HELPING KINDERGARTENERS MAKE SENSE OF NUMBERS TO 100

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Abstract

The authors share what was learned about kindergarteners' abilities to make sense of numbers to 100 when one of the authors, Linda Jaslow, took over a kindergarten class from February through the end of the school year. Through examples of how she engaged her students in nine weeks of problem solving and discussions focused on making sense of the number system, we provide evidence that the children grew substantially in their ability to count and show understanding when counting by 10's and using 10's during problem solving. Suggestions for tasks to promote continued growth are also provided. Throughout this teaching experience, Mrs. Jaslow was reminded of the complexity of making sense of our number system, and this article showcases her instructional decision making that was based on inquiry into children's thinking. By valuing children's existing ideas, Mrs. Jaslow could use that thinking to help guide her instruction.

Introduction

When young children are asked to build a train of cubes and find the number of cubes in that train, their counting can be quite creative! They may accurately count the first few cubes and then continue the verbal counting sequence to a seemingly random stopping point. During their counting, they may skip cubes, reuse cubes that have already been counted, or fail to link their counts to any cubes at all. This creative counting is an indicator of the complexity of learning about numbers. To make sense of numbers, children must learn not only the verbal counting sequence (1, 2, 3,…), but also the way to connect each count with an object (one-to-one correspondence) and the fact that the last spoken number corresponds to the number in the counted set (cardinality). After many counting experiences, children gain these initial understandings of our number system. However, what happens when children begin counting to larger numbers or when they start grouping and counting by 10's? What do they learn about numbers and, in particular, the role that 10 plays in the structure of our number system?

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For numbers greater than 10, developing understanding becomes more complex than for smaller numbers. First, the verbal number sequence becomes longer and harder to memorize. Second, quantities associated with large numbers are bigger, thus providing more opportunity for miscounting. To simplify counting a large number of objects, children sometimes group them, for example, into 10's. They then need to link each count to a group of 10 objects. They also need to monitor two attributes of a number simultaneously, switching fluidly between counting individual objects and counting groups of 10 objects [1].

In this article, we share what we learned about kindergarteners' abilities to make sense of numbers to 100 when one of the authors, Linda Jaslow, took over a kindergarten class from February through the end of the school year. This class was in an inner city school in which approximately 65% of the students were Hispanic and 35% were African-American. We also illustrate how her inquiry into children's thinking enabled her to value their existing ideas and support their growth.

Mrs. Jaslow's instructional philosophy is consistent with the *Principles and Standards for School Mathematics* and draws heavily from Cognitively Guided Instruction [2-4]. Cognitively Guided Instruction (CGI) is a research-based framework of children's mathematical thinking, as well as a philosophy that instruction should elicit and build on children's existing understandings, including those developed outside of school. By posing carefully selected problems and allowing children to solve these problems in ways that make sense to them, teachers can learn about children's existing ideas, consider what those ideas mean in terms of children's understandings, construct subsequent problems to appropriately challenge and extend those understandings, and then repeat the cycle. In short, both mathematical goals and children's thinking guide teachers' instructional decision making. The following is a first-hand account of what Mrs. Jaslow learned when she inquired into her kindergarteners' thinking, and then used that thinking to help guide her instruction.

Mrs. Jaslow's Adventures in Kindergarten

I had never taught kindergarten and had no idea what kindergarten students were capable of doing. In this district, kindergarteners were expected to count to 100 by the end of the school year. With about nine weeks of school left, I learned that many children had one-to-one correspondence only with small numbers (up to 5) and that few could count to numbers larger than 29. I began analyzing what facilitated children's understanding of larger numbers. I decided that they first needed to learn the 10's counting sequence (10, 20, 30, 40, 50,...) because I naively thought that if children could remember the names and order of the decades, they should be able to count by 1's past 29.

Getting Started

I set out to help the children count by 10's and was shocked to learn that they could all do so already. Now I was really puzzled—if they could count to 100 by 10's and they could count to 29, why were they unable to take that next step and say, "30"? I came to realize that counting by 10's was a rote chant unconnected to any quantities. Although the children may have had a sense of 10, they probably lacked meaning for the other numbers in the 10's counting sequence. I decided that this disconnection was similar to their experience in learning to count by 1's in that they knew the rote verbal sequence before they developed the ability to link each count to a quantity (one-to-one correspondence). In essence, I needed to help the kindergarteners develop *ten-to-ten* correspondence so that counting by 10's was more than a rote chant.

To build meaning into counting by 10's, I designed story problems that would require the use of numbers larger than 10 and encourage grouping by 10. The children were accustomed to solving story problems because almost all of my instruction on number was presented in a story context. I selected familiar contexts so that the children could draw on their informal knowledge about these contexts to help them reason quantitatively. I generally read a problem aloud to the children, made a variety of manipulatives available (e.g., unifix cubes, color tiles), and asked them to solve the problem in any way that made sense to them. I also encouraged, but did not require, children to represent their thinking on paper and to write number sentences related to the problem. After the children had time to solve a problem individually, several children shared their strategies with the whole class, and together we discussed how to clearly record strategies and which number sentences best represented the problem.

