Constructing Task: Sharing Candy Bars Differently

Adapted from Contexts for Learning Mathematics Fractions, Decimals, and Percents by Fosnot, Catherine Twomey et.al.

This task continues where the previous task left off. Many students may be curious about whether or not there is a way to share the candy bars equally before you even introduce this part of the lesson. If this is the case, let them think that they're driving the lesson. It will probably promote more inquiry in later tasks. The lesson begins with a quick series of computation problems that promote partial products with whole numbers and fractions.

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.NF.3 Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

MCC5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*
MCC5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
(for descriptors of standard cluster, see beginning of unit)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS

- How can we describe how much someone gets in a fair-share situation if the fair share is less than 1?
- How can we describe how much someone gets in a fair-share situation if the fair share is between two whole numbers?
- How can fractions be used to describe fair shares?
- How does the size of the whole determine the size of the fraction?

MATERIALS

- Copy of Sharing Candy Bars task (1 set of four pages per pair or small group)
- Pencil
- Accessible manipulatives

GROUPING

Pair/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments: This task was developed from Contexts for Learning Mathematics, by Fosnot and Jacob. A recording sheet is provided, but is not necessary for this task, especially if students are using a math journal or learning log. Students should draw representations of their mathematical thinking as well as use words and numbers to explain their thinking for three reasons:

SMP2. Reason abstractly and quantitatively.
SMP3. Construct viable arguments and critique the reasoning of others.
SMP4. Model with mathematics.

Students should be allowed to draw representations of their thinking. This allows them to "talk through" their process which in turn enables students the opportunity to attend to precision as they explain and reason mathematically.

BACKGROUND KNOWLEDGE

Students engaging in this task have an understanding of fair shares. If students lack this understanding, they will benefit from the previous task, as well as activities from Teaching Student Centered Mathematics, by John Van de Walle, pg. 136.

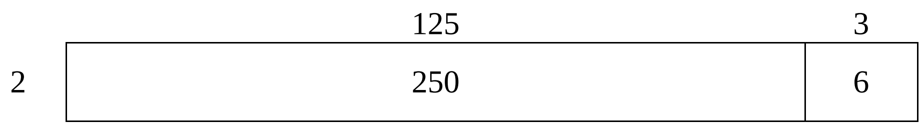
Teacher Notes:

Have the computation discussion with students and draw empty arrays to show their thinking. See below.

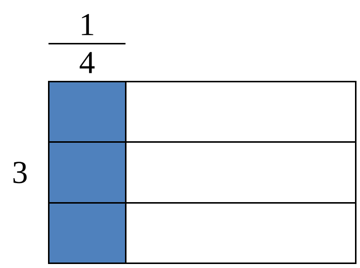
10 x 128
2 x 128
128 x 12
33 x 10

33 x 9
3 x 1/5
7 x 1/8
3 x ¼
4 x 1/5

For the problem 128 x 2, if a student said, "I did 125 x 2 and got 150, then did 3 x 2 and got 6. 150 + 6 = 156." You would draw something similar to the open array below.



The same thing would be true for the computation discussions involving fractions:
For the problem 3 x ¼, if a student said, "I did ¼ three times and got ¾, you might draw something like the following:



Review what was learned from the previous task. Give time for students to make sense and comments on others' work.
Introduce the problem and be sure everyone is clear with the context. Use the pictures from the last task to include at the end of this task to help develop this context.
Facilitate a preliminary discussion with the class, before students begin to work on the problem. Allow students to share their initial thoughts, then ask them to work in pairs to investigate the following:
Was the distribution of candy bars fair – did everyone in the class get the same amount?
How much of a candy bar did each person get, assuming the pieces were cut equally?
Ask students to investigate if it would be fairer if groups the first and third groups combined and shared their candy bars, and if the second and fourth groups combined and shared their candy bars. Place the papers from the last task together as stated above.

Encourage student struggles to become exciting challenges! As students cut up the candy bars, you may notice them:

Cutting halves and using landmark fractions.

For groups 2 and 4, there are 7 candy bars for 10 people. Students might cut 5 candy bars in half and cut the remaining candy bars into fifths, resulting in ½ + 1/5 per person.

