Reverse Engineering

UNIT

12

Contents

Mathematical Concepts

- Scales coordinate units to facilitate measure.
- The origin of the scale is 0, and units are marked by numerals to indicate distance traveled, not the count of the unit.
- Unit relations are represented on a scale, so that unit-of-unit and subunit-of-unit relations are visible. For example, a foot represents a composition of 12 inches (a unit-of-units). Each inch is partitioned further by composing 2-splits, such as $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ inch, creating a subunit of unit relation.
- Fractions represent partial units.
- $\frac{1}{b}$ represents 1 copy of a unit partitioned into b congruent (equi-partitioned) parts.
- $\frac{a}{b}$ represents a copies of a unit partitioned into b congruent parts.
- $\frac{a}{b}$ can be interpreted as the quantity obtained by traveling from zero to the location $\frac{a}{b}$.
- $\frac{a}{b}$ can also be interpreted as the quantity obtained by a iterations of a length $\frac{1}{b}$.
- Any value on the scale can serve as its origin (zero-point).

Prior Mathematical Understandings

- Length measure models of arithmetic operations: multiplication, addition and subtraction (Length Measure, Grade 2)
- 2-split and compositions of 2-splits of units of length, with arithmetic operations (Length Measure, Grade 2)

Unit Overview

The goal of the lesson is to reverse engineer a standard ruler to produce a copy that is 2 times as long as the standard ruler. The purpose of working at a larger scale is to foster thinking about relations between units and subunits. The larger scale makes it practically easier to compose 3- and 2-splits of each 2-inch unit (called a Binch or Big Inch in this unit) that mimic the divisions of the inch on a standard ruler.

Students start with 2-foot long strips of paper, a unit called a Big Foot (BF). Students first split the strip into 12 equal partitions. This split involves the composition of a 3-split and two 2-splits, in any order. To get a sense of how this might work, students are first challenged to 3-split the paper strip and to label the endpoint of each of the resulting partitions. The 3-split models multiplication of the unit by $\frac{1}{3}$. Students use the BF ruler to model addition and subtraction with thirds. Next, students 2-split each third, $\frac{1}{2} \times \frac{1}{3}$ BF, resulting in $\frac{1}{6}$ BF. Students label each of the endpoints of each of the sixths, establishing equivalences, such as between $\frac{1}{3}$ and $\frac{2}{6}$. They use these equivalences to add and subtract thirds, sixths, and mixtures of thirds and sixths. Then, students 2-split again, $\frac{1}{2} \times \frac{1}{6}$ BF to produce $\frac{1}{12}$ BF. Conventionally, we call the resulting 12th partition an inch when we begin with a standard foot instead of a Big Foot. Here we call the resulting unit a Binch (Big-inch). Students compare their construction to a ruler, noticing that their ruler is a scaled version of the standard ruler. This relates to the geometry units on similarity and scaling. Students go on to establish equivalences, such as among $\frac{1}{3}$ BF, $\frac{2}{6}$ BF, and $\frac{4}{12}$ BF by labeling them. Students use these equivalences to add and subtract thirds, sixths, and twelfths.

Students carefully examine an inch within a standard ruler and discern what the marks on the ruler mean. Then, working with 2-inch strips, they try to reproduce the $\frac{1}{2}$ and $\frac{1}{4}$ partitions, and use their reproduction to mark one of the Binches on their BigFoot ruler. Students compare and contrast 4 different ways of symbolizing units on a ruler, and they discuss which of the forms of symbolization makes the most sense for measuring length.

Unit Overview

Formative assessment includes having students use other length units to model multiplication by $\frac{1}{3}$ and composition of successive multiplications by $\frac{1}{2}$ and $\frac{1}{3}$. Students justify whether or not $\frac{1}{3}$ of $\frac{1}{2}$ unit will result in the same length as $\frac{1}{2} \times \frac{1}{3}$ unit. Students also add and subtract fractional lengths with like and unlike denominators.

