Constructing Task – Measuring for a Pillow

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.NF4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show* $(2/3) \times 4 = 8/3$, *and create a story context for this equation. Do the same with* $(2/3) \times (4/5) = 8/15$. *(In general,* $(a/b) \times (c/d) = ac/bd$.*)*

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

MCC5.NF.5 Interpret multiplication as scaling (resizing), by:

a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

MCC5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

(for descriptors of standard cluster, see beginning of unit)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS

- How can comparing factor size to 1 help us predict what will happen to the product?
- How can we model an area with fractional pieces?

- How can modeling an area help us with multiplying fractions?

MATERIALS

- Measuring for a Pillow Task
- Pencil, ruler
- Grid paper
- Fabric and thread or construction paper (optional)
- Accessible manipulatives

GROUPING

Pair/Individual

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Comments:

This task was developed to give students a real world application of determining what will happen to products when one factor remains the same and the other changes. It is not necessary to complete the sewing aspect of this task, but if you do, it could elicit student inquiry into other mathematical investigations. This task is meant to involve students in a deeper investigation of the concept of the array with fractions.

Students should be allowed to draw representations of their thinking. Using grid paper may facilitate this. Creating these representations allows them to "talk through" their process which in turn enables students the opportunity to attend to precision as they explain and reason mathematically.

BACKGROUND KNOWLEDGE

Students engaging in this task should be familiar with arrays and part-whole thinking as applied to multiplication. See Teaching Student Centered Mathematics, Vol. 2, Slicing Arrays Activity, pg. 66.

Teacher Notes:

Before beginning this task, have a computation discussion with your students using the following computations. It is important for students to have plenty of quiet think time for each individual computation as it is presented. Likewise, after the quiet think time, students should share their strategies before moving to the next problem.

10 x 10
10 x 5
10 x 20
5 x 20

After each is complete, if no student offers any thoughts about the products, provoke students to think about the products in each pair:

- Do you notice anything about the products in the pairs?
- Why do you think that's happening?

- What happened to the factors in each pair?
- Do you think this might happen all the time?

In your investigation over the next day or so, you may see and use this mathematical concept.

Part I
Introduce the task. Make sure students understand the concept of the task and what they are expected to do. Allow students to share ideas about the task with the group. Make sure students have construction paper (or fabric) and rulers to measure the correct dimensions. Sample intro idea:

You have been working with your book buddies in Kindergarten for several months now. It might be nice to give your book buddies a nice reading pillow for them to use while reading for the rest of the year. When thinking about pillows to lean against, a 12 ½ x 9 inch pillow for seems like a good size for a Kindergarten student.
Questions for students to focus on as they investigate:
How big (what is the area of) this pillow?
Is this large enough?

Allow students to work in pairs to measure and cut the (fabric) construction paper to the proper dimensions and begin the task.

Listen to student thinking and provide support with thought provoking questions like the ones below. Students may use several strategies to solve this problem. Look for students who rely heavily on manipulatives. These students should share first in the closing part of the lesson. You may see students using some of these strategies:

After cutting the paper to the dimensions listed above, students may use 1 inch color tiles to tile the paper, noting that 9 tiles fit one dimension and 12 ½ fit the other dimension. Students may note that 9 x 12 is 108 and 9 halves is 4 1/2 , so the total is 112 ½ square inches.

After cutting the paper to the dimensions listed, students may "cut" the paper with lines showing split arrays of 9 x 12 and 9 x ½ or 9 x 10, 9 x 2, and 9 x ½, then find the sums of these partial products.

FORMATIVE ASSESSMENT QUESTIONS

- How can you tell that your answer is correct?
- How do you know that mark goes there? Show me your thinking.
- How did you find the area of the pillow? Is there another way to think about this?
- Did you develop a shortcut to find your answers?
- What kind of representation will you use to show your thinking?
- Did you identify any patterns or rules? Explain what you have found!

After enough time has been devoted to the task, post the work around the room and have students take some time to view and make comments on others' work. Students may ask questions, or make mathematical commentary on post-it notes and stick them to the work. Pay attention to students'

talk and make note of what is discussed during this time as it may give you some ideas about who should share and in what order they should share.

When students have finished the tour, come back to the large group and begin the closing of the lesson. The goal of this closing is to help students make connections about areas of rectangles with fractional dimensions. Help students reach this goal, not by telling, but by asking thought provoking questions about the work.

