Purposeful Pedagogy and Discourse Instructional Model: Student Thinking Matters Most

In studying the Common Core State Standards for Mathematics (CCSSM), and in particular the Standards for Mathematical Practice (SMP), it becomes clear that what we do in the classroom will change both from the perspective of the teacher and the student. The teacher will need a deep and connected understanding of the mathematics content and, during instruction, will need to provide experiences that allow the students to construct meaning for themselves through carefully crafted tasks and conversations. Students will need to reason, communicate, generalize and challenge the mathematical thinking of themselves and others. Student thinking matters most.

The *purposeful pedagogy and discourse instructional model* that we are using in the Arkansas CCSS Mathematics Professional Development Project, is based on the research of four sets of researchers:

- Jacobs, Lamb, and Philipp on professional noticing and professional responding;
- Smith, Stein, Hughes, and Engle on orchestrating productive mathematical discussions;
- Ball, Hill, and Thames on types of teacher mathematical knowledge;
- Levi and Behrend (Teacher Development Group) on Purposeful Pedagogy Model for Cognitively Guided Instruction.

This model is intended to support teachers to deliver strong mathematical content using critical best classroom practices as well as to develop a learning environment where their students regularly use the 8 Standards for Mathematical Practice.

Assessing Students, Professional Noticing, and Teacher Mathematical Knowledge

At the core of our model is *assessing* students (TDG-CGI model), which refers to taking a close look at student understanding. While assessing students, we apply the concept of *professional noticing* (Jacobs et al.).

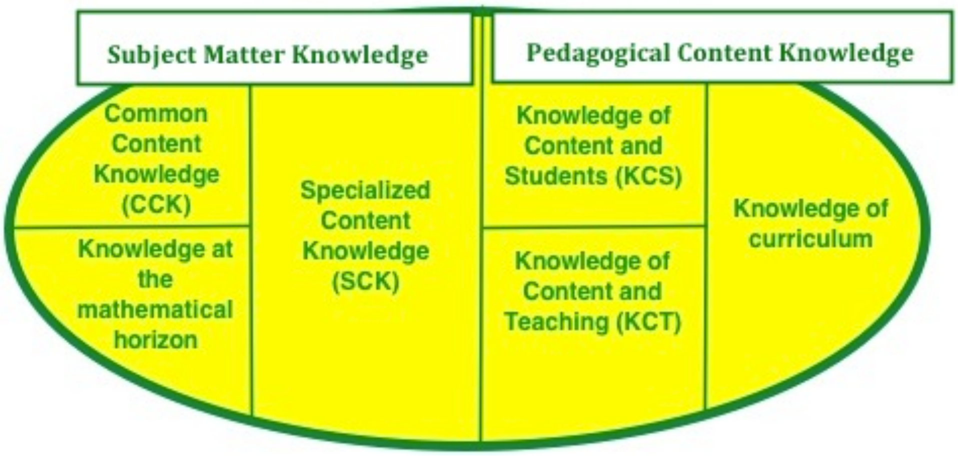
Professional noticing is comprised of 3 teacher skills:

- Attending to children's strategies,
- Interpreting children's understanding, and
- Deciding on how to respond on the basis of children's understanding.

In order to assess a students' understanding, we must look at the details of their thinking (what did they do) and then mathematically interpret these details. While this may seem trivial, students' strategies are complex and many deep mathematical operations and properties are embedded implicitly in their work. It takes time to identify the important details in students' thinking and then mathematically interpret the relationships and properties of operations that are embedded. The ability to notice will help the teacher identify the mathematics available for exploration during the lesson(s) to follow. Since student thinking matters most, in the Arkansas professional development courses the beginning of most classes will involve just making sense of and deepening our understanding of the details of students' strategies and the mathematical ideas embedded in their strategies.

The deeper and more connected a teacher's mathematical knowledge is, the easier it is to see and interpret the details of student thinking. Teaching mathematics requires a variety of types of knowledge as shown in Figure 1.