Academic Language:

- Origin (Zero-point)
- Equivalent (or equal)
- Scale
- Denominator
- Numerator
- Copier or Repeater (numerator)
- Split or Splitter (denominator)

Materials & Preparation

Read

- ☐ **Unit 12**
  Start by reading the unit to learn the content and become familiar with the activities.
- ☐ **Mathematical Background**
  Reread the Mathematical Background to anticipate the kinds of ideas and discussions you will likely see during instruction.
- ☐ **Measurement Construct Map**
  Read the construct map and look at the multimedia map to help you recognize the mathematical elements in student thinking, and to order these elements in terms of their level of sophistication, especially ToLM (Theory of Length Measurement) and composites of 3- and 2-splits.

Gather

- ☐ Student math journals
- ☐ Calculating machine tape for creating tape measures
- ☐ Glue
- ☐ Markers for labeling tape measures
- ☐ Chart paper, markers
- ☐ Sentence strips (They can be divided in half so they are not quite so wide)

Prepare

For each student: 4 Big-Foot strips ($\frac{1}{2}$ in. × 24 in.).

For each student: 4 two-inch strips.

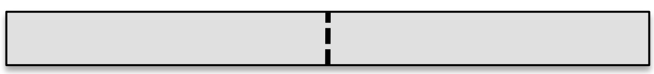
A standard ruler for each student of pair of students.

The innovation in this unit is to compose splits of different partitions. Previously, we established compositions of 2-splits, such as $\frac{1}{2}\times\frac{1}{2}\times 1$ unit as resulting in $\frac{1}{4}$ unit. Here we model compositions of 3-splits and 2-splits.

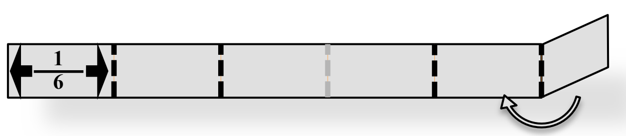
A 3 split of a unit, symbolized by $\frac{1}{3}$ of 1 unit (or as $\frac{1}{3}\times$ 1 unit), produces three equal partitions. These can be approximated by folding paper, although it is more difficult to do so than the previous 2-split.

To illustrate the composition of a 3-split and a 2-split, consider the following:

$\frac{1}{3}\times\frac{1}{2}\times 1$ unit: Beginning with 1 unit, we 2-split it, resulting in 2 congruent partitions.



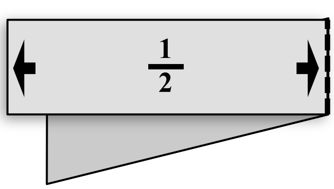
Then each of the 2 congruent partitions is split into 3 more congruent partitions, resulting in 6 partitions of the unit. The distance between the endpoints of each partition is called $\frac{1}{6}$ unit.

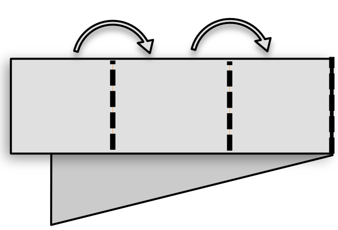


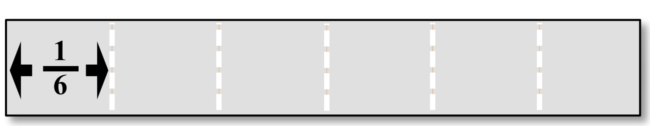
Because we are working with paper strips, another way of thinking about this action is as follows:

$\frac{1}{2}\times 1$ unit $=\frac{1}{2}$unit

$\frac{1}{3}\times\frac{1}{2}$unit $=\frac{1}{6}$ unit







Introducing Reverse Engineering

Whole Group

1. **Here is a familiar tool, the foot ruler.**

    a. Look carefully at the ruler with your partner. What do you notice? Write down three things that you notice about the ruler.
    b. Elicit what students notice and what everyone seems to agree about.