I initially posed a multiplication story problem involving 10's because I wanted my class to make connections between groups of 10 objects and the 10's counting sequence. Recognizing that many children could count only to 29, I began with a problem involving numbers less than 30: "There are two children at your table. How many fingers are there?" I used this context because it built on the children's existing knowledge that they have ten fingers. Furthermore, although the children generally solved problems by representing all quantities and then counting by 1's, I wondered whether, in this context, they would use their knowledge that fingers come in groups of 10 to help them count by 10's. None did! Every child solved the problem by counting one set of 10 fingers by 1's and then a second set of 10 fingers by 1's.

During our whole-class sharing, I decided to push on the children's understanding of 10's. Cecilia and Stasha came to the front of the class, held up their two sets of hands, and counted the first and then the second set of fingers by 1's. They determined that there were twenty fingers, and the class agreed. I then asked them how many fingers Cecilia had and how many fingers Stasha had. The class easily responded that they each had ten fingers. I asked if there was another way to count how many fingers we had if Cecilia had 10 fingers and Stasha had 10 fingers. Aisha responded that they could count the fingers by saying, "10, 20." To push them a little further, I had a third child join the first two and asked the class how we could count the fingers. Immediately, Miguel responded, "10, 20, 30," pointing to each of the girls in turn.

To provide opportunities for the children to build on these emerging understandings, I continued to pose multiplication story problems about groups of 10 (e.g., "There are 5 vases of flowers. There are 10 flowers in each vase. How many flowers are there?") and addition problems about 10's (e.g., "There are 10 cows, 10 horses, and 10 pigs on the farm. How many animals are there?"). I also posed problems with dimes, to reinforce the idea of 10's in a context in which counting by 10's is common (e.g., "Zandra has 3 dimes. How much is that worth?"). When constructing these problems, I chose numbers in the 20–60 range to encourage children to develop their counting skills for numbers greater than 29 and to ensure that the problems remained accessible to those children who were still struggling to count by 1's. I was nervous about having kindergarteners work with such large numbers, but I decided that even if the problems had no other effect, they would give the children practice in counting and one-to-one correspondence.

To solve these problems, the children used a variety of strategies that reflected a range of understandings of number. Some drew all items and counted by 1's (see Figure 1), whereas others counted on from 10, not drawing the first set (see Figure 2). Other children drew all individual items, but counted groups by 10 (see Figure 3). Finally, some children did not represent items at all and instead recorded how they had counted by 10's (see Figure 4).

I believed that these problems had the intended effect on the children's understanding. Over time, many children learned that they could count groups of 10 by counting by 10's, and others simply practiced counting by 1's to numbers greater than 29. More counting practice occurred during class discussions in which I purposefully chose children to share a range of strategies. If the sharer used a strategy of counting by 1's, then the whole class helped him or her count by 1's. If the sharer counted by 10's, we counted with him or her as well. Thus, even those children who were not yet ready to count their own groups by 10's could participate in the class discussions.

