Constructing Task: Sharing Candy Bars

Adapted from Contexts for Learning Mathematics Fractions, Decimals, and Percents by Fosnot, Catherine Twomey et.al.

Children learn mathematics by using what they know to make sense of new mathematical ideas. Equal sharing problems offer students the opportunity to use what they know about partitioning and division to learn fractions.

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.NF.2
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

MCC5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)*

MC5.NF.6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS

- How can we describe how much someone gets in a fair-share situation if the fair share is less than 1?
- How can we describe how much someone gets in a fair-share situation if the fair share is between two whole numbers?
- How can fractions be used to describe fair shares?

MATERIALS

- Copy of Sharing Candy Bars task (1 per pair or small group), or blank index cards to represent the candy bars
- Pencil
- Accessible manipulatives

GROUPING

Pair/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Comments:

This task was developed from *Contexts for Learning Mathematics*, by Fosnot and Jacob. A recording sheet is provided, but is not necessary for this task, especially if students are using a math journal or learning log. Students should draw representations of their mathematical thinking as well as use words and numbers to explain their thinking for three reasons:
SMP2. Reason abstractly and quantitatively.
SMP3. Construct viable arguments and critique the reasoning of others.
SMP4. Model with mathematics.
Students should be allowed to draw representations of their thinking. This allows them to "talk through" their process which in turn enables students the opportunity to attend to precision as they explain and reason mathematically.

BACKGROUND KNOWLEDGE

Students engaging in this task have an understanding of fair shares. If students lack this understanding, they will benefit from the previous task, as well as activities from Teaching Student Centered Mathematics, by John Van de Walle, pg. 136.

Teacher Notes:

Introduce the problem and be sure everyone is clear with the context. You may wish to use the pictures included at the end of this task to help develop this context.

Facilitate a preliminary discussion with the class, before students get to working on the problem. Allow students to share their initial thoughts, then ask them to work in pairs to investigate the following:

Was the distribution of candy bars fair – did everyone in the class get the same amount? How much of a candy bar did each person get, assuming the pieces were cut equally? Possible struggles students may have can be turned into wonderful inquiries! As students cut up the candy bars, you may notice them:

Cutting each candy bar into a familiar fraction first, such as halves or thirds, then cut the leftovers into slivers. This strategy may cause struggles with what to name the pieces (what is 1/5 of 1/3? for example)

Cutting each candy bar into a number of pieces that is the same as the number of people in the group. For example, if 4 candy bars are shared among 5 people, each of the 4 candy bars is cut into 5 pieces. So, 1/5 of each candy bar goes to each of the 5 people. This may cause students to struggle with the idea that the size of the whole matters. Everyone gets 4/20 of the pieces, but this is also 4/5 of one candy bar.

Using the long division algorithm to find a decimal quotient ($4 \div 5 = 0.8$). This strategy may promote discussion, so please allow students the freedom to make sense of this in the closing part of the lesson.

FORMATIVE ASSESSMENT QUESTIONS

- How can you tell that your answer is correct?
- How far away from a whole is your fraction? How do you know?
- What if we said these pieces you've drawn right here were now halves instead of eighths? How would your answer change?
- Did you develop a shortcut to find your answers?
- Did you identify any patterns or rules? Explain!

After enough time has been devoted to the task, ask pairs of students to make posters to prepare for the closing of the lesson. Posters should be clear for others in the class to understand their thinking, but should not just be the figuring that was initially done copied over again. The posters should be clear and concise presentations of any important ideas and strategies students wish to present.

Some ideas to encourage discussion about in the presentations of student work:

The size or amount of the whole matters
With unit fractions, the greater the denominator, the smaller the piece is
When naming the piece, the whole matters

This task is not enough for children to have a full understanding of fractions as division, nor is it enough for students to gain understandings of multiplication, division, or comparing fractions. It is an introduction to the idea of fractions in an equal sharing context, which opens the door for students to grapple with understanding fractions as division. It also allows

for students to make sense of a real life situation that may promote inquiry into other mathematical ideas with fractions.

Questions for Teacher Reflection

- How did my students engage in the 8 mathematical practices today?
- How effective was I in creating an environment where meaningful learning could take place?
- How effective was my questioning today? Did I question too little or say too much?
- Were manipulatives made accessible for students to work through the task?
- One positive thing about today's lesson and one thing you will change.

The following are instructional guidelines for creating more, perhaps similar equal sharing problems for fifth grade students.