2. **You said that there were inches on the ruler. How many did we see?**

    a. Each of the 12 inches is exactly the same length, so what part of the ruler does each inch represent? How could I symbolize an inch as a part of a foot ($\frac{1}{12}$ ft.) Remember, the bottom number of the fraction, also called the denominator, shows the number of equal parts of the unit. It tells us that the unit has been split into 12 equal parts. And the top number shows the number of copies of this equal part.
    b. Use your finger starting at zero to travel $\frac{1}{12}$ ft., $\frac{3}{12}$ ft., $\frac{6}{12}$ ft., $\frac{9}{12}$ ft., $\frac{12}{12}$ ft., $\frac{13}{12}$ ft.

3. **We are going to try to re-make what we see on the foot ruler, but we are going to use this Big-Foot (BF) unit instead.**

    Our first challenge is to split the Big-Foot unit into 12 equal parts, just like the ruler. Can we make 12 equal parts if we keep splitting by 2? Talk with your elbow partner**.**

3-Split

It seems that we will need to think about another way to split the unit. To get us started, I am going to demonstrate how I can fold the BF into 3 equal congruent (equal length) parts. Later we can split the unit more (teacher demonstration).

Individual

1. **Let's all try it out (BF unit or some other unit).** Split the unit into 3 equal parts by experimenting with the paper. Talk with your elbow partner too.

Whole Group

2. **What do we call each part? ($\frac{1}{3}$)** We can write a number sentence to tell someone else what we did:
   $\frac{1}{3}$ of 1 BF = $\frac{1}{3}$ BF The denominator is the splitter. We have split the BF into three equal parts. The top number, the numerator, tells us how many copies we have. Thinking about the BF unit, 1/3 BF means to travel the distance from zero (the beginning of the ruler) to 1/3 BF.

3. **Talk with your elbow partner: 1 BF is _____ times as long as $\frac{1}{3}$ BF.** How could you demonstrate that you are right? (Students should demonstrate that 3 iterations of $\frac{1}{3}$ BF is congruent with 1 BF).
   a. Say/Demonstrate: To make certain that we really have split the unit into $\frac{1}{3}$ BF, we should be able to fit exactly 3 copies of the $\frac{1}{3}$ BF so that the result is the same length as 1 BF ft. unit.
   b. Let's see if we can do that: Teacher tapes a BF unit to the board and directly underneath is, $\frac{1}{3}$ BF. Then iterates with another copy of the $\frac{1}{3}$ BF. 3 times, leaving a mark after each translation, so that students can see that 3 iterations of $\frac{1}{3}$ BF. are congruent with 1 BF. The teacher should lead students

in a symbolization of the iteration: $3 \times \frac{1}{3}$ BF means iterate (translate) $\frac{1}{3}$ BF 3 times, so that $3 \times \frac{1}{3}$ BF = 1.

c. Demonstrate that a non-equal partition into "thirds" will not have this property. Three iterations will either be longer or shorter than 1 BF.

Teacher note. We are seeking to promote the generalization that $n \times \frac{1}{n}$ unit $= \frac{n}{n}$ or 1 unit. The only exception to the failure to travel a unit distance by iterating a non-equal partition 3 times occurs if splitting the unit into parts happens by chance to produce one partition that is one-third of the unit, while the other 2 partitions are, for example, .17 unit and .5 unit, and then 3 iterations are made with the one-third unit. Ensure that your demonstration is with a non-equal 3-split that is not one-third!

Individual

4. **Label the $\frac{1}{3}$ units on your BF ruler. Where does 0 go?**

   a. Close your eyes, starting at one end of the ruler. Travel $\frac{1}{3}$ BF. Now continue to travel another $\frac{1}{3}$ BF. How far have we traveled? $\frac{2}{3}$ BF

   b. Write a number sentence that tells someone else what we did: $\frac{1}{3}$ BF + $\frac{1}{3}$ BF = $\frac{2}{3}$ BF or 2 x $\frac{1}{3}$ BF = $\frac{2}{3}$ BF.