Figure 1: Domain map for mathematics knowledge (Hill & Ball)

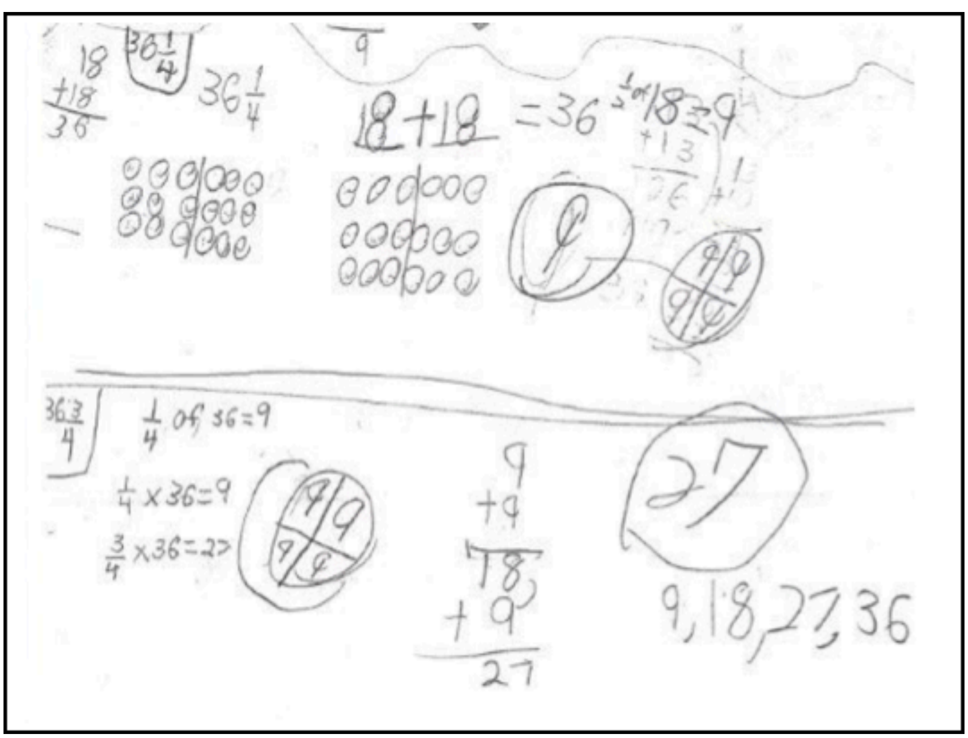


One type of teacher mathematical knowledge is *specialized content knowledge* – the mathematics behind the mathematics. For example, it is not enough to know we can divide fractions by inverting the second fraction and multiplying. A teacher must understand the mathematics that allows that strategy to work. Teachers must also understand how children will approach various problems, how their thinking develops, and how students' thinking is different than adults' thinking. This knowledge is called *knowledge of content and students*. All of this comes together to create the critical part of professional noticing, identifying the details of children's thinking and mathematically interpreting the details, which allows us to assess students' thinking, which of course matters above all else.

Exercising Professional Noticing

A fourth grade student solved the following problem: Kathy is making ____ cupcakes. She put ____ cups of frosting on each cupcake. How many cans of frosting will she need to make her cupcakes?
Two sets of numbers: (36, ¼) (36, ¾)

Figure 2: Student work on 36 x 1/4



What did this student do? What big mathematical ideas are embedded in her strategy? Take a few minutes to follow her trail of thinking. How would you mathematically notate her reasoning? See Figure 2.

What is it that teachers have to know to be able to understand the mathematics of this students thinking? It is not enough to know the properties of operations, teachers need to have a deeper understanding of these properties and be able to interpret this important mathematics embedded in student informal strategies.

To solve the problem using the first set of numbers, the student first transformed the problem with commutative property 36 x ¼ = ¼ x 36. She then solved by first finding that ½ of 36 = 18, and then finding that ½ x 18 = 9.

What mathematics allows for this sequence of thinking?

36 x ¼ = ¼ x 36	Commutative property
¼ x 36 = (½ x ½) x 36	Decomposing
(½ x ½) x 36 = ½ x (½ x 36)	Associative Property

The student then used the relationship between ¼ and ¾ to solve the problem with the other set of numbers.