For groups 1 and 3, there are 10 candy bars for 12 people. Similarly, students may cut 6 candy bars in halves and cut the remaining candy bars into thirds. This would give each person ½ + 1/3 of a candy bar to each person.

Using the idea of division represented as a fraction. Today they see that they have 7/10 and 10/12 to compare. They may not be able to use common denominators to compare, but they will have some of their own ways to compare and they should be encouraged to do this. For example, they may take the information from yesterday's lesson and decide to compare 3/5 and 4/5 to 7/10. Using connecting cubes or other manipulatives, students should be given the chance to make sense of the mathematics within the context of this problem. They may notice that 4/5 is the same as 8/10, and 3/5 is the same as 6/10, so 7/10 is right in the middle.

Using the long division algorithm to find a decimal quotient and then comparing. This strategy promotes nice equivalents for students to compare. It may be necessary to remind students who use this strategy that any decimals they find represent tenths.

It's possible that some students may begin to think of ratios such as $\frac{3\frac{1}{2}}{5}$

Do not discourage this, since it is mathematically true and is an equivalent relation. This kind of thinking should be supported, since it shows an ability to think flexibly and will help develop fraction sense.

FORMATIVE ASSESSMENT QUESTIONS

- How can you tell that your answer is correct?
- How far away from a whole is your fraction? How do you know?
- What if we said these pieces you've drawn right here were now halves instead of eighths? How would your answer change?
- Did you develop a shortcut to find your answers?
- Did you identify any patterns or rules? Explain!

After enough time has been devoted to the task, ask pairs of students to make posters to prepare for the closing of the lesson. Posters should be clear for others in the class to understand their thinking, but should not just be the figuring that was initially done copied over again. The posters should be clear and concise presentations of any important ideas and strategies students wish to present. The method they used to compare the situations and their justifications for why they think the combined group sharing is fairer (or not) should be included in their poster.

Some ideas to encourage discussion about in the presentations of student work:

The size or amount of the whole matters
With unit fractions, the greater the denominator, the smaller the piece is
When naming the piece, the whole matters

As students share ideas, listen for important ideas that may be worth spending extra time discussing, such as:

How to compare fractions
Interesting ways to redistribute candy bars
How they know that when combining groups of candy bars (numerators) and groups of people (denominators), you get a fraction in between.

Look for:
Evidence of halves and unit fractions, and an understanding that the redistributing produces an amount in between.
Comparing fractions and realizing that redistributing produces a fraction in between the original two fractions.
Using division to produce decimals to compare the quantities.

Final thoughts:
This investigation is fairer than the first, but is still not fair. Some students still get more candy than others.
The investigation does not need to end here. In fact, students may wonder if it would be fairer to share the 17 candy bars together with all of the 22 students. It is worth the time to do so.
Emphasize to students that it would not be efficient to cut the candy bars into 22 little pieces. Ask students if 17/22 is about ½, ¾, or 2/3? Where could one cut be made that would be a nice approximation?

Questions for Teacher Reflection

How did my students engage in the 8 mathematical practices today?
How effective was I in creating an environment where meaningful learning could take place?
How effective was my questioning today? Did I question too little or say too much?
Were manipulatives made accessible for students to work through the task?
One positive thing about today's lesson and one thing you will change.

The following are instructional guidelines for creating more, perhaps similar equal sharing problems for fifth grade students.

Instructional guidelines for Equal Sharing Problems and Introducing Fractions (from Extending Children's Mathematics, Fractions and Decimals, Empson, Susan B., and Levi, Linda)

- Equal sharing problems with answers that are mixed numbers and fractions less than 1. Focus on problems with 4, 8, 3, 6, 10, and 12 sharers, but include other numbers of sharers as well, such as 15, 20, and 100.