Whole Group

5. **Teacher elicits solutions to labeling the ruler and traveling in $\frac{1}{3}$ BF units.** Teacher reminds students that addition joins units or parts of units. And addition means also: Continue to travel, as in travel $\frac{1}{3}$ BF, pause (for effect), continue to travel (+) and stop after traveling to the endpoint of $\frac{1}{3}$ BF: $\frac{1}{3}$ BF + $\frac{1}{3}$ BF = $\frac{2}{3}$ BF. Or, 2 of $\frac{1}{3}$ BF = $\frac{2}{3}$ BF.

Partner

6. **Working with a partner, show $\frac{2}{3}$ BF (one person) + $\frac{2}{3}$ BF (the other person) = $\frac{4}{3}$ BF. Show $\frac{4}{3}$ BF + $\frac{5}{3}$ BF**

Whole Group

7. **Teacher asks selected students to demonstrate their solutions to the addition problems. At teacher option, equivalent expressions for multiplication are shown when the same unit is repeated, as in 2 of $\frac{2}{3}$ BF = $\frac{4}{3}$ BF, also expressed as 2 x $\frac{2}{3}$ BF = $\frac{4}{3}$ BF.**

Composing 2- and 3-Splits, Equivalent Fractions

Whole Group

1. **Fold the BigFoot strip so that only $\frac{1}{3}$ BF is showing.**
   a. If we take $\frac{1}{2}$ of $\frac{1}{3}$ BF, what will be the result? How many congruent parts?
   b. How do we write an equation that tells someone else what we did? $\frac{1}{2}$ of $\frac{1}{3}$ BF or we write $\frac{1}{2} \times \frac{1}{3}$ BF. Notice that we put the unit on the right side and split it by 2.

$$\frac{1}{2} \times \frac{1}{3} \text{ BF} = \frac{1}{6} \text{ BF}.$$

2. **Talk with your elbow partner:** 1 BF is ____ times as long as $\frac{1}{6}$ BF (6). How could you demonstrate that you are right?
3. **Say/Demonstrate: To make certain that we really have split the unit into $\frac{1}{6}$ BF, we should be able to fit 6 copies of the $\frac{1}{6}$ BF unit so that the result is the same length as 1 BF ft. unit.**
   a. Let's see if we can do that: Teacher uses the BF unit previously taped to the board and directly underneath the $\frac{1}{3}$ BF, place a copy of $\frac{1}{6}$ BF.
   b. Invite a student to come up and use another copy of the $\frac{1}{6}$ BF to iterate 6 times, leaving a mark after each translation on the BF, so that students can see that 6 iterations of $\frac{1}{6}$ BF are congruent with 1 BF.
   c. The teacher should lead students in a symbolization of the iteration: $6 \times \frac{1}{6}$ BF means iterate $\frac{1}{6}$ BF 6 times, so that $6 \times \frac{1}{6}$ BF = 1.
   d. Demonstrate that a non-equal partition into "sixths" will not have this property. Six iterations will either be longer or shorter than 1 BF. (Note: unless one happens to be $\frac{1}{6}$ BF.)

Teacher note. We are seeking to promote the generalization that $n \times \frac{1}{n}$ unit $= \frac{n}{n}$ or 1 unit.

Individual

4. **Try this out with your BF.** Label each 6th. Find all the equivalent fractions that you can. Label them on your ruler.

   Teacher note. Students should find and label $\frac{2}{6}$ BF = $\frac{1}{3}$ BF, $\frac{3}{6}$ BF = $\frac{1}{2}$ BF, $\frac{4}{6}$ BF = $\frac{2}{3}$ BF, 1 BF = $\frac{3}{3}$ BF = $\frac{6}{6}$ BF

Whole Group

5. **Select solutions to equivalence for general conversation.** Be sure to emphasize that, for example, $\frac{2}{3}$ BF means the same thing as $\frac{4}{6}$ BF because they are at the same location on the ruler—the same distance from zero.