Instructional guidelines for Equal Sharing Problems and Introducing Fractions (from Extending Children's Mathematics, Fractions and Decimals, Empson, Susan B., and Levi, Linda)

- Equal sharing problems with answers that are mixed numbers and fractions less than 1. Focus on problems with 4, 8, 3, 6, 10, and 12 sharers, but include other numbers of sharers as well, such as 15, 20, and 100.
- Represent children's solutions with equations, with an emphasis on linking addition and multiplication and on equations that reflect a multiplicative understanding of fractions. For example, if students solved a problem about 8 children sharing 5 burgers you might write the following equations:
    - 1/8 + 1/8 + 1/8 + 1/8 + 1/8 = 5/8 ("Lura drew 5 hamburgers and gave each person an eighth of each hamburger. She put the pieces together and said that 1/8 plus 1/8 plus 1/8 plus 1/8 plus 1/8 is 5/8. Does this equation show what Lura did?)
    - 5 x 1/8 = 5/8 ("Shelly drew 1 hamburger and split it into 8 pieces. She said that each person would get 1/8 of this hamburger. The other hamburgers would look the same as this and she said 5 groups of 1/8 is the same as 5/8.")
    - 5 divided by 8 is 5/8 ("Krystal said that she knows that when 5 things are shared by 8 people, each person gets 5/8.")
- Represent the word problem situation using equations.
- 8 children are sharing 5 hamburgers equally. How much hamburger does one child get?
    - 5 ÷ 8 = ☐
    - 8 x ☐ = 5

DIFFERENTIATION

Extension

- Allow students to investigate other shares and sharers as identified above. To challenge students, especially with large numbers of sharers, insist that students represent their fractions in multiple ways. For example, our team of 100 5th grade students is sharing the challenge of running a 40 mile race for charity. How many miles is each student's responsibility? Students could shade in a 10x10 grid, show the fraction as 40/100 and (0.40), then show it again as 4/10 (0.4) and again as 2/5.

Intervention

- Use smaller numbers of sharers. For example, give students one or two candy bars that have 2-3 sharers. The use of student created or commercial manipulatives, with teacher guidance and questioning will help students develop the concept of fractions as division.

Technology

http://nlvm.usu.edu/en/nav/category_g_2_t_1.html the national library of virtual manipulatives has several activities for students to practice operations and understanding of fractions.

http://calculationnation.nctm.org/Games/ this site, from NCTM, has engaging and sometimes addictive games for practicing calculations based on strategy.

http://www.k-5mathteachingresources.com/ this site offers simple contextual problems to use to extend and support students in their understanding of fraction computation and all problems are correlated to CCSS.

Sharing Candy Bars

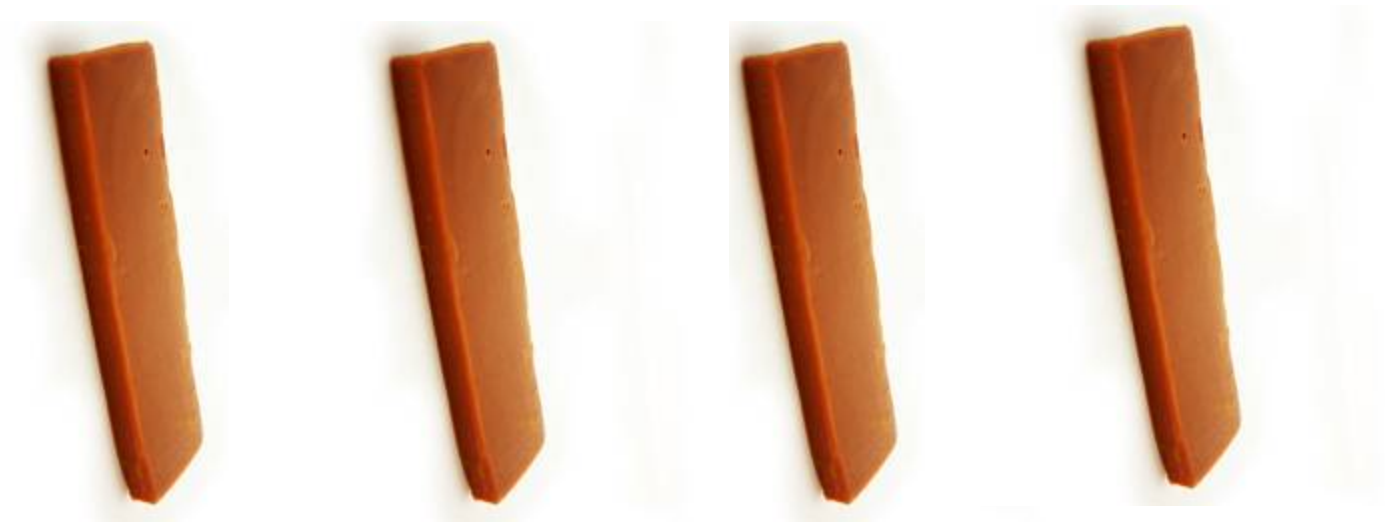
A fifth grade class is split into four groups. Students in the class brought in candy bars for a fraction celebration. When it was time for the celebration, the candy bars were shared as follows:

The first group had 4 people and shared 3 candy bars equally.
The second group had 5 people and shared 4 candy bars equally.
The third group had 8 people and shared 7 candy bars equally.
The fourth group had 5 people and shared 3 candy bars equally.