- Represent children's solutions with equations, with an emphasis on linking addition and multiplication and on equations that reflect a multiplicative understanding of fractions. For example, if students solved a problem about 8 children sharing 5 burgers you might write the following equations:
- 1/8 + 1/8 + 1/8 + 1/8 + 1/8 = 5/8 ("Lura drew 5 hamburgers and gave each person and eighth of each hamburger. She put the pieces together and said that 1/8 plus 1/8 plus 1/8 plus 1/8 plus 1/8 is 5/8. Does this equation show what Lura did?)
- 5 x 1/8 = 5/8 ("Shelly drew 1 hamburger and split it into 8 pieces. She said that each person would get 1/8 of this hamburger. The other hamburgers would look the same as this and she said 5 groups of 1/8 is the same as 5/8.")
  - 5 divided by 8 is 5/8 ("Krystal said that she knows that when 5 things are shared by 8 people, each person gets 5/8.")
- Represent the word problem situation using equations.
  - 8 children are sharing 5 hamburgers equally. How much hamburger does one child get?
    - 5 ÷ 8 = ☐
    - 8 x ☐ = 5

<u>DIFFERENTIATION</u>

Extension

- Allow students to investigate other shares and sharers as identified above. To challenge students, especially with large numbers of sharers, insist that students represent their fractions in multiple ways. For example, our team of 100 5th grade students is sharing the challenge of running a 40 mile race for charity. How many miles is each student's responsibility? Students could shade in a 10x10 grid, show the fraction as 40/100 and (0.40), then show it again as 4/10 (0.4) and again as 2/5.

Intervention

- Use smaller numbers of sharers. For example, give students one or two candy bars that have 2-3 sharers. The use of student created or commercial manipulatives, with teacher guidance and questioning, will help students develop the concept of fractions as division.

Technology

http://nlvm.usu.edu/en/nav/category_g_2_t_1.html the national library of virtual manipulatives has several activities for students to practice operations and understanding of fractions.

http://calculationnation.nctm.org/Games/ this site, from NCTM, has engaging and sometimes addictive games for practicing calculations based on strategy.

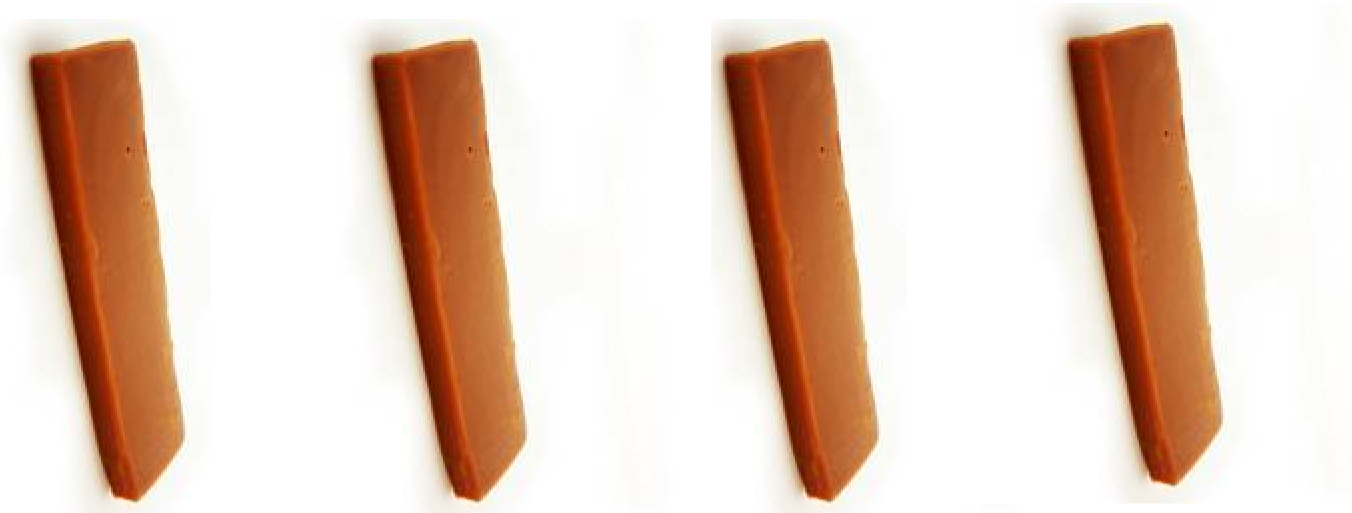
http://www.k-5mathteachingresources.com/ this site offers simple contextual problems to use to extend and support students in their understanding of fraction computation and all problems are correlated to CCSS.