Questions for Teacher Reflection

- How did my students engage in the 8 mathematical practices today?
- How effective was I in creating an environment where meaningful learning could take place?
- How effective was my questioning today? Did I question too little or say too much?
- Were manipulatives made accessible for students to work through the task?
- One positive thing about today's lesson and one thing you will change.

Part II

Make sure students have completed and shared work from part I.
When beginning part II, make sure students understand the context of the problem and any new vocabulary is understood.

The problem in part II is that students think the pillow is much too small (even for kindergarten students), so they look to increase the pillow size (area).

As students are getting ready to begin this task, give them time to share ideas about how to approach the problem (strategies and general ideas). When they are ready, give them the task and have manipulatives available – including grid paper.

Questions for students to think about as they investigate part II:
Should the area be doubled or tripled? Show your mathematical thinking.
What should the dimensions be for your new pillow?
How do these new dimensions compare to the original pillow dimensions from part I?

Allow pairs of students to begin working on the task.

Listen to student thinking and provide support with thought provoking questions like the ones below. Students may use several strategies to solve this problem. Look for students who rely heavily on manipulatives. These students should share first in the closing part of the lesson. You may see students using some of these strategies:

Students may wish to cut and tape paper to new dimensions and tile with one inch tiles.
After cutting the paper to the dimensions listed, students may "cut" the paper with lines showing split arrays, then find the sums of these partial products.

FORMATIVE ASSESSMENT QUESTIONS

- How can you tell that your answer is correct? Show me your thinking.
- How did you find the area of the pillow? Is there another way to think about this?
- Did you develop a shortcut to find your answers?
- What kind of representation will you use to show your thinking?
- Did you identify any patterns or rules? Explain what you have found!

After enough time has been devoted to the task, set the work around the room and have students take some time to view and make comments on others' work. Students may ask questions, or make mathematical commentary on post-it notes and stick them to the work. Pay attention to students' talk and make note of what is discussed during this time as it may give you some ideas about who should share and in what order they should share.

When students have finished the tour, come back to the large group and begin the closing of the lesson. The goal of this closing to help students make connections about areas of rectangles with fractional dimensions. Help students reach this goal, not by telling, but by asking thought provoking questions about the work.

Questions for Teacher Reflection
How did my students engage in the 8 mathematical practices today?
How effective was I in creating an environment where meaningful learning could take place?
How effective was my questioning today? Did I question too little or say too much?
Were manipulatives made accessible for students to work through the task?
One positive thing about today's lesson and one thing you will change.

DIFFERENTIATION

- **Extension**
  Students should work on contextual problems such as those found in Teaching Student Centered Mathematics, by John Van de Walle, pgs 167-172. Possible student representations are also presented in these pages.

- **Intervention**
  Students requiring intervention should also use contextual problems such as those found in Teaching Student Centered Mathematics, by John Van de Walle, pgs 167-172. Students should be talking their way through the problems with teacher support and questioning.

TECHNOLOGY

http://www.bbc.co.uk/schools/ks2bitesize/maths/number/ a lot of games and activities for students to use to practice working with fractions and decimals as well as whole numbers.

http://www.counton.org/games/map-fractions/falling/ in this game, students find fractions of whole numbers to collect leaves falling from a tree. The player with the most leaves at the end of the game is the winner.

http://nlvm.usu.edu/en/nav/category_g_2_t_1.html another activity from the national library of virtual manipulatives helps students see the multiplication of fractions as a familiar array.

Constructing Task – Measuring for a Pillow – Part I

You have been working with your book buddies in Kindergarten for several months now. It might be nice to give your book buddies a nice reading pillow for them to use while reading for the rest of the year. When thinking about pillows to lean against, a 12 ½ x 9 inch pillow for seems like a good size for a Kindergarten student.

Questions:

How big (what is the area of) this pillow?

Is this large enough?

Constructing Task – Measuring for a Pillow – Part II

After cutting the fabric for the pillow, it doesn't look like the pillow will be large enough. You can't decide if you should double the area of the pillow, or triple it. Show the possible dimensions of pillows for each case. Show your mathematical thinking with sketches, words and numbers. Which do you think is a better fit for your book buddy? Explain.