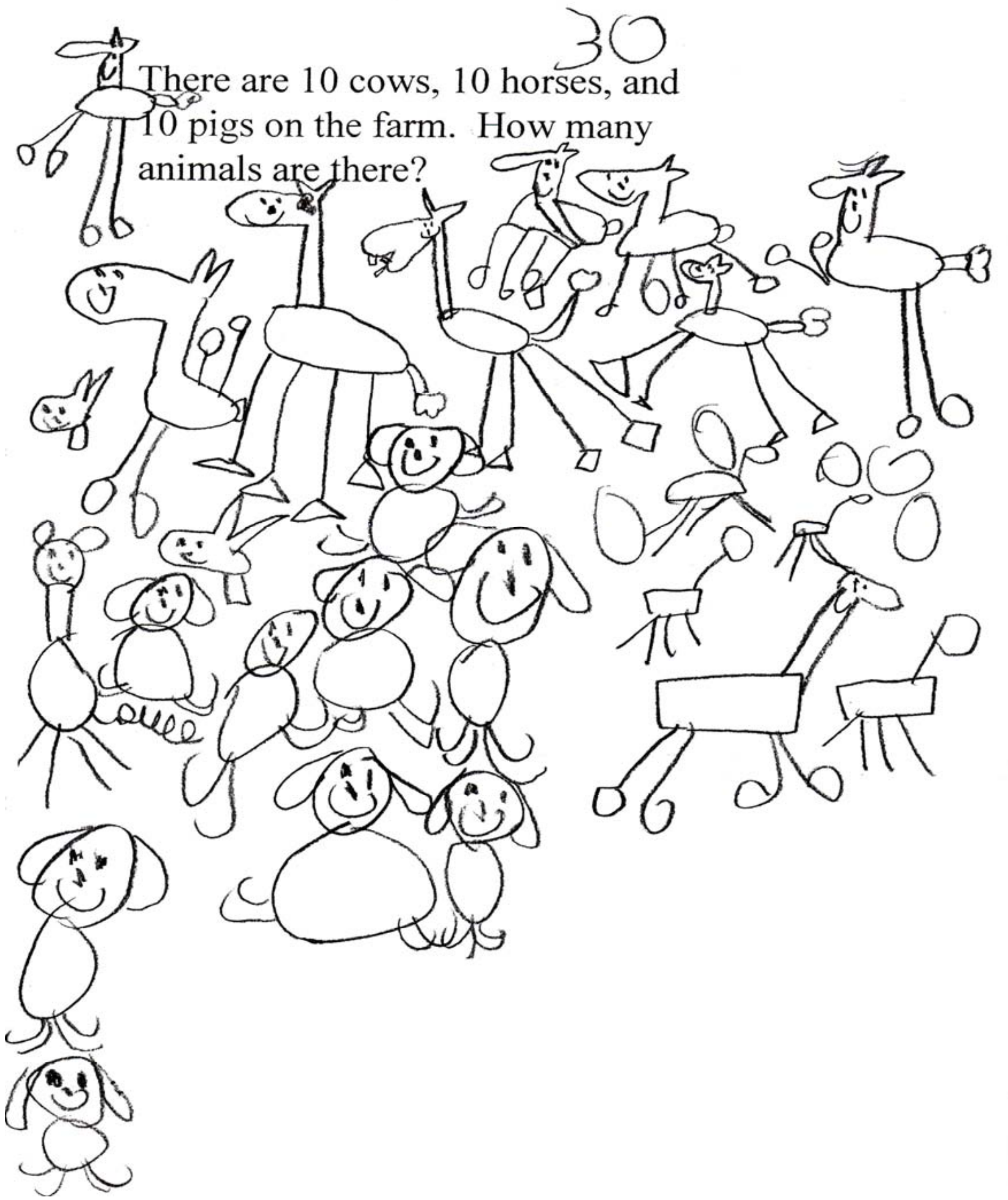

Figure 1. Representing all items and counting by 1's.

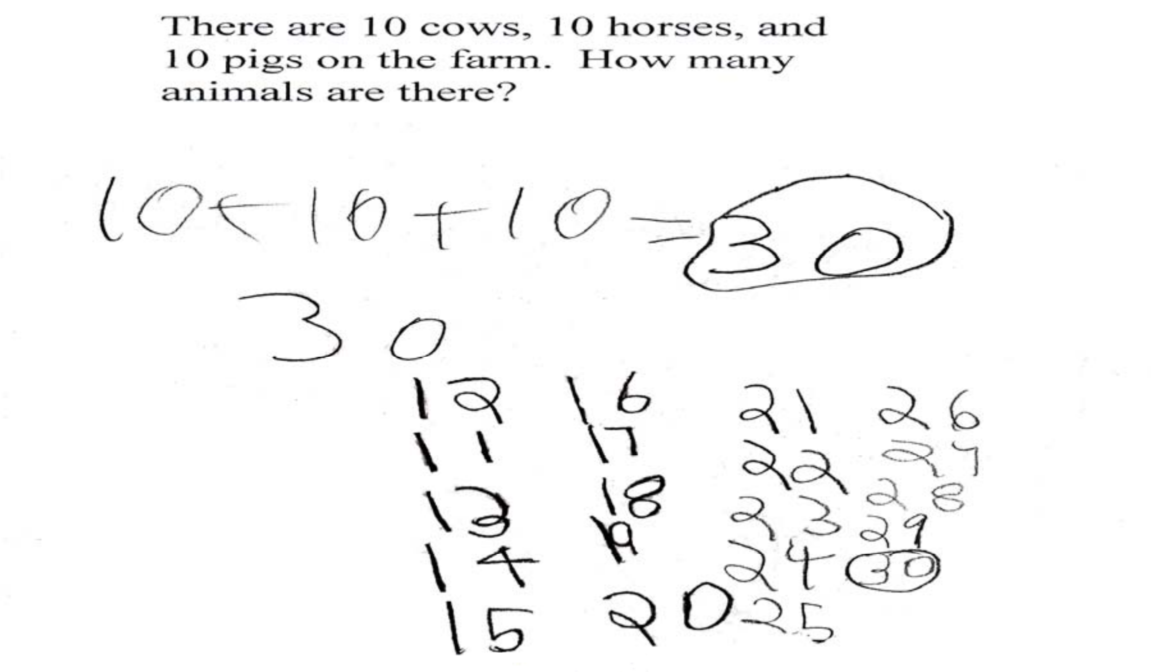

Figure 2. Counting on from 10.

