 (1)
1075 (25)

Relational Thinking –
Partitioning using Tens and Ones (Distributive Property)

- Break apart factor into tens and ones.

25 x 43 =
(10 x 43) + (10 x 43) + (5 x 43) =
430 + 430 + 215 =
860 + 215 =
1075

25
x 43
430 (10 x 43)
430 (10 x 43)
+ 215 (5 x 43)
1075

Relational Thinking –
Partitioning Both Factors (Distributive Property)

- Break apart both numbers into place values (expanded form).

25 x 43 =
(Adapted Area Model)

	40	3
20	800	60
5	200	15

1000 + 75 = 1075

25
x 43
15 (3 x 5)
60 (3 x 20)
200 (40 x 5)
+ 800 (40 x 20)
1075

Relational Thinking – Partitioning Additively (Distributive Property)

- Break apart factor(s) into easier parts using *addition*.

25 x 43 =
(25 x 40) + (25 x 3) =
(25 x 4 x 10) + (25 x 3) =
(100 x 10) + 75 =
1000 + 75 =
1075

25
x 43
1000 (25 x 40) = (25 x 4 x 10)
+ 75 (25 x 3)
1075

Relational Thinking – Partitioning Multiplicatively (Associative Property)

- Break apart factor(s) into easier parts using *multiplication*.

25 x 43 =
(5 x 5) x 43 =
5 x (5 x 43) =
5 x 215 =
1075

Relational Thinking – Compensation

- Using a larger a number to make a friendly number.

25 x 43 =
25 x (50 – 7) =
(25 x 50) – (25 x 7) =
1250 – 175 = 1075

Factor x Factor = Product

Multiplication

Modeling –

By Ones

- Draw to represent problem.

25 x 43 =

(1) 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||
(2) 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||
(3) 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||
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(25) 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||

1075 tally marks

Counting –

Skip Counting/Repeated Addition

- Repeatedly adding the same number.

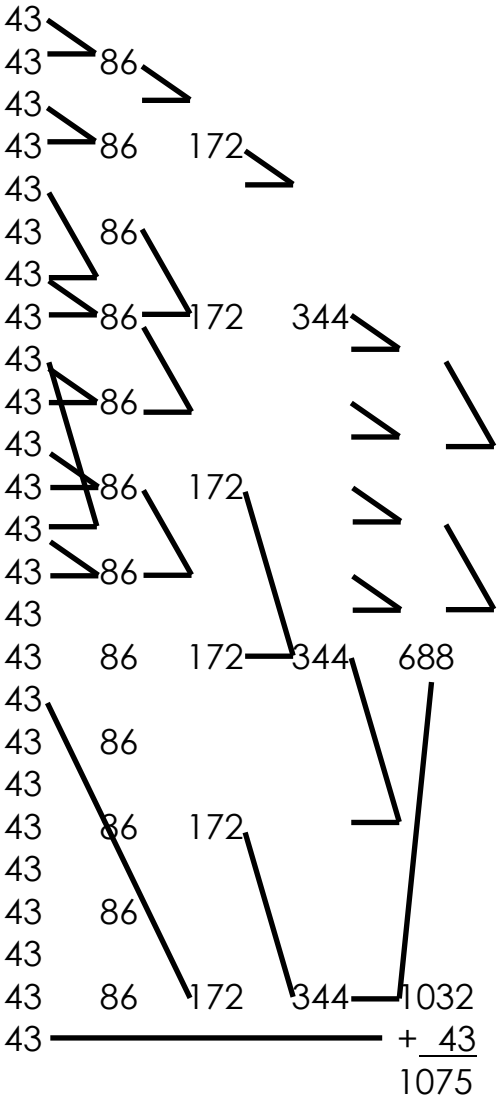
43 + 43 = 86
86 + 43 = 129
134 + 43 = 172
172 + 43 = 215
215 + 43 = 258
258 + 43 = 301
301 + 43 = 344
344 + 43 = 387
387 + 43 = 430
430 + 43 = 473
473 + 43 = 516
516 + 43 = 559
559 + 43 = 602
602 + 43 = 645
688 + 43 = 688
731 + 43 = 731
774 + 43 = 774
817 + 43 = 817
860 + 43 = 860
903 + 43 = 903
846 + 43 = 846
989 + 43 = 989
1032 + 43 = 1032
1075 + 43 = 1075

Counting –

Simple Doubling

- Repeatedly adding groups of same numbers.