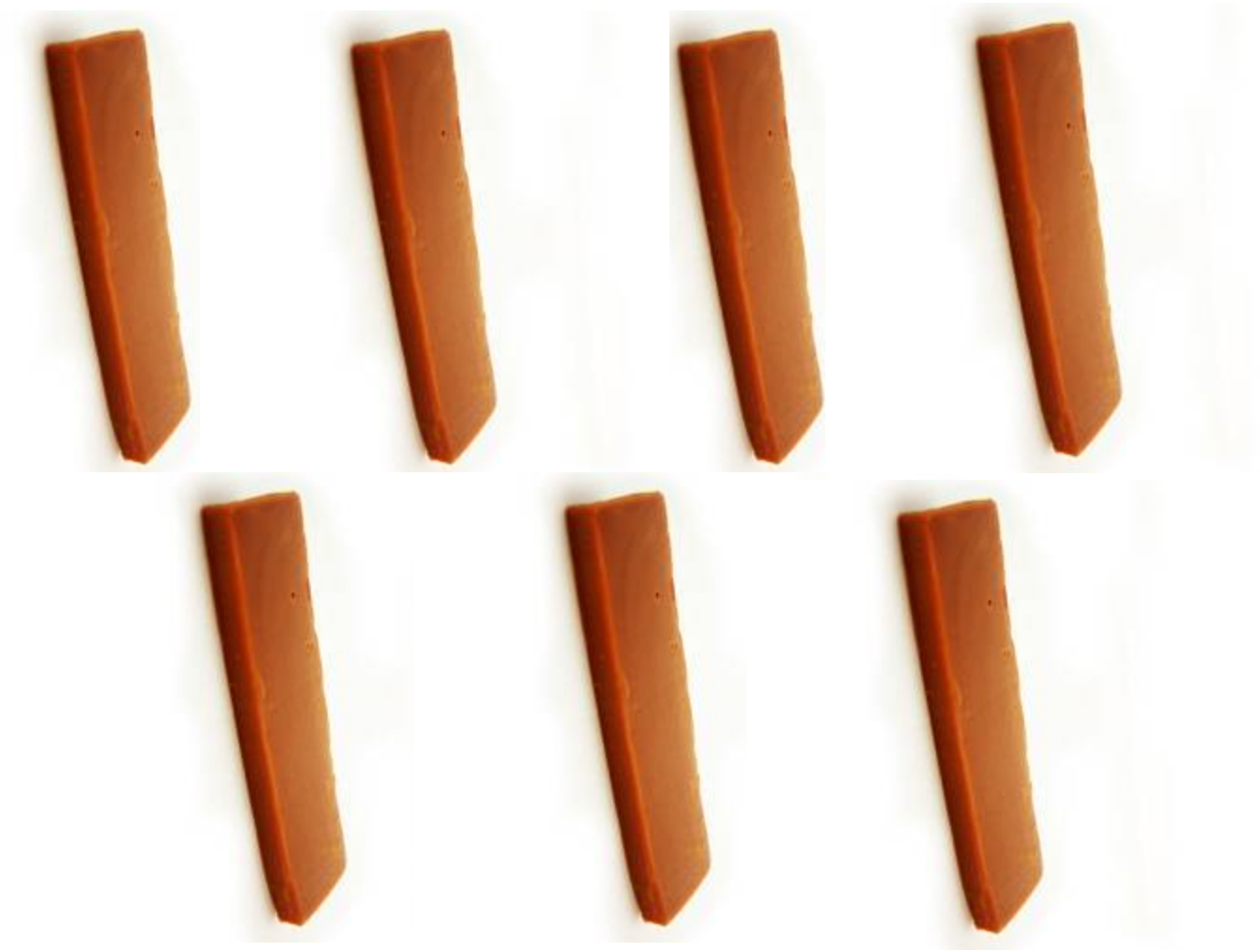
When the celebration was over the children began to argue that the distribution of candy bars was unfair, that some children got more to eat than others. Were they right? Or, did everyone get the same amount?

Four people share these three candy bars.

Five people share these four candy bars.



Eight people share these seven candy bars.



Five people share these three candy bars.