Relating Lengths

UNIT

15

Mathematical Concepts

- There is a reciprocal relation between the measures of two lengths if the same unit is used to measure both.
- Relations of 2: If the measure of length A is 2 times as long as the measure of B, then the measure of length B is $\frac{1}{2}$ times as long as the measure of A. (If A = 2B, then B = $\frac{1}{2}$ A)
- Relations of 3: If the measure of length A is 3 times as long as the measure of B, then the measure of length B is $\frac{1}{3}$ times as long as the measure of A. (If A = 3B, then B = $\frac{1}{3}$ A).
- Any relation: If the measure of length A is $\frac{c}{d}$ times as long as the measure of B, then the measure of length B is d/c times as long as the measure of A. (If A = $\frac{c}{d}$ B, then B= $\frac{d}{c}$ A)

Unit Overview

The unit employs the natural language of "times as long" to support relational thinking. Students first use a cm. ruler to cut strips of paper with lengths measured in cm. Then students use relations of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{2}{3}$ times as long as the measured strip of paper to draw another length. Students are guided to consider the reciprocal relation between A (the paper strip) and B (the drawn line). For example, if B is $\frac{1}{2}$ times as long as A, then A is 2 times as long as B.

Contents

Materials and Preparation

Read

- ☐ **Unit 15**
  Start by reading the unit to learn the content and become familiar with the activities.
- ☐ **Mathematical Background**
  Reread the mathematical background carefully to help you think about the important mathematical ideas within the unit.

Prepare

- ☐ Provide students with rulers marked in cm.

Materials and Preparation

The core ideas about measurement emphasized in this unit are those of relation

Magnitude

A length has a magnitude—an extent.

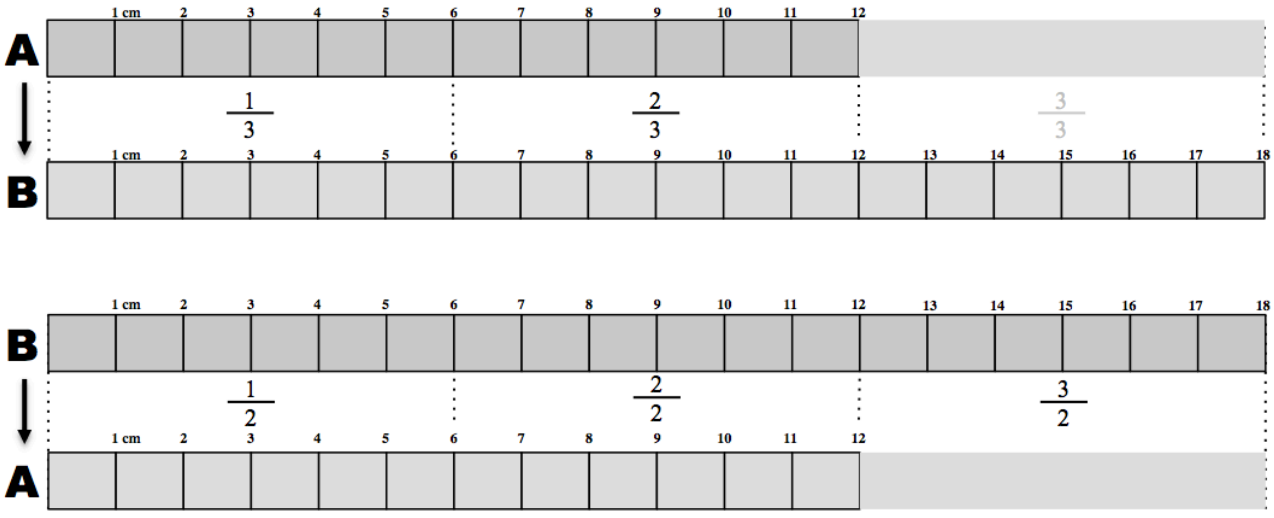
Measure of Magnitude as an Accumulation of Units

The measure of the magnitude of a length is the ratio of that length to the length of a particular unit of measure. We can count units to establish this ratio. For example, a 10-inch length is 10 times as long as a 1-inch length, so it has a measure of 10 inches.

Multiplicative Comparisons of Lengths via Common Measures

If the measure of a length, A, is 10 units, and the measure of another length B is 20 units, we can conclude that A is $\frac{1}{2}$ times as long as B, and that means that B is the reciprocal times as long as A (2 times as long).

The multiplicative comparisons can be complex but all have the same structure. If common measure establishes that A is $\frac{2}{3}$ times as long as B, then B must be $\frac{3}{2}$ times as long as A.



A is $\frac{2}{3}$ times as long as **B**. **B** is $\frac{3}{2}$ times as long as **B**.

Make a Length

Whole Group Demonstration

1. **The teacher cuts a strip that is 10cm long, labels it A and asks:**

   How could I draw a line that is 2 times as long as this 10 cm. length? Talk with your elbow partner about this. Try to draw the line and label it as B.

2. **The teacher elicits student responses, with a focus on justifying why a line that is 20 cm is 2 times as long as the 10 cm line.**

3. **The teacher asks students to complete the following:**

   B is _____ times as long as A.