6. **We can see that $\frac{1}{3}$ BF is a different length than $\frac{1}{6}$ BF. So, if we join $\frac{1}{3}$ BF to $\frac{1}{6}$ BF, we say that we are adding $\frac{1}{6}$ BF to $\frac{1}{3}$ BF.**

   a. Demonstrate to students. Or, we can say that first we travel $\frac{1}{3}$ BF, then we continue to travel another $\frac{1}{6}$ BF. But so far, when we have added, we have always used the same partial unit. Remember we said that $\frac{2}{3}\text{BF} + \frac{2}{3}\text{BF} = \frac{4}{3}\text{BF}$. Is there a way that we could use what we just did with equivalence to keep using the same partial unit measure for both of the lengths that we are joining? Talk with your elbow partner about this.

   b. Elicit student solutions and if necessary, suggest that $\frac{1}{3}$ BF means the same thing as $\frac{2}{6}$ BF and so $\frac{2}{6}\text{BF} + \frac{1}{6}\text{BF} = \frac{3}{6}\text{BF}$.

   c. Reflect on the use of equivalence to find common measure.

Partners

7. **For each of these, one partner uses the BF strips (one from each partner) to show what happens and the other partner completes the number sentence and where necessary, changes it:**

$\frac{2}{6}\text{BF} + \frac{5}{6}\text{BF} =$__________

$\frac{3}{3}\text{BF} + \frac{1}{3}\text{BF} =$__________

$\frac{4}{6}\text{BF} + \frac{2}{3}\text{BF} =$__________

$\frac{1}{3}\text{BF} + \frac{1}{2}\text{BF} =$__________

Whole Group

8. **Teacher selects student solutions for presentation.** It is helpful to demonstrate multiple valid solutions when this would be productive (e.g., $\frac{4}{6}$ BF and $\frac{2}{3}$ BF with a result of $\frac{4}{3}$ BF or $\frac{8}{6}$ BF.)

Unit Overview
Materials and Preparation
Reverse Engineering
3-Split
2 and 3-Splits
2-2 and 3-Splits
BigFoot Inches
Zero Point
Formative Assessment

Composing 2-, 2- and 3-Splits ($\frac{1}{12}$), Equivalent Fractions

Whole Group

1. **Fold the BigFoot strip so that only $\frac{1}{6}$ BF is showing.**

   a. If we take $\frac{1}{2}$ of $\frac{1}{6}$ BF, what will be the result? How many congruent parts? Why do you think so? Why not 8 congruent parts?

   b. How do we write an equation that tells someone else what we did?

   c. We write $\frac{1}{2}$ of $\frac{1}{6}$ BF or we write $\frac{1}{2} \times \frac{1}{6}$ BF. The resulting equation is:

$$\frac{1}{2} \times \frac{1}{6}\text{BF} = \frac{1}{12}\text{BF}$$

2. **Talk with your elbow partner: 1 BF. is ____ times as long as $\frac{1}{12}$ BF.** (12) How could you demonstrate that you are right?

3. **Say/Demonstrate: To make certain that we really have split the unit into $\frac{1}{12}$ BF, we should be able to fit 12 copies of the $\frac{1}{12}$ BF unit so that the result is the same length as 1 BF ft. unit.**

   a. Let's see if we can do that: Teacher uses the BF unit previously taped to the board and directly underneath the $\frac{1}{6}$ BF, place a copy of $\frac{1}{12}$ BF.

   b. Invite a student to come up and use another copy of the $\frac{1}{12}$ BF to iterate 12 times, leaving a mark after each translation on the BF, so that students can see that 12 iterations of $\frac{1}{12}$ BF are congruent with 1 BF.

   c. The teacher should lead students in a symbolization of the iteration: $12 \times \frac{1}{12}$ BF means iterate $\frac{1}{12}$ BF 12 times, so that $12 \times \frac{1}{12}\text{BF} = 1$.

   d. Demonstrate that a non-equal partition into twelfths will not have this property. Twelve iterations will either be longer or shorter than 1 BF.

Teacher note. We are seeking to promote the generalization that $n \times \frac{1}{n}$ unit $= \frac{n}{n}$ or 1 unit.

Individual

4. **Try this out with your BF.** Label each 12th. Find all the equivalent fractions that you can. Label them on your ruler.

