

Ribbon Paths: Introducing Angle as Turn

UNIT

4

Mathematical Concepts

- The measure of a length is obtained by iterating a unit and accumulating the number of iterations of that unit, so that the total length is n times as long as the unit length.
- Distances between landmarks are measured lengths of the straight path that connects them.
- An angle is a turn about a fixed point.
- An angle is measured by the amount and direction of rotation.

Unit Overview

Walking Straight Paths

Students use a new measure of length, a yardstick, because it is clearly more suitable than a foot to describe distance in larger scale space outdoors. The teacher brings students outside with a clipboard or other means to take notes. Students work in pairs starting at a landmark that the teacher designates to create a straight path. They write directions that other students can follow. Students exchange directions and attempt to re-create the path that the writer of the directions had in mind.

Angles as Turns

Students are introduced to bends in paths by angles as turns. They symbolize and enact 1 whole turn, both clockwise (to the right) and counterclockwise (to the left). This process is repeated for $\frac{1}{2}$ and $\frac{1}{4}$ turns.

Ribbon Paths

Using a yardstick to define the measure of straight-line segments, and direction and measured rotations to describe turns, students both follow and construct directions between landmarks in large-scale space. Students represent their walks as a map, with paper-clip lengths standing in for yards, and with angles indicated by the angle-as-rotation notation. Students construct their paths with ribbons and flags to mark starting points, turns, and ending points.

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Mathematical Background

Ribbon Paths Unit 4

Materials & Preparation

- ☐ Surveyor Tape and Flags
- ☐ Yardsticks
- ☐ #1 Size Paper Clips

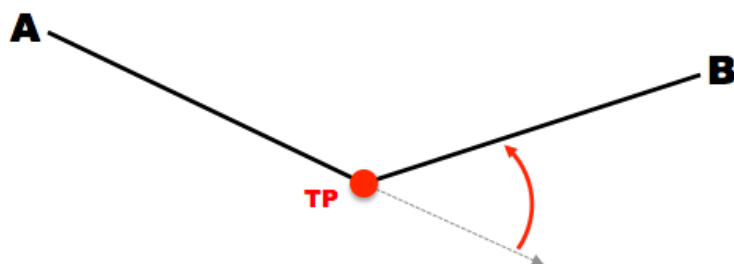
Mathematical Background

Angles

Properties of an angle include a fixed point, a starting direction (an initial heading), a turning direction (right-clockwise or left-counter clockwise) and a magnitude (amount) of turn. The measure of the angle is the part of one whole turn. In this unit we describe measure by referring to 1 whole turn, half turn, and quarter turn.

Angles as Turns

Starting at A, the dashed line shows the direction of travel of the walker—the path that would be made by no change in the walker's direction. But instead of continuing to walk straight, the walker turns at TP (the turning point). The arc-arrow shows the direction (left, or counter-clockwise) and the amount of turn. After turning, the rest of the path is made by walking straight from TP to B.



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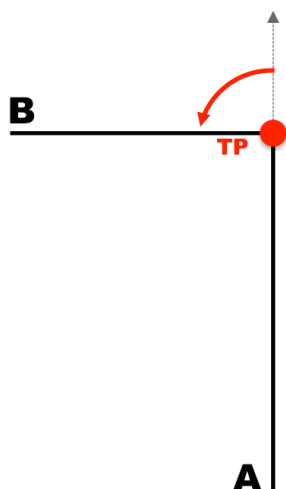
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Walking Straight Paths



The situation set up by the teacher in the classroom is an L-shaped path, one requiring a straight line path, a 90 degree or $\frac{1}{4}$ turn to the right or left, and another straight line path.

Students develop instructions to walk this path, then try out their instructions using flags and ribbon outside.