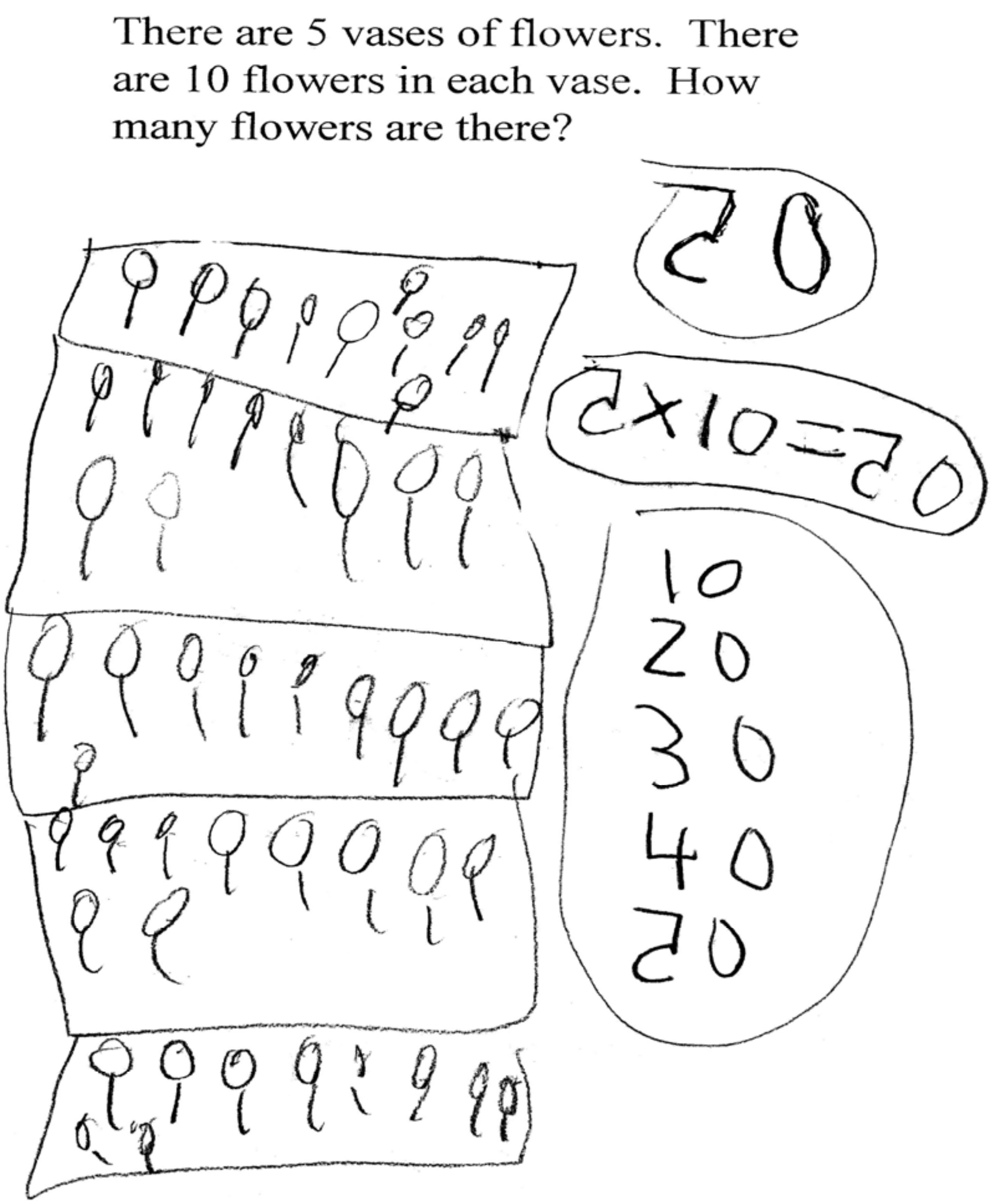

Figure 3. Representing all items, but counting by 10's.

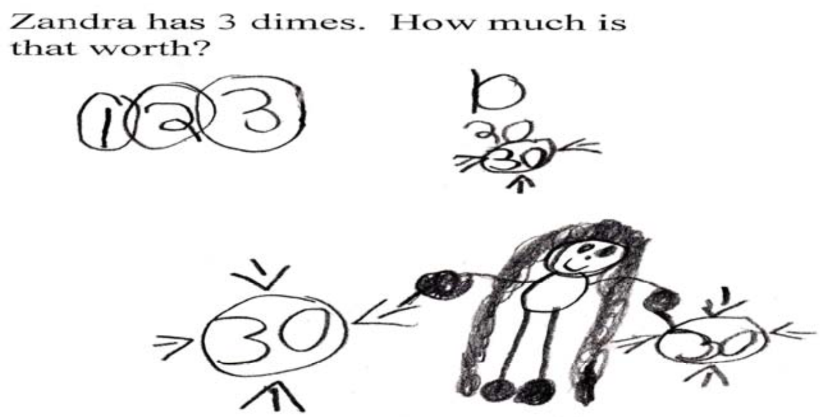

Figure 4. Counting by 10's.

