Performance Task: The Hiking Trail

Adapted from Contexts for Learning Mathematics Fractions, Decimals, and Percents by Fosnot, Catherine Twomey et.al.

This task develops the concept of equivalent fractions students will need to add fractions with unlike denominators later in the unit. The purpose of this task is to encourage student development of strategies to find equivalent fractions.

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.NF.3 Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

MCC5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS

- How can looking at patterns help us find equivalent fractions?
- How are equivalent fractions helpful when solving problems?
- How does the size of the whole determine the size of a fraction?
- How can we tell if some fractions are related?

- How can learning about fractions that are related help us solve problems?

MATERIALS

- Copy of the Task The Hiking Trail (1 per pair of students or small group)
- Pencil
- Ruler (60 inch tape measure or yardstick)
- Accessible manipulatives

GROUPING

Pair/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments:

This task was adapted from *Contexts for Learning Mathematics Fractions, Decimals, and Percents* by Fosnot, Catherine Twomey et.al. A recording sheet is provided, but is not necessary for this task, especially if students are using a math journal or learning log. Students should draw representations of their mathematical thinking as well as use words and numbers to explain their thinking for three reasons:

SMP2. Reason abstractly and quantitatively.
SMP3. Construct viable arguments and critique the reasoning of others.
SMP4. Model with mathematics.

Students should be allowed to draw representations of their thinking. This allows them to "talk through" their process which in turn enables students the opportunity to attend to precision as they explain and reason mathematically.

BACKGROUND KNOWLEDGE

This task was developed from *Contexts for Learning Mathematics*, by Fosnot and Jacob. Students engaging in this task have a deep understanding of fractions and the beginnings of fraction sense fostered in previous tasks. If students need additional support in developing this fraction sense, support students with activities from *Teaching Student- Centered Mathematics*, by John A Van de Walle and LouAnn Lovin., pgs. 144 – 146 (activities 5.6 – 5.10).

Teacher Notes:

Present one computation at a time and facilitate a discussion with students, asking them to find and explain the strategy they used to find:

100/2
100/4
200/4
200/8
400/16

Introduce the problem and be sure everyone is clear with the context. Let students know that they will be designing a hiking trail for a four day Hike-a-thon. The trail is 6.0 km and is in the GA Mountains. The committee has decided what kind of informational markers and how often they should be placed. Your task is to figure out where to put informational markers along the way.

- A Camping area and Food Wagons should be at each fourth of the trail.
- Resting Points should be at every eighth of the trail.
- Water Stations should be at every tenth of the trail.
- Juice and Snack Tables should be at every fifth of the trail.
- Recycling and Trash Bins should be placed at every marker
- Kilometer Markers should be placed along the trail, so that hikers know how much of the course they've completed. These markers should be placed at every twelfth, sixth, and half of the course, as well as at all of the other locations above. These markers should show how many kilometers have been completed.

Give pairs of students sixty inch measuring tapes or yard sticks. Have the pairs draw a sixty inch hiking trail on some butcher paper.
Students may use a variety of strategies including, but not limited to:

- Halving. They may take half of the halves to find fourths, and take half of the fourths to find eighths.
- Dividing by the denominator. Students may think of 1/5 of 6.0 = 6.0/5
- Adding parts. Students may think about 3/8 as 1/8 more than 2/8.
- Use equivalence ideas developed in the ratio table task earlier. Students may say 6/8 = ¾ since 3 ***x 2*** = 6 and 4 ***x 2*** = 8

FORMATIVE ASSESSMENT QUESTIONS

- *How can you tell that your answer is correct?*
- *How do you know that marker goes there? Show me your thinking.*
- *How can you tell that your markers are in the correct place? Is there another way to think about this?*
- *Did you develop a shortcut to find your answers?*
- *Did you identify any patterns or rules? Explain what you have found!*

After enough time has been devoted to the task, hang the work around the room and have students taken some time to view and make comments on others' work. Students may ask questions, or make mathematical commentary on post-it notes and stick them to the work. Pay attention to students' talk and make note of what is discussed during this time as it may give you some ideas about who should share and in what order they should share.

When students have finished the tour, come back to the large group and begin the closing of the lesson. The goal of this closing is to help students make generalizations about equivalent fractions. Help students reach this goal, not by telling, but by asking thought provoking questions about the work.

DIFFERENTIATION

Extension

- Students who are ready for an extension of this lesson can connect it to geography by using a map of the GA Mountains and using map/math skills to highlight a 6.0 km trail.

Intervention

- Students requiring intervention should have access to manipulatives and, like all other students, share their thinking. All misconceptions are potential learning points for all students. In addition, students requiring intervention should be given a distance of 60 km, rather than 6.0 km, to build their understanding of fractions of whole numbers without the potential decimal as a response. In the closing, the connection between 60 km and 6.0 km (as well as the solutions to the problem) should be made by students through careful teacher questioning.

Technology

http://www.bbc.co.uk/schools/ks2bitesize/maths/number/ a lot of games and activities for students to use to practice working with fractions and decimals as well as whole numbers.

http://education.nationalgeographic.com/education/mapping/interactive-map/?ar_a=1 this is an interactive map where students can create a trail, with markers, in the mountains of Georgia.

http://www.k-5mathteachingresources.com/ this site offers simple contextual problems to use to extend and support students in their understanding of fraction computation and all problems are correlated to CCSS.

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The Hiking Trail

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