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Whole Group

1. Students first write instructions for walking a straight path.

The teacher introduces a new unit of length measure, the yardstick and says: “We have been using this (hold a foot-strip unit) to measure. Today we are going to make some L-shaped paths outside (draw an inverted L shape on the board). Notice that our path has two parts. It starts here (gesture to bottom of L) and goes to here (mimic walking along the path). We will use this unit (called a yard) to measure the length of each part of our path. We will start by writing directions to walk from here (gesture to bottom) to here (gesture to top).

Q: What are the advantages of using this yardstick unit (gesture to the yardstick, saying that it is called one yard) instead of a foot ? (fewer iterations, lower counts)

Q: If we were traveling straight outside, how could we use the yardstick to tell someone else how far to travel? (iteration, tiling)

Instruction

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Partners

2. **Continue with: Today we are going to write instructions that someone else could follow to walk the first part of this path, if we started outside with a flag marking the beginning of the path.**

(Teacher hands out *Path Instructions* worksheet.)

Teacher Note

The teacher challenges students to write down instructions that could be used by a hypothetical newcomer to travel the first part of the L path. The description should include where to start (e.g., stand at the flag facing the school building), the line of sight (e.g., looking at a convenient landmark), and how far to travel (e.g., the number of yards. It is up to you whether or not the number of yards traveled is a whole number or includes, perhaps, $\frac{1}{2}$ yards.) It may be helpful for the teacher to enact instructions that are proposed by students.



Whole Group

3. **Select some examples of directions for the first segment.** Highlight the role of walking straight and using the yardstick to create a path of a certain length. Have at least one pair of students enact constructing the first segment of the path in the classroom.

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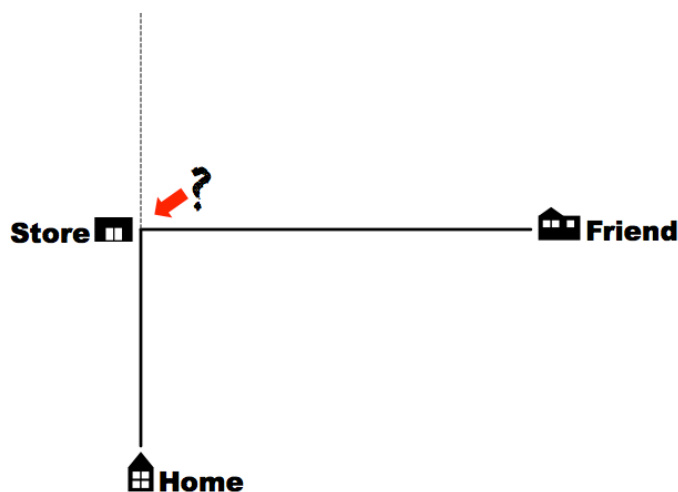
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Paths with Bends: Introduction of Full, Half, and Quarter Turns

Whole Group

4. Pretend we were walking from our house to a store and then to our friend's house. When we reach the store, if we keep going straight ahead, we will do this: Teacher draws representation of continued travel with a dashed line on board, we won't be able to make the L.

 What do we have to do at the end of this first part of the path to get to our friend's house? Establish that we need a new kind of instruction, something we are going to call a *turn*.



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5. **If we are going to write instructions for making a turn, we need to tell where we are standing, which we call the turning point, which way we are facing (looking), which we call our heading, then say which direction (left or right) to turn and how much to turn.** The teacher asks a student to stand on a point, labeled with an X, and face the front of the room.

“Name” is standing on this X, which is his turning point. What is “Name” looking at? (What direction is he facing?) “Name”, raise your right hand. I’d like you to turn all the way around to the right so that when you are finished, you are facing the front of the room again. We call that one full turn to the right. What is Gerald looking at now? Gerald, now raise your left hand. Show me one full turn to the left. Gerald turns toward his left hand, all the way around and back to the beginning. That is one full turn to the left. What is Gerald looking at now?”