   Teacher note. Students should find and label in twelfths their previous locations: $\frac{2}{6}$ BF $= \frac{1}{3}$ BF, $\frac{3}{6}$ BF $= \frac{1}{2}$ BF, $\frac{4}{6}$ BF $= \frac{2}{3}$ BF, 1 BF $= \frac{3}{3}$ BF $= \frac{6}{6}$ BF.

Whole Group

5. **Select solutions to equivalence for general conversation.** Be sure to emphasize that, for example, $\frac{2}{3}$ BF means the same thing as $\frac{4}{6}$ BF and $\frac{8}{12}$ BF because they are at the same location on the ruler—the same distance from zero.

6. **We can see that $\frac{1}{12}$ BF is a different length than $\frac{1}{6}$ BF. So, if we join $\frac{1}{12}$ BF to $\frac{1}{6}$ BF, we say that we are adding $\frac{1}{12}$ BF to $\frac{1}{6}$ BF.**

   a. Demonstrate to students. Or, we can say that first we travel $\frac{1}{6}$ BF, then we continue to travel another $\frac{1}{12}$ BF. Is there a way that we could use what we just did with equivalence to keep using the same partial unit measure for both of the lengths that we are joining? Talk with your elbow partner about this.

   b. Elicit student solutions (e.g., $\frac{1}{6}$ BF $+ \frac{1}{12}$ BF $= \frac{2}{12}$ BF $+ \frac{1}{12}$ BF). Reflect on the use of equivalence to find common measure.

Partners

7. **For each of these, one partner uses the BF strips (one from each partner) to show what happens and the other partner completes the number sentence and where necessary, changes it:**

$\frac{6}{12}\text{BF} + \frac{6}{12}\text{BF} =$ ___________

$\frac{3}{3}\text{BF} + \frac{1}{6}\text{BF} =$ ___________

$\frac{4}{6}\text{BF} + \frac{2}{12}\text{BF} =$ ___________

$\frac{1}{3}\text{BF} + \frac{3}{12}\text{BF} =$ ___________

Whole Group

8. **Teacher selects student solutions for presentation. It is helpful to demonstrate multiple valid solutions when this would be productive.**

9. **Wrap-up: Let's unfold our BigFoot ruler and compare it to the standard ruler.** What is the same? What is different? What stands in for an inch on our BigFoot Ruler?

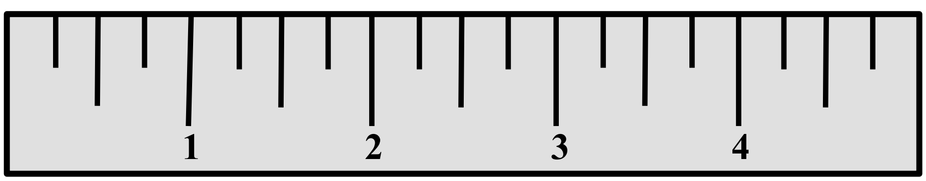
Partitioning BigFoot "Inches"

Whole Group / Partners

1. **Look again at an inch on your ruler and a copy that we gave you of some of the marks on a standard ruler.**

   a. What do the marks mean? How could you make them on one of the "Binches" we provided—there are 12 Binches in a BigFoot.

   b. Work with your partner and try to find a way to make the same partitions of each unit that you see in this copy. For each partition that you make, write an equation so that someone else would know what you did and what the result was.

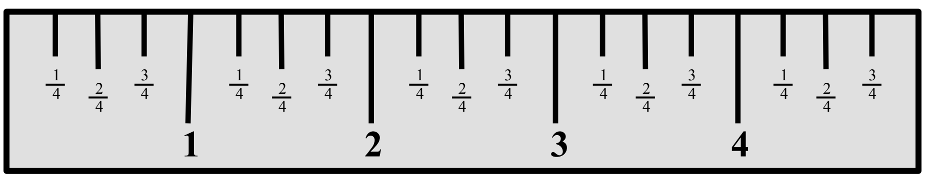


Whole Group

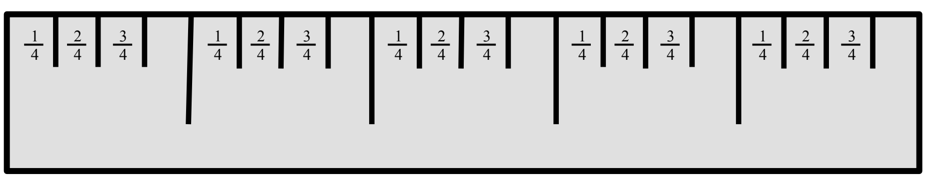
2. **The teacher elicits solution strategies and equations.**

