Investigating Multiplication

Mathematical Concepts

- $\frac{a}{b}$ means a iterations of the b-split of a unit.
- $\frac{a}{b} \times 1-$ unit expands the magnitude of the unit length when $a > b$.
- $\frac{a}{b} \times 1-$ unit shrinks the magnitude of the unit length when $a < b$.
- $\frac{a}{b} \times 1-$ unit leaves the magnitude of the unit length unchanged when $a = b$.

Unit Overview

The goal of the lesson is to re-visit the meaning of $\frac{a}{b}$ unit as a iterations of $\frac{1}{b}$ unit and to investigate what happens to a unit length or to a composite length under multiplication. In the lesson, students compare and contrast multiplication of a unit length by fractions less than, equal to, and greater than one. Teachers support the following generalization: Multiplication of the unit length when $\frac{a}{b} > 1$ results in an expansion of the length. Mutliplication of the unit length when $\frac{a}{b} < 1$ results in contracting or shrinking the length. Multiplication by $\frac{a}{b}$ where $a = b$ leaves the length unchanged. The lesson concludes with an extension of multiplication by $\frac{a}{b}$ to composite units (e.g., 2 BF).

UNIT

13

Contents

Read

- ☐ **Unit 13**
  Start by reading the unit to learn the content and become familiar with the activities.
- ☐ **Mathematical Background**
  Reread the Mathematical Background to anticipate the kinds of ideas and discussions you will likely see during instruction.

Prepare

- ☐ Either have students cut 10 strips that are 1 BF long or provide them.
- ☐ Have students cut 6 strips that are 2 BF long or provide them.
- ☐ Extra strips of 1 BF and 2 BF, as needed.
- ☐ For formative assessment, per student, 5 unit strips of a different length than the BF.

The core ideas about measurement emphasized in this unit are those related to the relation between the size of a unit and its measure, and the previously established notion of equivalent measures as the same distance from the origin of a measure.

Magnitude

A length has a magnitude—an extent.

Measure as Accumulation of Units

The measure of the magnitude of a length is the ratio of that length to the length of a particular unit of measure. We can count units to establish this ratio. For example, a 10-inch length is 10 times as long as a 1 inch length, so it has a measure of 10 inches.

Inverse Relation between Length of Unit and Measure of a Magnitude

Given a magnitude and units of measure that are of different lengths, shorter unit lengths produce greater measures than do longer unit lengths. For example, if measured in centimeters, the 10-inch length has a measure of approximately 25 centimeters.

Interpreting $\frac{a}{b}$ unit as *a* copies (iterations) of 1/b units

Recall that $\frac{a}{b}$ is interpreted as *a* copies of a *b*-split of a unit. This means, for example, that $\frac{2}{4}$ means 2 iterations of a 4-split of the unit.

Multiplication of a Unit Length

Multiplication of a length measured in units, such as 4 u, by a quantity less than 1, such as $\frac{1}{3}$ or $\frac{2}{3}$, shrinks the length. It decreases its magnitude, so that $\frac{1}{3} \times 4 \text{ u} = \frac{4}{3}$ u or $\frac{2}{3} \times 4 \text{ u} = \frac{8}{3}$ u.

Multiplication of a unit length by a quantity greater than 1, such as $\frac{5}{3}$, expands the unit length. It increases its magnitude. For example, $\frac{5}{3} \times 4 \text{ u} = 2 - \frac{20}{3}$ u.

Multiplication of a unit length by 1 leaves the magnitude of the unit length unchanged. For example, $\frac{2}{2} \times 4 \text{ u} = \frac{8}{2}$u.

Equivalence and Multiplying by 1

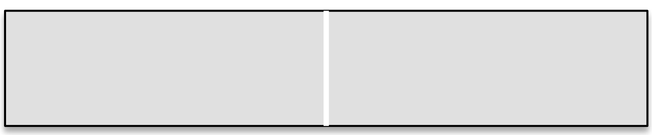
Multiplying by $\frac{a}{b}$ where $a = b$ means that the unit of measure is changing but the magnitude of the length remains unchanged, because the change in the splitting factor (b) is compensated by the same change in the number of iterations. So, for any a and b, where $a = b$, multiplication by $\frac{a}{b}$ is equivalent to any other multiplication by $\frac{a}{b}$ where $a = b$.