6. **Have students enact 1 whole turn (a 360 degree rotation) to the right, first raising their right hands.** Make sure that students pivot as nearly as possible around an imagined point on the floor, so that they clearly distinguish turns from moves. Make sure that every student can make one complete rotation to his or her right. Ask what they notice (including, one whole turn results in a return to exactly the same heading). Then the teacher asks students to enact 1 whole turn to the left.

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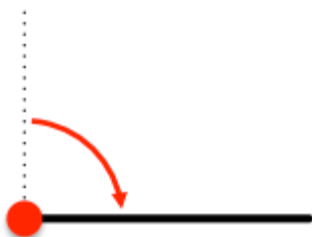
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7. **The teacher asks students: How can we show (write) on paper the difference between 1 whole turn to the right (clockwise) and 1 whole turn to the left (counter-clockwise)?** Let students make various proposals and talk about what each one helps us see about the turns. Then the teacher introduces a convention of using an arc and an arrow to show the direction and amount of turn. For example, 1 whole turn clockwise (to the right by a person positioned at the center of the circle) is represented as:



8. **To show other turns, we use dashes to show the direction we were facing or traveling and a solid line to show where we turned to.** What kind of turn is this (draw and gesture to diagram below)? ($\frac{1}{4}$ turn clockwise or to the right.)



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9. Ask students and write representations:

Q: Who can show me what a *half* turn to the left might look like?

Q: Why do you think we call that a *half* turn to the left?

Q: Next, when I tell him, Gerald is going to make a half turn to the right. Which way will he be looking when he is done?
Gerald, will you show us?

Q: Everyone stand up and face toward the board. Now look down and notice where you are standing. That is your turning point, and when I tell you, you are going to turn to the right on that point. Now slowly, make one whole turn to the right. (Where should we be looking?) When you are finished, make a turn right $\frac{1}{2}$ whole turn. (Where are we looking now?) If you turn to the left $\frac{1}{2}$ whole turn, where will you be looking? Try it out: Turn to the left $\frac{1}{2}$ whole turn. Now turn right $\frac{1}{4}$ whole turn.

10. Introduce a convention for labeling turns: we write Turn, then the direction of the turn, then the amount of the turn. So if I turn to the right $\frac{1}{2}$ whole turn, I write: Turn Right $\frac{1}{2}$ whole turn or I write TR $\frac{1}{2}$ whole-turn. Use fraction notation.

Teacher Note

The symbolization T represents Turn and L or R represent the direction of turn from the point-of-view of a person's body. The unit of turn is a whole-turn, as in 1 whole-turn or $\frac{1}{2}$ whole-turn. As students become more comfortable with these ideas, TR could be replaced with R and whole-turn could be replaced with turn, so R $\frac{1}{2}$ turn, instead of TR $\frac{1}{2}$ whole-turn. But it is important that children keep in mind that the unit of measure is the whole turn—one complete rotation of the body.

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11. The teacher says: “OK, everyone stand up. I am going to write a turn command on the board and after you see it, you do it.” The teacher writes, without saying anything, each of the turn symbols at least once. Be sure to include whole turns to the left and right (TL 1 whole-turn, TR 1 whole-turn), half turns to the left and right (TR $\frac{1}{2}$ whole-turn, TL $\frac{1}{2}$ whole-turn), and quarter turns to the left (TL $\frac{1}{4}$ whole-turn and to the right (TR $\frac{1}{4}$ whole-turn).

12. Ask students: If we are at the store, how would we turn to face our friend’s house? In what direction should we turn? How much of a whole turn? How should we write that so someone else will know what we mean? (Guide or tell students, labeling the turn arrow as $\frac{1}{4}$ or one-fourth and show the difference between left and right quarter turns.)

Instruction

Ribbon Paths Unit 4

Ribbon Paths

Students complete instructions for the L path.