Professional Responding, Purposeful Pedagogy, and Orchestrating Classroom Discourse
Critical instructional decisions are based on the mathematical interpretation of students understanding. With specialized content knowledge and knowledge of content and students in place, we are ready to focus on our mathematical practice. The Purposeful Pedagogy Model (TDG; Cognitively Guided Instruction) and Orchestrating Classroom Discourse (Stein et al.) come together to give us a vision of such practice centered around the all important student thinking.

The Purposeful Pedagogy Model has three components: *assess* students, *set a learning goal*, and *design instruction*.

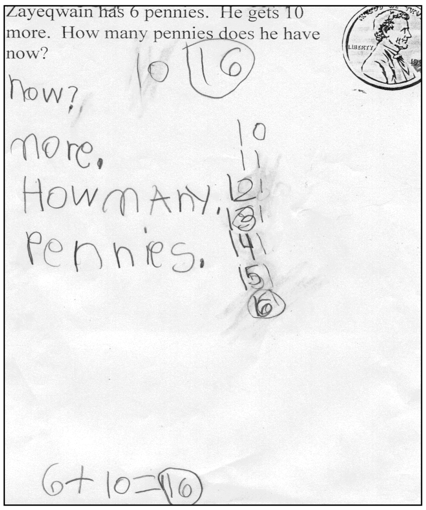
Elements for the *design of the instruction* are defined by the Orchestrating Classroom Discourse research.

Orchestrating Classroom Discourse outlines 5 practices for doing so:
1. *Anticipating* likely student responses to cognitively demanding mathematical tasks;
2. *Monitoring* students' responses to the tasks during the explore phase;
3. *Selecting* particular students to present their mathematical responses during the discuss-and-summarize phase;
4. Purposefully *sequencing* the student responses that will be displayed;
5. Helping the class *make mathematical connections* between different students' responses and between students' responses and the key ideas.

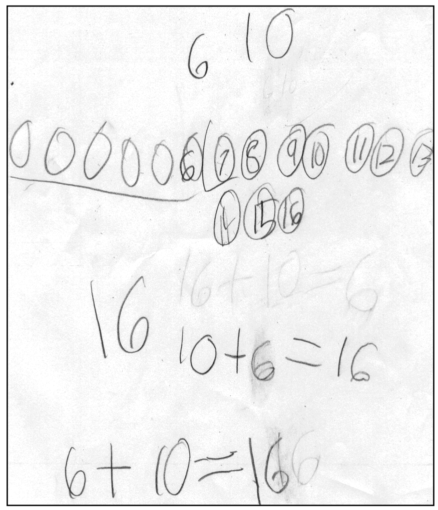
We will use the details of student understanding to *set learning goals* for our students, *design instruction*, and *orchestrate classroom discourse*. In doing so, we are engaging in the comprehensive practice of *professional responding*. This is best understood by taking a look at a classroom vignette from Kindergarten.

The students in this class have been solving problems that begin with 10 and add some more. The teacher has elected to present this problem by beginning with an amount other than 10 and then adding on 10 to see how students will respond. Before reading the classroom exchange and the teacher's *professional responding*, look at the student work from a kindergarten class for the following problem and answer these questions for yourself: Zayeqwain had 6 pennies. He gets 10 more. How many pennies does he have now?

- What did the students do?
- What is the mathematics embedded in their strategies?
- How are the strategies alike and different?
- Why do you think the teacher would have selected these two students to share?
- What conversation do you think the teacher would like to have?



Pretty



Moniqua

Classroom Vignette

The classroom teacher, Mrs. J asked the two students to share their solutions with the class and then engaged the class in a discussion around their strategies.

Pretty: There are 10 [pennies], (then she counted on) 11, 12, 13, 14, 15, 16.

Moniqua: There are 6, (then counted) 7, 8, 9, 10, 11, 12, 13, 14, 15, 16. And look I came up with 2 number sentences (excitedly)
6 + 10 = 16 and 10 + 6 = 16. See I can do it two ways!

Mrs. J: Look at these two strategies. Are they alike or different?

Sandia : They are alike. They both counted up.

Mrs. J.: I can see that they both used a counting up strategy. What do the rest of you think?

Theo: No, they are not alike. They started counting from a different number. Moniqua started counting from 6 and Pretty started counting from 10.

Mrs. J.: (pointing at Moniqua's number sentences) So, which one of Moniqua's number sentences go with the problem?