What Next?

When the children became more proficient in working with multiplication and addition with 10's, I began to wonder what they would do with this problem: "There are 20 butterflies. Twenty more butterflies join them. How many butterflies are there?" Would they use their emerging knowledge of 10's to help them solve the problem? Most children counted only by 1's. They counted out 20 objects, then another 20 objects, and finally counted all objects to get 40. Only two children explicitly used their knowledge of 10, saying, "10 + 10 + 10 + 10 =40."

Mrs. J.: Where did the 10 + 10 + 10 + 10 come from? I don't see any 10's in the problem.

Stasha: 10 + 10 = 20, and 10 + 10 = 20.

Mrs. J.: Okay, so what did you do next?

Stasha: I said, "10, 20, 30, 40."

Stasha did not know that 20 + 20 = 40, but she did know that 20 was comprised of two 10's. Because she frequently solved problems by counting by 10's, decomposing 20 into two 10's made this problem easier for her. I found this solution interesting, and it prompted me to wonder whether a problem involving only one 10 might allow more children to recognize the 10's in a number.

To explore this question, I posed the following problem: "You have 10 cookies. Stasha gives you 11 more. How many cookies do you have now?" I wondered whether the children would use their knowledge of 10's to decompose 11 into 10 + 1 and simplify their problem solving by reconceptualizing the problem as 10 + (10 + 1). Although they were generally successful with this problem, none thought of the problem in this way! Most counted by 1's to make a set of 10 and a set of 11, and then counted all 21 by 1's.

Because none of the children decomposed 11, I realized that even those children who understood 10 + 10 = 20 did not think about 11 as 10 + 1. Was I surprised! I was again reminded of the complexity of making sense of our number system. All the children could count by 1's to 20 and by 10's to 100. However, they were still building their understanding of the underlying structure of the number system and the critical role that 10 plays. If the children were to understand numbers to 100, they needed to recognize that 11 is the same as 10 plus 1, 24 is made of two 10's and four 1's, and so on. I now had a new direction for my instruction.

Extending Children's Understanding

To extend the children's understanding of the role of 10 in our number system, I began to pose story problems requiring the addition of a single-digit number to 10 (e.g., "There are 10 butterflies. 6 more come. How many butterflies are there?"); or, the subtraction of a single-digit number to get 10 (e.g., "There are 19 giraffes eating. 9 walk away. How many giraffes are still eating?"). Most children counted only by 1's, making the first set and then adding or taking away the second set, depending on the problem context (see Figures 5 and 6). Gradually, however, the children's strategies for addition became more sophisticated and about half the children began to count on from 10. For example, for the butterfly problem (10 + 6), they recognized 10 as a group, and then counted on: "11, 12, 13, 14, 15, 16," to get the answer (see Figure 7).