For example,

1 of 1 unit is the same distance traveled as $\frac{2}{2}$ of 1 unit.

1 unit

$\frac{2}{2}$ unit



$\frac{3}{3}$ is also the same distance traveled. Note that the unit of measure has changed (from 1u to $\frac{1}{2}$ u to $\frac{1}{3}$ u) but this is compensated for by iterating a greater number of times (from 1 to 2 to 3).

$\frac{3}{3}$ unit

These expressions (1u, $\frac{2}{2}$ u, $\frac{3}{3}$ u) are equal in the sense that they all represent the same distance traveled from the starting point to the ending point.

Multiplying Units of Length by Fractions

Partners / Small Groups

1. **Students use their BF tape measures to cut 1 strip of paper, and then use congruence to cut 4 additional 1 BF strips.**
2. **Students put 1 strip aside and keep it in sight. Students work with partners with the other 4 strips to find the result of:**

   a. $\frac{1}{3} \times 1$ BF
   b. $\frac{2}{3} \times 1$ BF
   c. $\frac{3}{3} \times 1$ BF
   d. $\frac{4}{3} \times 1$ BF

3. **Students note any pattern that they detect and make a conjecture about whether or not the pattern would be the same for other fractions.**

   Teacher Note. Students should keep relating the result to the unit strip. We aim to support the generalization that multiplying a measured length by a fraction will either (a) shrink the length, (b) expand the length, or (c) leave the magnitude of the length alone, although it may change the unit of measure.

Whole Group

4. **Students demonstrate their results and try to make a generalization about what their results suggest. The teacher can ask questions to support generalization by asking:**

   Q: What does "*x*" mean? (Connect to "of" to help students who may not be certain)
   Q: What does the denominator of each of these fractions mean? What did you do? (emphasize the split of the BF unit)
   Q: What does the numerator of each of these fractions mean? (Emphasize number of copies)
   Q: What seems to be happening when we multiply 1 BF by $\frac{1}{3}$ vs. by $\frac{4}{3}$? Which result makes the BF shorter? Longer?
   Q: Which does <u>not</u> seem to make BF longer or shorter? Why?

Partners / Small Groups

1. **Students use their BF tape measures to cut 5 strips of paper, each of which is 1 BF long.**
2. **Students keep the BF in sight. For each of the following, one partner predicts whether or not the BF will shrink, expand or remain unchanged. The other partner uses the strip to find the result.**

    a. $\frac{1}{4} \times 1$ BF
    b. $\frac{2}{4} \times 1$ BF
    c. $\frac{3}{4} \times 1$ BF
    d. $\frac{4}{4} \times 1$ BF
    e. $\frac{6}{4} \times 1$ BF

Whole Group

3. **Students demonstrate their results and try to make a generalization about what their results suggest. The teacher can ask questions to support generalization by asking:**

    Q: What seems to be happening when we multiply 1 BF by $\frac{1}{4}$ vs. by $\frac{6}{4}$? Which result makes the BF shorter? Longer? Why?

    Q: What happens when we multiply 1 BF by $\frac{4}{4}$? Does the length of the BF change? What is changing? (the unit of measure is now $\frac{1}{4}$ BF)

4. **True or False? $\frac{3}{3}$ BF $= \frac{4}{4}$ BF**

    Teacher note. It is important to address in what sense $\frac{3}{3}$ and $\frac{4}{4}$ "mean the same thing." They are the same in that 3 iterations of $\frac{1}{3}$ BF results in the same location as 4 iterations of $\frac{1}{4}$ BF. But $\frac{1}{4}$ BF and $\frac{1}{3}$ BF are clearly different partitions. "Equals" always refers to a relation between two or more elements and does not mean that they are exactly alike in every way.