   A is _____ times as long as B.

   Talk with your elbow partner and see if you agree.

4. **The teacher solicits student responses and leads a conversation to justify the responses based on splitting and iterating.**

   If B is 2 times as long as A, then how many A should fit into B?

   If A is $\frac{1}{2}$ times as long as B, then if we split B into 2 equal partitions, how long is each partition?

   Teacher note. Be sure to have strips of paper to physically demonstrate these relations.

Instruction

Problem 1: $\frac{1}{3}$, 3

Individual

1. **Students cut a strip of paper that is 18 cm long. Call this length A. Now draw a line that is $\frac{1}{3}$ times as long. Call it B.**

   A is _____ times as long as B.

   B is _____ times as long as A.

Whole Group

2. **The teacher elicits responses and students justify that A is 3 times as long as B because 3 copies of B cover the length of A. And, B is $\frac{1}{3}$ times as long as A, because if A is split into 3 equal partitions, then each partition is congruent with B.**

   Teacher note. Use paper strips to visually demonstrate these relationships.

Problem 2: $\frac{1}{4}$, 4

Individual

1. **Students cut a strip of paper that is 20 cm long. Call this length A. Now draw a line that is $\frac{1}{4}$ times as long. Call it B.**

   A is _____ times as long as B.

   B is _____ times as long as A.

Whole Group

2. **The teacher elicits responses and students justify that A is 4 times as long as B because 4 copies of B cover the length of A. And, B is $\frac{1}{4}$ times as long as A, because if A is split into 4 equal partitions, then each partition is congruent with B.**

   Teacher note. Use paper strips to visually demonstrate these relationships.

Problem 3: 3, $\frac{1}{3}$

Individual

1. **Students cut a strip of paper that is 5 cm long. Call this length A. Now draw a line that is 3 times as long. Call it B.**

   A is _____ times as long as B.

   B is _____ times as long as A.

Whole Group

2. **The teacher elicits responses and students justify that A is $\frac{1}{3}$ times as long as B because 3 copies of A cover the length of B. And, B is 3 times as long as A, because if B is split into 3 equal partitions, then each partition is congruent with A.**

   Teacher note. Use paper strips to visually demonstrate these relationships.

Problem 4: 4, $\frac{1}{4}$

Individual

1. **Students cut a strip of paper that is 8 cm long. Call this length A. Now draw a line that is 4 times as long. Call it B.**

   A is _____ times as long as B.

   B is _____ times as long as A.

Whole Group

2. **The teacher elicits responses and students justify that A is $\frac{1}{4}$ times as long as B because 4 copies of A cover the length of B. And, B is 4 times as long as A, because if B is split into 4 equal partitions, then each partition is congruent with A.**

   Teacher note. Use paper strips to visually demonstrate these relationships.

Problem 5: $\frac{2}{3}$, $\frac{3}{2}$

Individual

1. **Students cut a strip of paper that is 12 cm. long. Call this length A. Now draw a line that is $\frac{2}{3}$ times as long. Call it B.**

   B is _____ times as long as A.

   A is _____ times as long as B.

Whole Group

2. **The teacher elicits responses and students justify that B is $\frac{2}{3}$ times as long as A, because 2 copies of a 3-split of A is congruent with the length of B (8 cm.) And, A is $\frac{3}{2}$ times as long as B, because 3 copies of a 2-split of B (4 cm.) is congruent with the length of A (12 cm.).**

   Teacher note. Use paper strips to visually demonstrate these relationships.

Problem 6: Challenge Problem

1. **Using the cm. ruler, draw a length B that is $\frac{3}{4}$ times as long as a length A. Show that A must be $\frac{4}{3}$ times as long as B.**

Formative Assessment

Provide students with strips of paper that are as long as the lines.

1. Here is a line:

   Draw another line that is 4 times as long. You can use the strips of paper provided to help you measure.

   Complete this statement:

   The first line is _____ times as long as the line that you drew.

2. Here is a line:

Draw another line that is $\frac{2}{3}$ times as long. You can use the strips of paper provided to help you measure.

Complete this statement:

The first line is _____ times as long as the line that you drew.

Formative Assessment

3. The lengths of A and B are measured in inches. If A is $\frac{2}{3}$ times as long as B, then B must be _____ times as long as A. Explain.

Formative Assessment Record — Relating Lengths Unit 15

Student _____ Date _____

For each student, indicate

Level	Description	Notes
Understands reciprocal relation	On items 1-2, obtains correct result. On item 3, responds $\frac{3}{2}$ and explains why with words or drawings.	
Reciprocal relation for easily visualized relation ($\frac{2}{3}$, $\frac{3}{2}$)	Items 1, 2 obtains correct result.	
Partial Understanding of Reciprocal	Item 1 correct results. Item 2 correct drawing but does not describe relation as $\frac{3}{2}$ in number sentence.	
Reciprocal relation for unit fraction.	Correct response to item 1.	
Incomplete Understanding	On item 1, draws correct response but does not symbolize relation as $\frac{1}{4}$	
Other		
NL	No interpretable responses.	

Academic Language: Indicate academic words the student is familiar with by recording them here.