Partners

1. **Students complete instructions (with yards and turns) to walk the entire L path.** Students record their additional instructions on the worksheet.
2. **Using flags to mark the starting point, the turning point, and the ending point, students go outside and follow their directions using ribbon (surveyor tape) to outline the path.**



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Teacher Note

You can have students follow directions written by other students. This makes the importance of explicit guidance clear. As students work, you can assist their thinking about angle-as-turn by using different color ribbon to mark the continuing path and then literally rotating one colored ribbon (the straight path) onto the second ribbon that represents the other length. For advanced students, you can offer additional challenges of making the L path bigger or deciding about how to walk a rectangle.



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Whole Group

3. **Coming back to the classroom, select a few student solutions for consideration.** Draw the corresponding paths on the board, with labels of yards for distances and directed arrows with fractions of whole turns to represent turning angles.
4. **Challenge students to decide how to modify some of the student directions to make a bigger L path.**

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Teacher Note

Students will often not be certain about which parts of the directions should be modified. But coming to understand that the turn remains the same but the lengths (number of yards traveled) change to make the ribbon path bigger or smaller helps students distinguish between angle-as-turns and length.

For example, a path consisting of starting at flag A, walk straight 5 yards to flag B, turn right $\frac{1}{4}$ whole turn, walk straight 5 yards, could be enlarged by changing the number of yards walked to 8.

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Small Group and Partners

- 5. Work together to draw what happens when you follow these directions:**

Path 1 From the starting point, the person walks 10 yards, turns to the right $\frac{1}{4}$ whole turn, and then walks another 5 yards.

Path 2 From the starting point, the person walks 10 yards, turns to the left $\frac{1}{4}$ whole turn, and then walks another 10 yards.

Path 3 From the starting point, the person walks 10 yards and then turns to the right $\frac{1}{2}$ whole turn, and then walks 10 yards.

Whole Group

- 6. Students share their drawings for each challenge.** At teacher discretion, students enact the paths.

Formative Assessment

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The teacher passes out copies of “On my Way” and explains that the job is to write instructions for the path from the start, the school building, to the store. It is very important to stay on the road!

Students use small (#1 sized) paper clips to represent yards and write the series of paper clip lengths and turns that will get them to the goal. Note that the final part of the path to the school cannot be measured with a whole number of paper clips. This is a good opportunity to see if any students spontaneously volunteer that the final path segment includes a partial unit.

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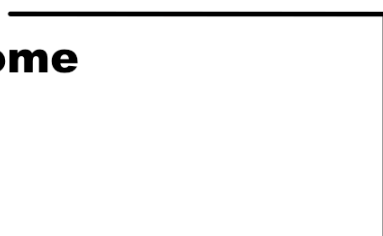
On My Way Worksheet

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Name: _____



Home



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START AT _____

FACE (LOOK TOWARD) _____

WALK STRAIGHT _____ YARDS

TURN _____

WALK STRAIGHT _____ YARDS

TURN _____

WALK STRAIGHT _____ YARDS

Formative Assessment Record

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Student _____ Date _____

Indicate the levels of mastery demonstrated by circling those for which there is clear evidence:

Level	Description	Notes
ToMA⁰ 3A	Recognize and compare amount of turns (quarter turns, half turns, three-quarter turns). Performance: Student successfully depicts all quarter turns on the path.	
ToMA⁰ 2C	Coordinate angle as turn and angle as figure. Performance: Student depicts points in path where there should be bends but does not accurately represent $\frac{1}{4}$ turns. May depict only 1 turn successfully.	
ToMA⁰ 1A	Informal conceptions of angle. Performance: Student does not recognize appropriate points on path for angle as turn.	
Scales Length?	<i>Circle those that apply:</i> Yes No Other _____	
ToML4D	Partition and compose partitions by factors of 2. Performance: Student measures length with $\frac{1}{2}$ clip-unit.	
ToML3F	Iterate a unit to measure. Performance: Uses paper clip to measure scaled length	
NL	Does not know how to measure or cannot make any sense of the task.	

Academic Language:

Indicate academic words the student is familiar with by recording them here.

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