Class: 6 + 10 = 16.

Mrs. J.: Why?

Claudette: Because Zayeqwain has 6 pennies and then he gets 10 more.

Mrs. J.: Do any of these number sentences represent Pretty's strategy?

Claudette: 10 + 6 = 16.

Mrs. J.: Why?
Maria: Because she started with 10 first and then added her 6 seconds.
Mrs. J.: Is that okay to do?
Class: Yes. No. (mixed answers)
Mrs. J.: Will they both get the same answer?
Cecilia: I just counted it on my fingers. They are both 16. (The class is surprised.)
Mrs. J.: Really? Do you think this was just an accident, or do you think this will always happen?
Cecilia: It won't always happen, just on this problem.

Mrs. J decided to stop after this exchange and let her students' ideas percolate. About a week later, when she posed a similar problem (4 + 10), five additional students switched the order of the numbers to solve the problem and counted on from 10 instead of 4, utilizing the commutative property of addition. After further discussion, many of the students were beginning to think that this might be something that would always work.

How is this episode related to the *purposeful pedagogy and discourse instructional model*? The teacher posed a problem to her class and allowed the students to solve the problem the way that made sense to them. She identified student work that had the potential to help her students discover and make sense of an important mathematics concept. Specifically, when Pretty counted on from the larger number, the teacher understood Pretty's strategy was based on the commutative property. The teacher also noticed Moniqua's number sentences, 6 + 10 = 16 and 10 + 6 = 16. Based on her analysis and observation, she made an instructional decision to use this as an opportunity to have class discussion about the commutative property and how number sentences relate to the structure of the problem. As opposed to telling the students that this was a "turn around fact" or to "just count on from the larger number," she put the students in the position to consider these complex ideas for themselves by facilitating the dialogue to help them make meaning connected to their existing thinking.

While this type of exchange requires the classroom teacher to think very purposefully about instructional decisions and to think deeply about the mathematics embedded in students' solutions, the effort is worthwhile. The evidence comes from Cognitively Guided Instruction, an instructional model that emphasizes these very practices. Visits to CGI classrooms in Arkansas will reveal that children are thinking more deeply and flexibly about mathematics. They are not simply solving problems that have no meaning to them; they are becoming young mathematicians capable of explaining their thinking, which matters most, and grappling with and making sense of the complexity of the mathematics.

How do we now take the information we have about students' thinking and *professionally respond* in a way that is based on students' understanding and designed to facilitate children's thinking along a learning trajectory? We must select or design appropriate mathematical tasks or problems.

Mathematical tasks should be selected that will facilitate children's development. Once we have identified the task, we should consider the following questions:

- What do we anticipate students will do with the task?
- Will this task provide the experiences needed to further students' development?
- Which of the strategies we expect are likely to help the most in making sense of the mathematics in the goals we have set for them?

The next stage is to pose the task or problem and allow the students to solve the problem in a way that makes the most sense to them. Our job is to monitor students to identify what students are doing, guide them as they work, and decide which students' papers should be shared.

Back to Teacher Mathematical Knowledge

Once we have identified the best student strategies to meet the learning goals, we need to decide in which order to share students' strategies and what mathematical connections should be the focus of the classroom discussion. Again, the teacher's mathematical knowledge, specifically her *knowledge of content and teaching* (Hill & Ball, Figure 1), will be critical in making decisions by being able to envision how the mathematics available through the students' strategies connect to one another and to the mathematics concepts that are desired.

At this juncture, the teacher's knowledge of the mathematics meets the need to design or plan the discourse to take students deeper into the mathematics. This involves both the sequencing of the presentation and also the selection and phrasing of the questions posed during the discourse. There are likely multiple productive paths, but there are certainly some unproductive or problematic paths as well, and the teacher will need to choose well. Student thinking matters most.

Seeing It All Together

The research of these four sets of researchers come together to create the instructional model that we are using in the courses for the Arkansas CCSS Mathematics Professional Development Project. While this model, being a blend of the work of so many different projects, may seem complex at first, it is perhaps more straight forward when viewed using the graphic organizer below. The key ideas that hold the model together are the importance of noticing the details of student thinking, interpreting those details, and using that information to design instruction comprised of discourse around student strategies aimed at a specific mathematical goal. In other words, what details do we see in our children's work, how do we interpret their thinking, and where mathematically do we go from there? In maintaining this focus throughout the professional development courses, it is our hope to support teachers in their journey toward achieving mathematical proficiency for their students as described in the CCSSM. And always remember, student thinking matters most.