25 x 43 =



Dividend ÷ Divisor = Quotient

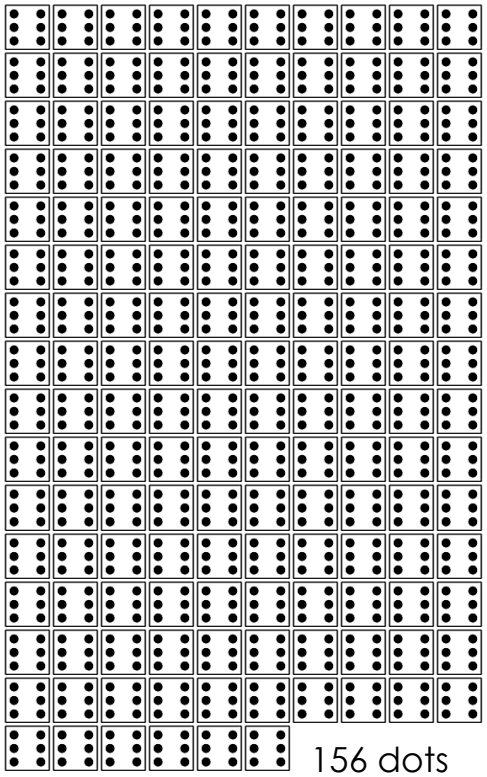
Division

Modeling –

By Ones or Tens and Ones

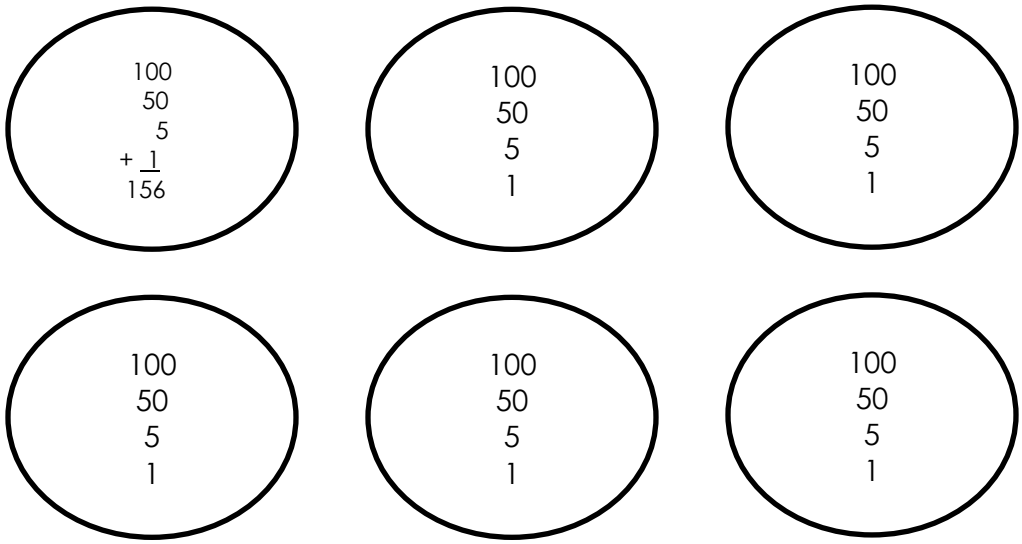
- Draw to represent problem.

936 ÷ 6 =
6 in each group



156 dots

936 ÷ 6 =
6 groups



Counting –

Repeated Subtraction/Addition/Skip Counting

- Repeatedly subtract or add divisor.

36 ÷ 9 =

36 – 9 = 27
27 – 9 = 18
18 – 9 = 9
9 – 9 = 0
4 nines to make 36

36 ÷ 9 =

0 + 9 = 9
9 + 9 = 18
18 + 9 = 27
27 + 9 = 36
4 nines to make 36

Dividend ÷ Divisor = Quotient

Efficient Division

Relational Thinking –
Building Up to Dividend

- Add and/or multiply divisor to build up to dividend.

936 ÷ 6 =

1	6
2	12
3	18
5	30
6	36
10	60
100	600
50	300
150	900
156	936

Relational Thinking –
Partitioning the Dividend with Tens and Ones

- Break apart dividend into tens and ones

936 ÷ 6 =
(900 ÷ 6) + (30 ÷ 6) + (6 ÷ 6) =
150 + 5 + 1 =
156

936 ÷ 6 =
900 ÷ 6 = 150
30 ÷ 6 = 5
6 ÷ 6 = 1
156

Relational Thinking –
Complex Doubling

- Double groups of divisor repeatedly.

936 ÷ 6 =

6 (1)
+ 6 (1)
12 (2)
+ 12 (2)
24 (4)
+ 24 (4)
48 (8)
+ 48 (8)
96 (16)
+ 96 (16)
192 (32)
+ 192 (32)
384 (64)
+ 384 (64)
768 (128)
+ 96 (16)
864 (144)
+ 48 (8)
912 (152)
+ 24 (4)
936 (156)

Relational Thinking –
Partitioning the Divisor

- Break apart numbers into easier parts.

936 ÷ 6 =
936 ÷ (3 x 2) =
(936 ÷ 3) ÷ 2 =
312 ÷ 2 =
156

936 ÷ 6 =

$$\begin{array}{r} 12 \\ 300 \\ 3\overline{)936} \\ -\,900 \\ \hline 36 \\ -\,36 \\ \hline 0 \end{array} = 312 \qquad \begin{array}{r} 6 \\ 150 \\ 2\overline{)312} \\ -\,300 \\ \hline 12 \\ -\,12 \\ \hline 0 \end{array} = 156$$

Relational Thinking –
Partitioning the Dividend

- Rely on known facts and multiples of 10.

936 ÷ 6 =

$$\begin{array}{rll} 6\overline{)936} & & \\ -\,600 & 100 \times 6 & \\ \hline 336 & & \\ -\,300 & 50 \times 6 & \\ \hline 36 & & \\ -\,30 & 5 \times 6 & \\ \hline 6 & & \\ -\,6 & 1 \times 6 & \\ \hline 0 & & \end{array} \quad 156 = 156$$

936 ÷ 6 =
(Adapted Area Model)

	6
100	600
50	300
+ 6	36
156	936

Relational Thinking –
Friendly Numbers/Compensating

- Breaking apart or using a larger a number to make a friendly number.

936 ÷ 6 =
(1200 – 300 + 36) ÷ 6 =
(1200 ÷ 6) – (300 ÷ 6) + (36 ÷ 6) =
200 – 50 + 6 =
150 + 6 =
156

Relational Thinking –
Ratio

- Dividing the divisor and dividend by a fraction equivalent to 1

936 ÷ 6 =

$$\frac{936}{6} \div \frac{3}{3} = \frac{312}{2} \div \frac{2}{2} = \frac{156}{1} = 156$$