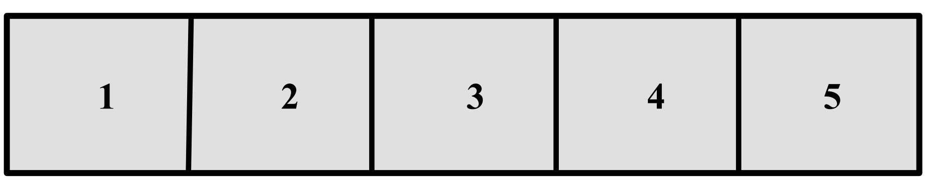
   a. For each equation, have at least one student demonstrate with paper. For example, the longer vertical line marks the 2-split, $\frac{1}{2}$ of 1 Binch, or $\frac{1}{2} \times 1$ Binch. The shorter vertical line marks the either the composition of 2-splits or a 4-split. Either $\frac{1}{2}$ of $\frac{1}{2}$ Binch, also expressed by $\frac{1}{2} \times \frac{1}{2}$ Binch, $\frac{1}{4}$ of 1 Binch, also expressed by $\frac{1}{4} \times 1$ Binch.

   b. Be sure to discuss the choices made by these designers. Other designers might prefer:



   c. These choices could be compared to others that have some clear drawbacks, such as these:





3. **Use the parts of your Binch to mark a Binch on your BigFoot ruler.** Compare your marks to those on the ruler.

Zero Point with Standard Rulers

Whole Group

1. **Measuring a length starting at 1 inch.**
    a. Now that you know how a standard ruler is made, let's use it to measure some lengths. But, when carpenters use rulers, the ends often wear so much that they're not sure anymore where 0 is. So, they often start their measurement at 1.
    b. If you start your measurement at 1 and draw a line that is 6 inches long, predict where on the ruler your line will end.

Individual

2. **Now try it.** Be sure to travel from one and move 6 inches. Where did you wind up? Check to see if you drew a line that is 6 inches long.
3. **Measuring a length starting at 2 inches.**
    a. If you start at 2 inches and draw a line that is 6 inches long, predict where on the ruler your line will end.
    b. Now try it. Be sure to travel from 2 inches and move 6 inches. Where did you wind up? Check to see if you drew a line that is 6 inches long.
4. **Measuring a length starting at a partial unit.**
    a. If you start at $2\frac{1}{2}$ inches and draw a line that is 6 inches long, predict where on your ruler your line will end.
    b. Now try it. Be sure to travel starting at $2\frac{1}{2}$ inches and moving 6 inches. Where did you wind up? Check to see if you drew a line that is 6 inches long.

Whole Group

5. **Share solution strategies.** Use the broken ruler item for a formative assessment of zero-point strategies.

Formative Assessment

1. With these orange units ($\frac{1}{2}$" by 3"), for each, show and tape your results to the page.

(a) $\frac{1}{3} \times 1$ unit

(b) $\frac{1}{2} \times \frac{1}{3}$ unit

Formative Assessment

(c) $\frac{1}{3} \times \frac{1}{2}$ unit

(d) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{3}$ unit

(e) $\frac{1}{3}$ unit $+ \frac{1}{3}$ unit

(f) $\frac{1}{3}$ unit $+ \frac{1}{6}$ unit

Formative Assessment

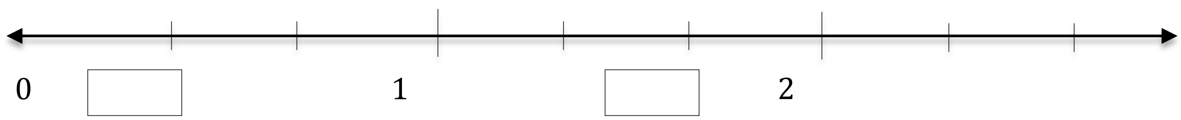
2. Put these into order from smallest length to longest length:

$\frac{5}{3}$ ft. $\frac{2}{6}$ ft. $\frac{1}{2}$ ft. $\frac{2}{3}$ ft. $\frac{4}{6}$ ft.