Partners / Small Groups

5. **Students use their BF tape measures to cut 6 strips that are 2 BF long. They mark 0, 1 BF and 2 BF on the cut strips.**

6. **Students keep the 2 BF strip in sight. For each of the following, one partner predicts whether or not the 2 BF strip will shrink, expand, or remain unchanged. The other partner uses the strips to find the result.**

   a. $\frac{1}{4} \times 2$ BF
   b. $\frac{1}{6} \times 2$ BF
   c. $\frac{3}{6} \times 2$ BF
   d. $\frac{6}{6} \times 2$ BF
   e. $\frac{7}{6} \times 2$ BF

Whole Group

7. **Students demonstrate their results and again see if the pattern observed with 1 BF holds for 2 BF.**

   Q: Did we see the same pattern again? Why is it happening again?
   Q: Do we have to actually use the strips to know if 2 BF will grow or shrink or remain the same?

8. **True or False? $\frac{6}{6}$ of 2 BF = $\frac{12}{12}$ of 2 BF. Here is a familiar tool, the foot ruler.**

   a. Look carefully at the ruler with your partner. What do you notice? Write down three things that you notice about the ruler.
   b. Elicit what students notice and what everyone seems to agree about.

9. **You said that there were inches on the ruler. How many did we see?**

   a. Each of the 12 inches is exactly the same length, so what part of the ruler does each inch represent? How could I symbolize an inch as a part of a foot ($\frac{1}{12}$ ft.) Remember, the bottom number of the fraction, also called the denominator, shows the number of equal parts of the unit. And the top number shows the number of copies of this equal part.
   b. Use your finger starting at zero to travel $\frac{1}{12}$ ft., $\frac{3}{12}$ ft. , $\frac{6}{12}$ ft., $\frac{9}{12}$ ft., $\frac{12}{12}$ ft., $\frac{13}{12}$ ft.

10. **We are going to try to re-make what we see on the foot ruler, but we are going to use this Big-Foot (BF) unit instead.**

    Our first challenge is to split the BigFoot unit into 12 equal parts, just like the ruler. Can we make 12 equal parts if we keep splitting by 2? Talk with your elbow partner**.**

Provide students with a unit length that is not the same length as the BF.

Name ________________________________ **Date** ________________

Each strip is the length of a unit of measure called a Kazoie (K). Use a strip to show and write the result of:

1. $\frac{1}{3}$ x 1 K = ______________

2. $\frac{3}{3}$ x 1K = ______________

3. $\frac{4}{3}$ x 1 K = ______________

4. $\frac{3}{4}$ x 2 K = ______________

5. Show with strips why this statement is true or why it is false.

   $\frac{3}{3}$ K = $\frac{2}{2}$ K

6. When a length 1K is multiplied by $\frac{9}{5}$ ($\frac{9}{5} \times 1$ K), will the resulting length measured in K be longer or shorter or unchanged? Why do you think so? [DO NOT USE STRIPS]

Formative Assessment Record

Student ____ Date _______________

Indicate the levels of mastery demonstrated by circling those for which there is clear evidence:

Level	Description	Notes
Multiplication of length by $\frac{a}{b}$ interpreted as split-copy	On items 1-4, splits the unit appropriately and shows the a copies to obtain the correct result.	
Multiplication of length by $\frac{a}{b}$ interpreted as split-copy for a < or = b.	On items 1-3, splits units appropriately and shows the a copies to obtain the correct result.	
Emerging sense of the meaning of Multiplication by $\frac{a}{b}$	Performance on items 1-3 shows some evidence of split or of copy, but fails to coordinate them. Please note how student seems to be thinking about m	
Equivalence	Item 5. Shows that $\frac{2}{2}$ K is as long as $\frac{3}{3}$ K even though the splits are different. Does not show why or thinks that they can't be.	
Generalization	Item 6. Predict correctly and justifies by generalization established in class.	
NL	No interpretable responses.	