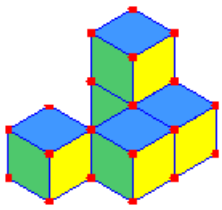


CONSTRUCTING TASK: How Many Ways?

STANDARDS FOR MATHEMATICAL CONTENT

MCC5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

MCC5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

MCC5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

BACKGROUND KNOWLEDGE

Students should have had experiences with the attributes of rectangular prisms, such as faces, edges, and vertices, in fourth grade. This task will build upon this understanding.

The "How Many Ways?" student recording sheet asks students to determine the area of the base of each prism using the measurements of base and height of the solid's BASE. The general formula for the area of a parallelogram is $A = bh$. Knowing the general formula for the area of a parallelogram enables students to memorize ONE formula for the area of rectangles, squares, and parallelograms since each of these shapes is a parallelogram.

The general formula for the volume of a prism is $V = Bh$, where B is the area of the BASE of the prism and h is the height of the prism. Knowing the general formula for the volume of a prism prevents students from having to memorize different formulas for each of the types of prisms they encounter.

There are six possible rectangular prisms that can be made from 24 snap cubes.

$1 \times 1 \times 24$
$1 \times 2 \times 12$
$1 \times 3 \times 8$
$1 \times 4 \times 6$
$2 \times 2 \times 6$
$2 \times 3 \times 4$

Students may identify rectangular prisms with the same dimensions in a different order, for example, $1 \times 4 \times 6$, $1 \times 6 \times 4$, $6 \times 1 \times 4$, $6 \times 4 \times 1$, $4 \times 1 \times 6$, $4 \times 6 \times 1$. All of these are the same rectangular prism, just oriented differently. It is okay for students to include these different orientations on their recording sheet. However, some students may need to be encouraged to find different rectangular prisms.

Common Misconceptions:
Students may have difficulty with the concept of the formula $V=Bh$ representing 3 factors. (length, width, height). They may leave out one of the components because of that misconception.

ESSENTIAL QUESTIONS

- Why is volume represented with cubic units?
- How do we measure volume?
- How can you find the volume of cubes and rectangular prisms?

MATERIALS

- "How Many Ways?" student recording sheet
- Snap cubes

GROUPING

Partner/Small Group Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will use 24 snap cubes to build cubes and rectangular prisms in order to generalize a formula for the volume of rectangular prisms.

Comments

To introduce this task ask students to make a cube and a rectangular prism using snap cubes. Discuss the attributes of cubes and rectangular prisms – faces, edges, and vertices. Initiate a conversation about the figures:

- What is the shape of the cube's base?
- What is the shape of the rectangular prism's base? The base of each is a rectangle (remember a square is a rectangle!).

Students should notice that the cube and rectangular prism are made up of repeated layers of the base. Describe the base of the figure as the first floor of a rectangular-prism-shaped building. Ask students, "What is the area of the base? Next, discuss the height of the figure. Ask students, "How many layers high is the cube?" or "How many layers high is the prism?" The number of layers will represent the height. DO NOT LEAD THE DISCUSSION TO THE VOLUME FORMULA. Students will use the results of this task to determine the volume formula for rectangular prisms on their own.

While working on the task, students do not need to fill in all ten rows of the "How Many Ways?" student recording sheet. Some students may recognize that there are only six different ways to create a rectangular prism using 24 snap cubes. For students who have found four or five ways to build a rectangular prism, tell them they have not found all of the possible ways **without telling them exactly how many ways are possible**. It is important for students to recognize when they have found all possible ways and to prove that they have found all of the possible rectangular prisms.

Once students have completed the task, lead a class discussion about the similarities and differences between the rectangular prisms they created using 24 snap cubes. Allow students to explain what they think about finding the volume of each prism they created. Also, allow students to share their conjectures about an efficient method to find the volume of any rectangular prism. Finally, as a class, come to a consensus regarding an efficient method for finding the volume of a rectangular prism.

Task Directions

Students will follow the directions below from the "How Many Ways?" student recording sheet.

1. Count out 24 cubes.

2. Build all the rectangular prisms that can be made with the 24 cubes. For each rectangular prism, record the dimensions and volume in the table below.
3. What do you notice about the rectangular prisms you created?
4. How can you find the volume without building and counting the cubes?

Shape #	Area of the BASE of the Solid $A = bh$		Number of Layers of the Base (Height of Solid)	Volume
	base	height		
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

FORMATIVE ASSESSMENT QUESTIONS

- What is the shape of the rectangular prism's base? How can you find the area of the base?
- What is the height of the rectangular prism? How do you know? (How many layers or "floors" does it have?)
- What is the volume of the rectangular prism? How do you know? (How many snap cubes did you use to make the rectangular prism? How do you know?)

DIFFERENTIATION

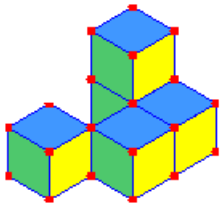
Extension

- Ask students to suggest possible dimensions for a rectangular prism that has a volume of 42 cm^3 without using snap cubes.
- Ask students to explore the similarities and differences of a rectangular prism with dimensions 3 cm x 4 cm x 5 cm and a rectangular prism with dimensions 5 cm x 3 cm x 4 cm. Students can consider the attributes and volumes of each of the prisms.
- Students can calculate the area of each surface of the solid and determine the total surface area.

Intervention

- Some students may need organizational support from a peer or by working in a small group with an adult. This person may help students recognize duplications in their table as well as help them recognize patterns that become evident in the table.
- Some students may benefit from using the "Cubes" applet on the Illuminations web site (see link in "Technology Connection" below). It allows students to easily manipulate the size of the rectangular prism and then build the rectangular prism using unit cubes.

Name_____ Date _____



How Many Ways?

1. Count out 24 cubes.
2. Build all the rectangular prisms that can be made with the 24 cubes. For each rectangular prism, record the dimensions and volume in the table below.
3. What do you notice about the rectangular prisms you created?
4. How can you find the volume without building and counting the cubes?

Shape #	Area of the BASE of the Solid $A = bh$		Number of Layers of the Base (Height of Solid)	Volume
	base	height		
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				