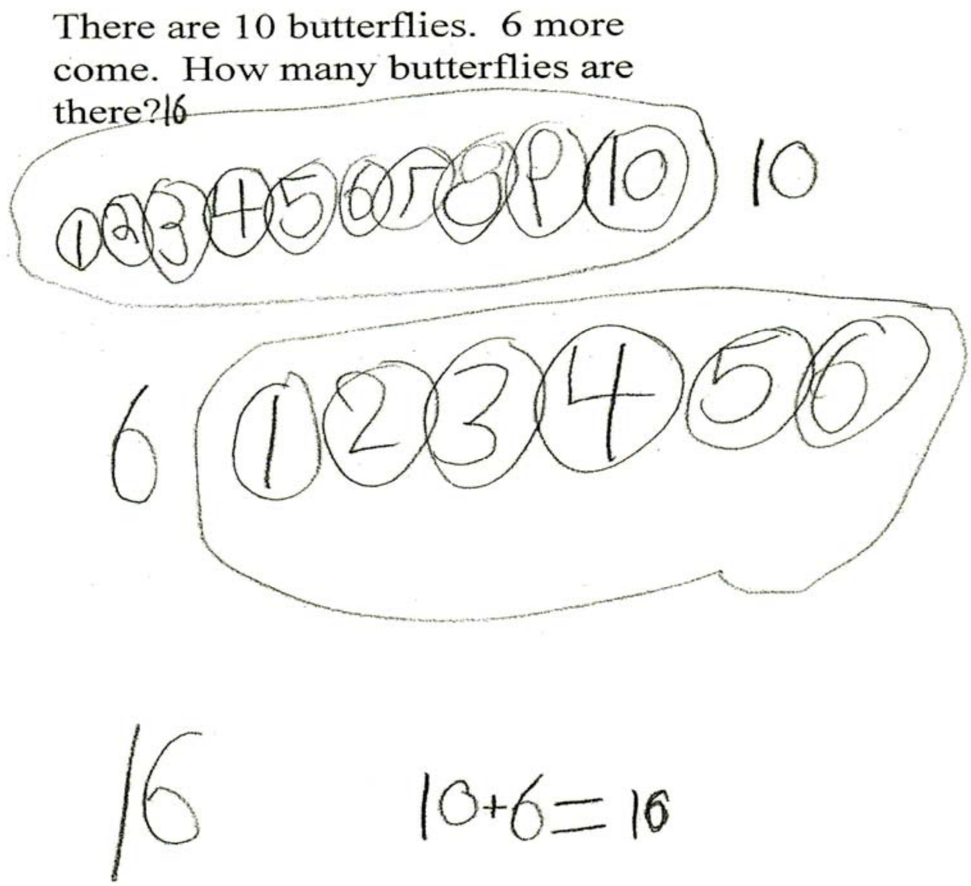

Figure 5. Modeling the action in the addition story problem and counting by 1’s.

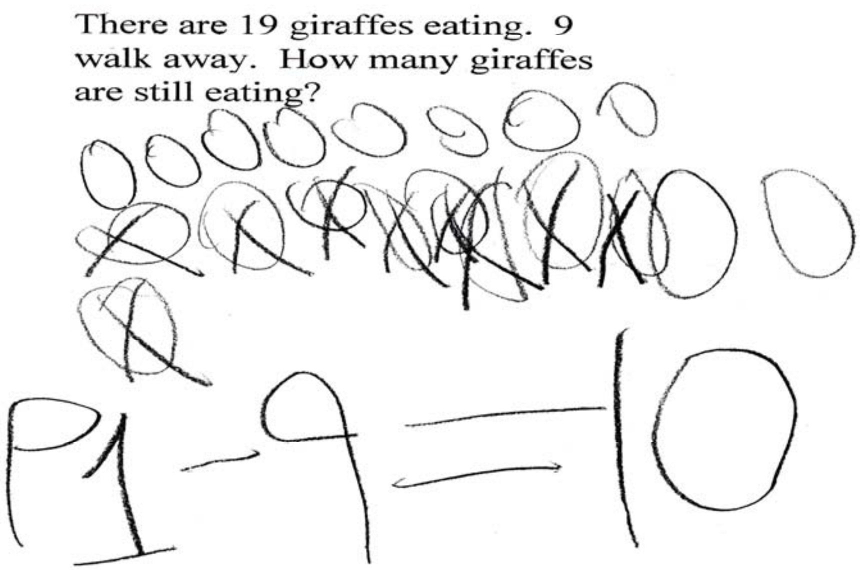

Figure 6. Modeling the action in the subtraction story problem and counting by 1's.

