Constructing Task: Fraction Addition and Subtraction

This task helps students develop the concept of addition with fractions using common and unlike denominators through the use of various manipulatives. It is important for students to solve multiple problems over time using concrete models and representing these models on paper. It is important for students to write the abstract fraction equations however students must be able to explain and justify their thinking through representations and models.

STANDARDS FOR MATHEMATICAL CONTENT

Cluster: Use equivalent fractions as a strategy to add and subtract fractions.
MCC5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

MCC5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS

- How can looking at patterns help us find equivalent fractions?
- How are equivalent fractions helpful when solving problems?
- How can a model help us make sense of a problem?
- How can making equivalent fractions and using models help us solve problems?
- When should we use models to solve problems with fractions?
- What connections can we make between the models and equations with fractions?

MATERIALS

- Pencil
- Journal, Learning Log, Student Problem Solving Notebook, etc.
- Accessible manipulatives, rulers may be helpful in this task, but are not necessary.
- Copy of task.

GROUPING
Pair/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments:
The goal of this task is for students to make sense of a problem with unlike denominators, where addition is the operation to use. The difficult part for the teacher is to not give students an algorithm (find common denominators) before the task. It is much more beneficial for students to struggle with the idea of how to go about combining two fractions with unlike denominators. This constructive struggle is where the learning happens, but the teacher must provide facilitating questions at the end of the lesson to bring the students' own understanding to light. A student journal question, asking what students learned, or what they are still struggling with when adding fractions with unlike denominators, would make a good formative assessment and it would give students time to reflect on their own thinking (metacognition) and understanding.

BACKGROUND KNOWLEDGE

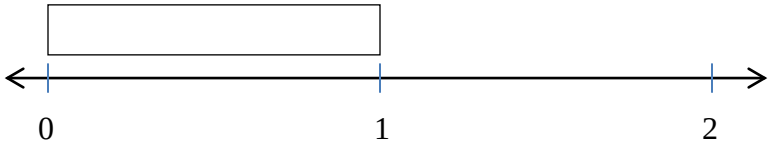
Students can use any representations they wish (SMP 5), but they will always use representations that are familiar to them. Pattern blocks make great fraction manipulatives, but they can be limiting since they imply part to whole more often than part to set (or group). It is necessary for students to have multiple models for fractions in order to facilitate flexibility in representations and computation.

Teacher Notes:
Part I
Cut a strip of paper, see fraction strips at the end of this task and identify/label this as the whole that students will use to complete the task. Cut about 5 more strips the same length. Present the students with the following equations and have them model the equation using blank fraction strips. Have students share their model.

¼ + ¼
1/3 + 1/3
2/3 + 2/3

Students are guided to use the paper strips, fold, cut and join, but some students may already know what the answers are. Encourage students to prove they are correct with the models and see if they can name some of these fractions in other ways. What fractions are equivalent to 1/2? (*show how you know with modeling)*
What fractions are equivalent to 4/3? *(show how you know with modeling)*

Use more examples if needed to increase students' understanding once students have created the strips, guide them to the next question which leads students to represent their models on a number line. Students may need guidance in creating the number line, locating where 1 would be, etc. Have students locate all of their leftover and newly created fractions on the number line. See below:



FORMATIVE ASSESSMENT QUESTIONS

- How can you tell that your answers are correct?
- How do you know that your points on the number line are labeled correctly?
- What other names do you think there are for some of your points on the number line?
- Did you identify any patterns or rules for adding these kinds of fractions? Explain what you have found!

After enough time has been devoted to the task, group partners with other partners to make groups of 4-6 students. Appoint a facilitator to guide the group and make sure everyone shares their thinking. Pay attention to students' talk and make note of what is discussed during this time as it may give you some ideas about who should share and in what order they should share with the whole group.
When students have finished, come back to the large group and begin the closing of the lesson. The goal of this closing to help students make generalizations about adding fractions. Help students reach this goal, not by telling, but by asking thought provoking questions about the work and asking students to share in an order that moves from least efficient to most efficient, or mostly concrete to more representational and abstract.

Part II
Begin this lesson by reviewing the work from the previous lesson. Review any generalizations made and introduce the next part of the task.
Make sure everyone understands the context of the task. Let students know that the task they completed previously was a warm-up for the real task. In this task, student will be asked to do the same thing, but with fractions that are different. Show the fractions and equations:

Sam had 1/2 of a chocolate bar and John gave him 1/3 more of a chocolate bar. How much chocolate bar does he now have?
Sam had 2/3 of a chocolate bar and John gave him 1/2 more of a chocolate bar. How much chocolate bar does he now have?
Sam had 2/3 of a chocolate bar and John gave him 1/2 more of a chocolate bar. How much chocolate bar does he now have?