Explain why you put them in the order you did:

Formative Assessment

3. What numbers go in the boxes on this numberline?



Formative Assessment

Name ____

Write as many equivalent fractions to $\frac{6}{12}$ as you can in 30 seconds.

Formative Assessment

Name ____

Write as many equivalent fractions to $\frac{3}{3}$ as you can in 30 seconds.

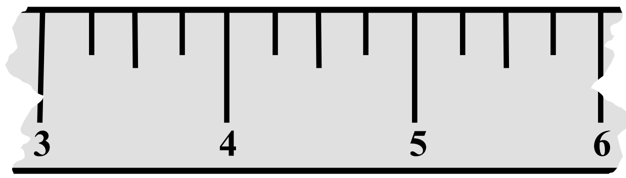
Formative Assessment

Name ____

Write as many equivalent fractions to $\frac{1}{3}$ as you can in 30 seconds.

Formative Assessment

Tom wants to measure the lengths of two lines, but his ruler fell on the floor and broke.



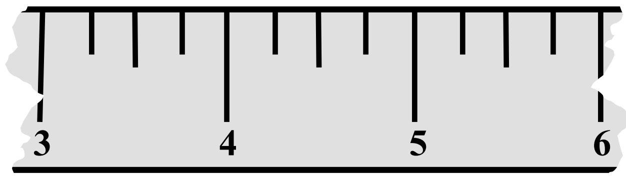
(a) Can he use this ruler to measure the short line below? Please circle either answer **A** or answer **B**, and then explain.

A. No, he cannot use the ruler to measure the short line.

Explain **why**:

B. Yes, he can use the ruler to measure the short line.

Explain **how**:



(b) Can he use the ruler to measure the long line below? Please choose either answer **A** or answer **B**, and then explain.

A. No, he cannot use the ruler to measure the long line.
Explain **why**:

B. Yes, he can use the ruler to measure the long line.
Explain **how**:

Formative Assessment Record

Student ____ Date ______________

Indicate the levels of mastery demonstrated by circling those for which there is clear evidence:

Level	Description	Notes
ToLM5D	*Account for change of origin when measurement does not start at zero.* Performance: Broken Ruler item. Yes for both, or Yes only for short line, or No (Circle which applies)	
ToLM5B	*Generate and compose splits of units involving odd factors, such as 3 or 5.* Performance: Item 1 (a-d) Represents each split and composition of splits. Circle if correct response: 1a, 1b, 1c, 1d.	
Addition of Fractions: Same Split	*Add fractions with identical unit splits.* Performance: Item 1e. Circle if correct response.	
Addition of Fractions: Different Splits	*Add fractions with different unit splits.* Performance: Item 1f. Circle if correct response.	
Equivalent Fractions	*Rapid generation of benchmark equivalent fractions.* Performance: Item 4. $\frac{6}{12} = (\frac{3}{6}, \frac{2}{4}, \frac{1}{2})$ Circle all that apply. Note others.________ Item 5. $\frac{3}{3} =$ _______ count number of valid solutions Item 6. $\frac{1}{3} = (\frac{2}{6}, \frac{4}{12})$ Circle all that apply. Note others.______________________	
Numberline Representations	*Familiarity with numberline representation of fractions.* Performance: Item 3. Identifies $\frac{1}{3}$ and $1\frac{2}{3}$ Circle all that are identified.	
Ordering Fractional Quantities	*Order magnitude of measured fractional quantities with different denominators.* Performance: Item 2, note order.	