Sharing Candy Bars Differently

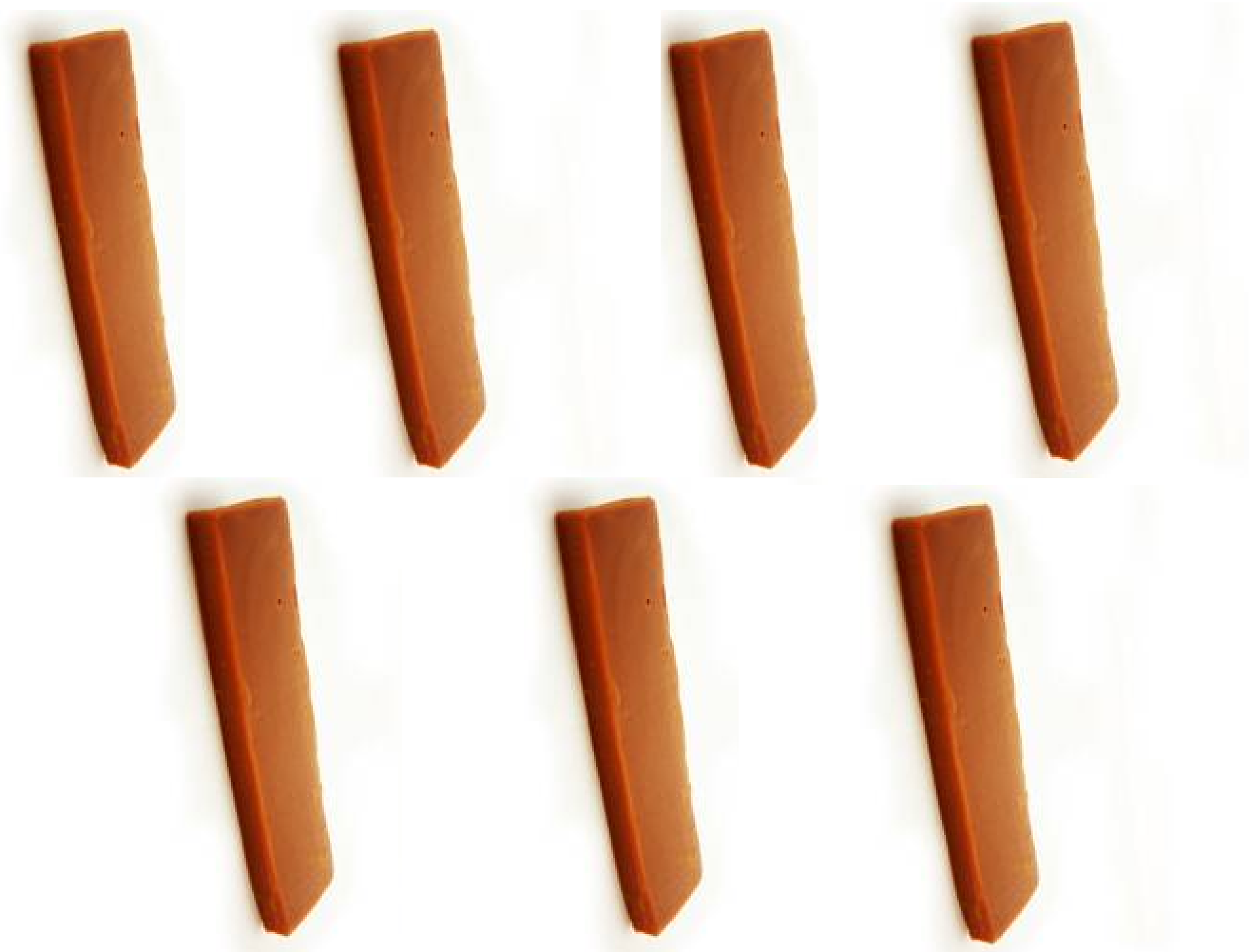
After looking at your first investigation, perhaps there is a way to make this sharing of candy bars more fair. Do you think it would be fairer if groups 1 and 3 combined and shared, and groups 2 and 4 combined and shared?

Group 1. Four people share these three candy bars.

Group 2. Five people share these four candy bars.



Group 3. Eight people share these seven candy bars.



Group 4. Five people share these three candy bars.