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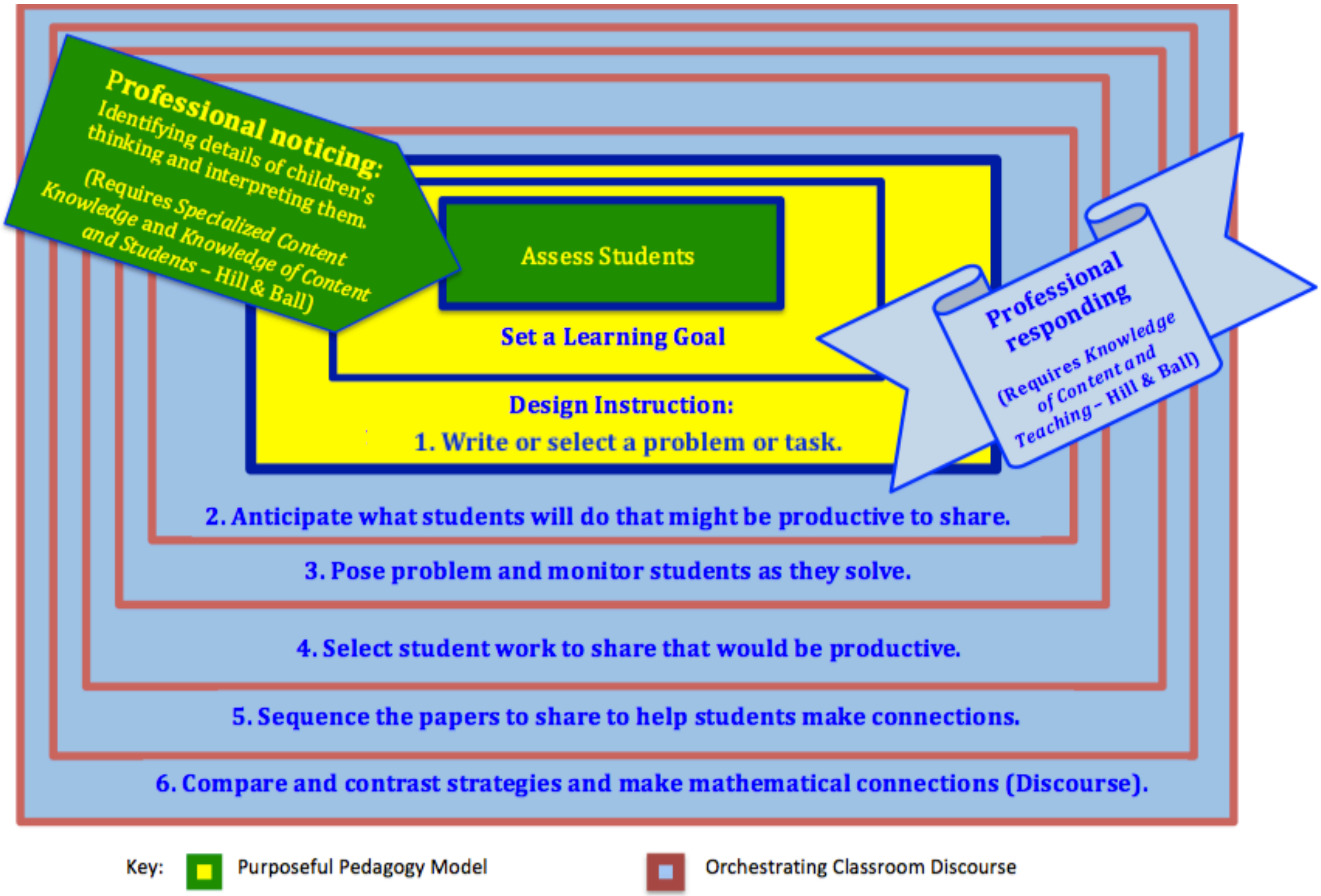

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