Constructing Task – Reasoning with Fractions

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with*
(2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)

MCC.NF.5 Interpret multiplication as scaling (resizing), by:
a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS

- How can comparing factor size to 1 help us predict what will happen to the product?
- How can we model an area with fractional pieces?
- How can modeling an area help us with multiplying fractions?

MATERIALS

- Reasoning with Fractions Task
- Accessible manipulatives
- Grid Paper

GROUPING
Pair/Individual

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments:
This task was developed to give students a real world application of determining what will happen to products when one factor remains the same and the other changes. This task is meant to involve students in a deeper investigation of the concept of the array with fractions.
Students should be allowed to draw representations of their thinking. Using grid paper may facilitate this. Creating these representations allows them to "talk through" their process which in turn enables students the opportunity to attend to precision as they explain and reason mathematically.

BACKGROUND KNOWLEDGE

Students engaging in this task should be familiar with arrays and part-whole thinking as applied to multiplication. See Teaching Student Centered Mathematics, Vol. 2, Slicing Arrays Activity, pg. 66.

Teacher Notes:
Before beginning this task, have a computation discussion with your students using the following computations. It is important for students to have plenty of quiet think time for each individual computation is presented. Likewise, after the quiet think time, students should share their strategies before moving to the next problem.

16 x 1
½ x 16
1 ½ x 16

Are any of the products smaller than the whole number factor?
Why do you think that's happening?
Do you think this might happen all the time, even with mixed numbers?

In this task, you will investigate this mathematical concept.

Part I
Introduce the task. Make sure students understand the context of the task and what they are expected to do. Allow students to share ideas about the task with the group. Make sure students have materials necessary for investigating this task.

The problem here is not just to find answers to naked computation problems, but to explain why some of the products get smaller than one of the factors and other products become larger. Evidence must be presented as to why this happens in each case.

Allow students to work in pairs to answer the questions posed.

Listen to student thinking and provide support with thought provoking questions like the ones below.

Students may use several strategies to solve this problem.

Some students may use 1 inch color tiles, initially, but may run into trouble explaining fractions using these manipulatives. It is possible for students to use these manipulatives by assigning a fractional length to each tile. For example, students may decide that the length of each tile represents ¼, rather than 1. This presents its own challenges, but the struggle is where the learning happens.

Other students may use grid paper in the same manner presented above. A variety of grid sizes may be useful for this task.

FORMATIVE ASSESSMENT QUESTIONS

- How can you tell that your answer is correct?
- How does this explain what is happening here? Show me your thinking.
- What kind of representation will you use to show your thinking?
- Did you develop a shortcut to find your answers?
- Did you identify any patterns or rules? Explain what you have found!

After enough time has been devoted to the task, bring pairs of students together to share in groups of 4 to 6 students. As students share, listen for different explanations and look for different representations.

When students have finished the sharing, come back to the large group and begin the closing of the lesson. The goal of this closing is to help students make connections about areas of rectangles with fractional dimensions. Help students reach this goal, not by telling, but by asking thought provoking questions about the work.

Questions for Teacher Reflection

- How did my students engage in the 8 mathematical practices today?
- How effective was I in creating an environment where meaningful learning could take place?
- How effective was my questioning today? Did I question too little or say too much?
- Were manipulatives made accessible for students to work through the task?
- One positive thing about today's lesson and one thing you will change.

Constructing Task – Reasoning with Fractions

Below are several fraction multiplication problems out of context. The goal for each is to determine why the product is larger (or smaller) than the underlined factor. Use familiar manipulatives to investigate. Create a representation, along with words and numbers, to show your thinking.

1 x 18

3 x 16

14 x ½

1/3 x 24

1 ½ x 14

2 1/3 x 24

Choose two from the last four problems and create a context for the each. Share your story problem with your partner.