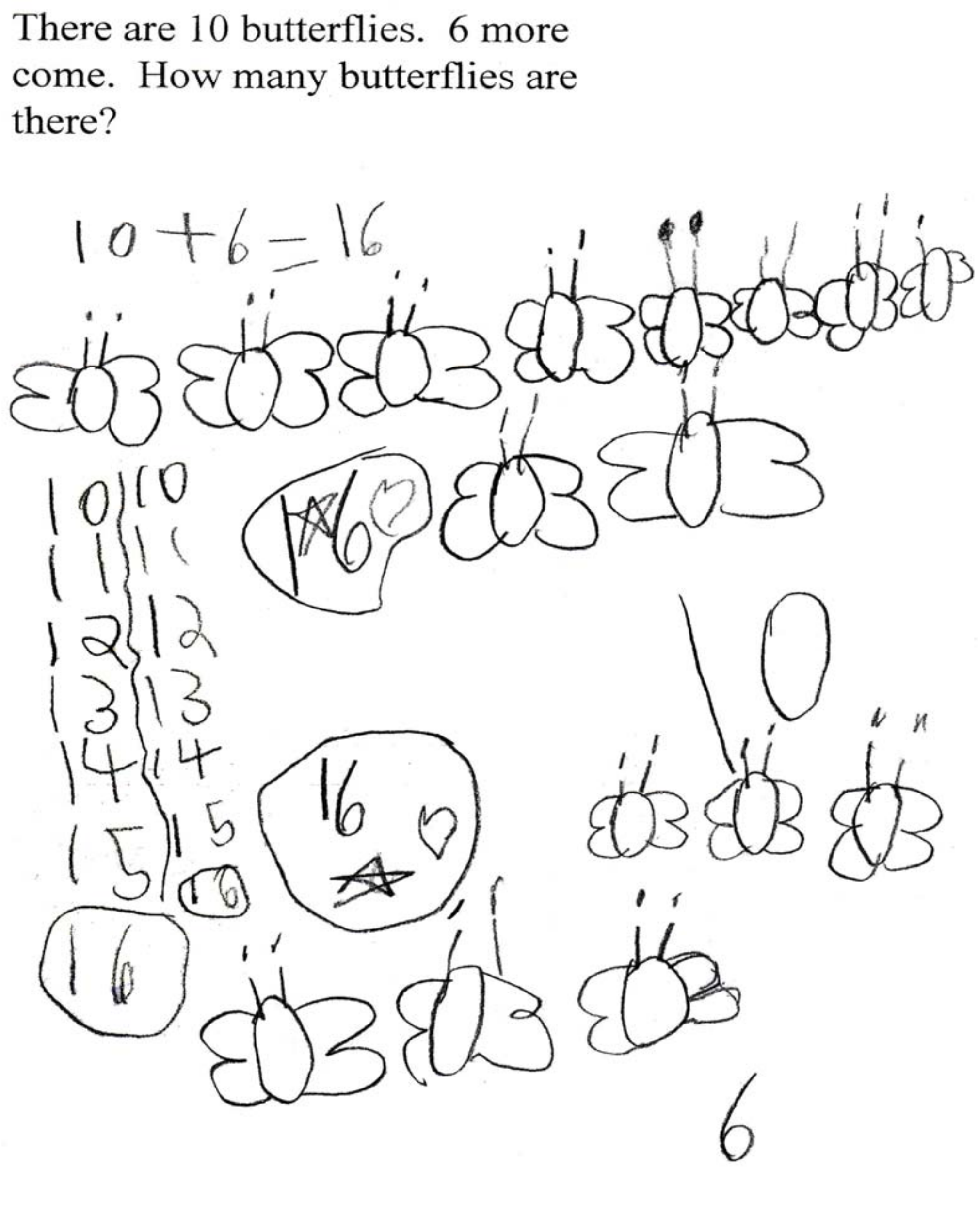

Figure 7. Modeling the action in the addition story problem, but counting on from 10.

These addition strategies reflected children's growth in sophistication of their problem-solving strategies and, in particular, in their abilities to group numbers. However, exactly what the children were learning was unclear. Were they focusing on decomposing 16 into a group of one 10 and six 1's, or were they focusing on solving addition problems by counting on from the larger number? The subtraction problems (subtracting a single-digit number to yield 10) helped me recognize that the latter explanation was more likely, and that the children needed more experience identifying 10 in teen numbers. To directly use knowledge of 10 to solve these subtraction problems, children would need to decompose a teen number into 10 and a single-digit number, but none did so. Instead, they represented all items and counted by 1's while they took away the required quantity.

At this point in my instruction, the school year was coming to an end. The children had grown substantially in nine weeks, but I had underestimated the complexity of learning about numbers to 100. On the one hand, the children's counting had improved. About 75% of the children could now count to 100 by 1's, even though we had done little rote counting and had focused our problem solving on numbers only to 60. Also, the children were beginning to show understanding when counting by 10's and using 10's during problem solving because (I believe) after making sense of the counting by 10's chant, they were able to recognize the underlying structure and extend their counting from 60 to 100. These counting abilities contrast with the children's counting when I arrived, at which time they could count (chant) by 10's, but could not count by 1's to numbers larger than 29! On the other hand, despite this growth, my class still had much to learn about our number system and, in particular, they needed more opportunities to decompose numbers into 10's and 1's. In the final sections, we reflect on possible future directions to extend these children's mathematical understanding.

Reflections and Future Directions

To support understanding of numbers to 100, Mrs. Jaslow engaged her children in nine weeks of problem solving and discussions focused on making sense of the number system. However, we recognize that this understanding, being quite complex, takes years to develop fully and that the children would need many more related experiences throughout elementary school. So what should come next for these children?

The use of story problems was the primary tool in the development of these children's understanding, and Mrs. Jaslow found two categories of story problems especially helpful: 1) grouping problems with 10 in each group; and, 2) problems designed to help children compose new numbers from a 10 and 1's (e.g., 10 + 6) and decompose numbers into 10 and 1's (e.g., 19 – 9 = 10). Upon reflection, we identified several ways to extend these problem categories to further foster children's understanding of numbers to 100, and each is described below (see Table 1).