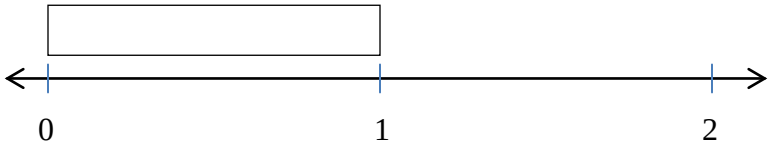
*****ALLOW TIME FOR A CONSTRUCTIVE STRUGGLE TO TAKE PLACE*****

Ask students what is different about these fractions/equations. Reflect back on the work from the last lesson again with questions like, "I remember when all we had was fourths: ¼ and ¼. That was TWO fourths. I wonder what we'll have to do to combine fractions that are not alike. Any ideas? Brainstorm with your students and ask what they could do, since these fractions are not alike.

Students will not likely know any answers right away for this part, but having had the experience in part one, they will likely be successful with this. Students will likely be able to show fractions joined together as unlike denominators but unable to immediately identify the sum. Refer students to the previous lesson and allow them to share, again, what they learned about identifying equivalent fractions. Finding an equivalent fraction for the two parts combined can guide students to made meaningful connections and generalizations.

Students may ask for another strip to cut into smaller pieces. Encourage them to use the pieces they have to make what they need. This builds understandings of how fractions are related (1/2 of 1/3 is 1/6 or a 1/2 piece cut in three makes 3 1/6 pieces) and helps build a fractional number sense. Experiences like this, with teacher guidance and questioning, will also build fluency with fraction computation.

Once students have created the strips for each equation, guide them to the next question which leads students to represent their models on a number line. Some students may still need guidance in creating the number line, locating where 1 would be, etc. See below:



Once students have completed their number line, have them label the point with an equation (1/2 + 1/3) as well as a fraction (10/12 or 5/6).

FORMATIVE ASSESSMENT QUESTIONS

- How can you tell that your answers are correct?
- How do you know that your points on the number line are labeled correctly?
- How can we change these problems into ones just like the easy ones we did in the last task?
- Is there one fraction name for your equation points on the number line?
- Did you identify any patterns or rules for adding these kinds of fractions? Explain what you have found!

After enough time has been devoted to the task, group partners with other partners to make groups of 4-6 students. Appoint a facilitator to guide the group and make sure everyone shares their thinking. Pay attention to students' talk and make note of what is discussed during this

time as it may give you some ideas about who should share and in what order they should share with the whole group.

When students have finished, come back to the large group and begin the closing of the lesson. The goal of this closing to help students make generalizations about adding fractions. Help students reach this goal, not by telling, but by asking thought provoking questions about the work and asking students to share in an order that moves from least efficient to most efficient, or mostly concrete to more representational.

DIFFERENTIATION

- **Extension**
  Students needing an extension should be given opportunities to investigate addition with fractions with different denominators. This is what they will do in part II, but simple problems with a context should be used and can be found in Teaching Student Centered Mathematics Vol. 2, Grades 3-5, by John A. Van de Walle on pg 162. Samples of possible student representations can also be found on pg 163.

- **Intervention**
  Students requiring intervention should use manipulatives and talk their way through the problem with teacher support and questioning.

Technology
http://www.bbc.co.uk/schools/ks2bitesize/maths/number/ a lot of games and activities for students to use to practice working with fractions and decimals as well as whole numbers.

http://www.counton.org/games/map-fractions/racing/ this is an interactive board game where players race bikes on a game board by adding fractions with like denominators.

http://www.counton.org/games/map-fractions/frosty/ this is an interactive three across board game where players add fractions with like and unlike denominators and place a virtual counter on the sum. The first to get three in a row wins the game.

http://www.k-5mathteachingresources.com/ this site offers simple contextual problems to use to extend and support students in their understanding of fraction computation and all problems are correlated to CCSS.

Fraction Addition and Subtraction

Part I

1. Cut out a strip of paper, and label it "1 whole." Cut 4 or 5 more of these same size strips. Fold, cut, and join the strips to create new strips that are the following lengths:

$\frac{1}{4} + \frac{1}{4}$ $\frac{1}{3} + \frac{1}{3}$ $\frac{2}{3} + \frac{2}{3}$

2. Use the strips you made in step 1, with diagrams and number lines to help you explain how to add and subtract fractions like these.

Fraction Addition and Subtraction Part II

1. Cut out a strip of paper, and label it "1 whole." Cut 4 or 5 more of these same size strips. Fold, cut, and join the strips to create new strips that are the following lengths:

$\frac{1}{2} + \frac{1}{3}$ $\frac{2}{3} - \frac{1}{2}$ $\frac{2}{3} + \frac{1}{2}$

2. Use the strips you made in step 1, with diagrams and number lines to help you explain how to add and subtract fractions like these. How is adding (and subtracting) these kinds of fractions different than the fractions in Part I?