Table 1
Story Problems to Help Children Understand Numbers to 100

	Grouping Problems	**Problems to Decompose Numbers Into 10’s and 1’s**
Problems Posed	• *Multiplication (10 in each group):* There are 5 vases of flowers. There are 10 flowers in each vase. How many flowers are there? • *Addition (around 10):* There are 10 cows, 10 horses, and 10 pigs on the farm. How many animals are there?	• *10 + a single-digit number:* There are 10 butterflies. 6 more come. How many butterflies are there? • *Subtracting from a teen to get 10:* There are 19 giraffes eating. 9 walk away. How many giraffes are still eating?
Potential Extensions	• *Division (Grouping by 10’s):* You have 50 stamps to put in your stamp book. Each page holds 10 stamps. How many pages will you need? • *Grouping by multiples of 10:* The teacher has 2 new boxes of markers, and each box has 30 markers. How many markers does the teacher have? The clown had 20 blue balloons, 20 red balloons, and 20 yellow balloons. How many balloons did the clown have? • *Mixing 10’s and 1’s: (beginning with a decade number)* Aisha has 3 bags of candy. Each bag has 10 pieces. She also has 4 loose pieces of candy. How much candy does Aisha have? On Monday, Alicia earned 10 citizenship points. On Tuesday, she earned 10 more points. On Wednesday, she earned 11 points. How many points has she earned? • *Mixing 10’s and 1’s: (beginning with a non-decade number)* The class counted 22 watermelon seeds. Then they counted seeds from 3 more watermelon pieces, and each had 10 seeds. How many seeds did they count in all? Michael has $23 in his piggy bank. He earned $10 on Saturday and $10 on Sunday. How much money does he have now?	• *Decade number (greater than 10) + a single-digit number:* Raphael had 40 toy cars. His uncle gave him 6 more toy cars for his birthday. How many toy cars does Raphael have now? • *Subtracting a single-digit number from a non-decade number (greater than 20) to get a decade number:* There are 34 butterflies. 4 fly away. How many butterflies are left?

Additional Grouping Problems

In addition to using multiplication and addition problems focused on grouping 10’s, teachers can pose division problems in which 10 items are grouped together (e.g., “You have 50 stamps to put in your stamp book. Each page holds 10 stamps. How many pages will you need?”) Children naturally solve this type of division problem by making groups of 10, and thus, discussing their strategies can help children make sense of counting and grouping by 10.

Using Multiples of 10

Another potential extension includes the use of multiples of 10 rather than 10 itself. For example, teachers might present a problem asking children to recognize the 10’s in numbers greater than the teens (e.g., “Raphael had 40 toy cars. His uncle gave him 6 more toy cars for his birthday. How many toy cars does Raphael have now?”). Even a child who knows that 16 is made of a 10 and a 6 may not know that 46 is made of four 10’s and a 6. Children need multiple opportunities to decompose numbers into the appropriate 10’s and 1’s.

Similarly, multiples of 10, rather than 10 itself, can be used in grouping problems (e.g., “The teacher has 2 new boxes of markers, and each box has 30 markers. How many markers does the teacher have?”). In this problem, children have opportunities to use the three 10’s in 30 to simplify problem solving.

Mixing 10’s and 1’s

Children who can count by 10’s and by 1’s independently may struggle when asked to do both in the same problem. Grouping problems can be extended to give children opportunities to consider groups of 10 and single items within the same problem (e.g., “Aisha has 3 bags of candy. Each bag has 10 pieces. She also has 4 loose pieces of candy. How much candy does Aisha have?”). Children who have trouble moving between 10’s and 1’s might correctly show three groups of 10 and four 1’s, but when determining the total, incorrectly count, “10, 20, 30, 40, 50, 60, 70,” by counting each individual item as 10. Grouping problems involving both 10’s and 1’s provide children opportunities to develop the necessary fluidity in moving between counting by 10’s and counting by 1’s.

A further extension is grouping problems children may solve by counting by 10’s from a non-decade number. Children first learn to increment/decrement by 10 from a decade number (i.e. 10, 20, 30,...), and to start counting by 10's from a non-decade number is more challenging and requires experience with such problems. For example, to provide children with an opportunity to count by 10's from 22, a teacher might pose this problem: "The class counted 22 watermelon seeds. Then they counted seeds from 3 more watermelon pieces, and each had 10 seeds. How many seeds did they count in all?"

Final Thoughts

We are not suggesting that the categories of story problems we describe are the only ones possible or desirable to use. In fact, children need opportunities to solve a wide variety of story problems that allow them to develop many mathematical concepts. We also recognize that some approaches to developing understanding of number do not depend on story problems, but we chose to highlight them, not only because they were powerful in helping these kindergarteners learn, but also because multiplication and division story problems, in particular, are often overlooked during instruction with young children. We encourage teachers to pose problems with strategically selected numbers even if their students are still struggling with counting. Children improved their counting skills and place-value understanding by working with larger numbers during problem solving, illustrating that consistent counting is not a prerequisite to engaging children in problem solving and other place-value activities.

A final caveat is in order. Although carefully designed story problems with relevant contexts and intentional number selections can be powerful instructional tools, the benefits do not reside solely in the design of the problems. Children must be allowed to solve problems in ways that make sense to them and be provided with opportunities to share their thinking. Teachers must consistently inquire into children's thinking and build on what they learn. Because there is no single best sequence of problems, teachers must pose a problem, listen to their children's thinking, consider their options, and then select an appropriate next problem. We encourage teachers to follow their curiosities about children's thinking and allow their instruction to constantly evolve on the basis of what they hear from their students. Children's mathematical thinking is often different from adults' mathematical thinking. At times, seemingly simple ideas may appear confusing to children and, at other times, young children will impress adults with the complexity of their own ideas.

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