Rogers Public Schools

5th Grade Instructional Strategies and Background Knowledge for CCSSM

The Common Core Instructional Strategy documents are a compilation of research, “unpacked” standards from many states, instructional strategies and examples for each standard at each grade level. The intent is to help teachers understand what each standard means in terms of what students should know and be able to do. It provides only a sample of instructional strategies and examples. The goal of every teacher should be to guide students in understanding and making sense of mathematics.

Resources used for these documents: Common Core State Standards; Elementary and Middle School Mathematics: Teaching Developmentally by John A Van De Walle; Department of Education “unpacking” of the standards from Arizona, Ohio and North Carolina.

Domain	**Operations and Algebraic Thinking**
Cluster	***Write and interpret numerical expressions.***
5th Grade Standards **5.OA.1** **5.OA.2**	**5.OA.1.** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. **5.OA.2.** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7).Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*

Instructional Strategies

Students should be given ample opportunities to explore numerical expressions with mixed operations. This is the foundation for evaluating numerical and algebraic expressions that will include whole-number exponents in Grade 6.

There are conventions (rules) determined by mathematicians that must be learned with no conceptual basis. For example, multiplication and division are always done before addition and subtraction.

Begin with expressions that have two operations without any grouping symbols (multiplication or division combined with addition or subtraction) before introducing expressions with multiple operations. Using the same digits, with the operations in a different order, have students evaluate the expressions and discuss why the value of the expression is different. For example, have students evaluate 5 × 3 + 6 and 5 + 3 × 6. Discuss the rules that must be followed. Have students insert parentheses around the multiplication or division part in an expression. A discussion should focus on the similarities and differences in the problems and the results. This leads to students being able to solve problem situations which require that they know the order in which operations should take place.

After students have evaluated expressions without grouping symbols, present problems with one grouping symbol, beginning with parentheses, then in combination with brackets and/or braces.

Have students write numerical expressions in words without calculating the value. This is the foundation for writing algebraic expressions. Then, have students write numerical expressions from phrases without calculating them.

5.OA.1 calls for students to evaluate expressions with parentheses ( ), brackets [ ] and braces { }. In upper levels of mathematics, evaluate means to substitute for a variable and simplify the expression. However at this level students are to only simplify the expressions because there are no variables.

Example:
Evaluate the expression 2{ 5[12 + 5(500 - 100) + 399]}
Students should have experiences working with the order of first evaluating terms in parentheses, then brackets, and then braces.
The first step would be to subtract 500 – 100 = 400.
Then multiply 400 by 5 = 2,000.
Inside the bracket, there is now [12 + 2,000 + 399]. That equals 2,411.
Next multiply by the 5 outside of the bracket. 2,411 x 5 = 12,055.
Next multiply by the 2 outside of the braces. 12,055 x 2= 24,110.

Mathematically, there cannot be brackets or braces in a problem that does not have parentheses. Likewise, there cannot be braces in a problem that does not have both parentheses and brackets.

This standard builds on the expectations of third grade where students are expected to start learning the conventional order. Students need experiences with multiple expressions that use grouping symbols throughout the year to develop understanding of when and how to use parentheses, brackets, and braces. First, students use these symbols with whole numbers. Then the symbols can be used as students add, subtract, multiply and divide decimals and fractions.

Examples:

- (26 + 18) ÷ 4 Answer: 11
- {[2 x (3+5)] – 9} + [5 x (23-18)] Answer: 32
- 12 – (0.4 x 2) Answer: 11.2
- (2 + 3) x (1.5 – 0.5) Answer: 5
- $6-\left(\frac{1}{2}+\frac{1}{3}\right)$ Answer: 5 1/6
- { 80 [2 x (3 ½ + 1 ½)] }+ 100 Answer: 108

To further develop students' understanding of grouping symbols and facility with operations, students place grouping symbols in equations to make the equations true or they compare expressions that are grouped differently.

Examples:

- 15 - 7 – 2 = 10 → 15 - (7 – 2) = 10
- 3 x 125 ÷ 25 + 7 = 22 → [3 x (125 ÷ 25)] + 7 = 22
- 24 ÷ 12 ÷ 6 ÷ 2 = 2 x 9 + 3 ÷ ½ → 24 ÷ [(12 ÷ 6) ÷ 2] = (2 x 9) + (3 ÷ ½)
- Compare 3 x 2 + 5 and 3 x (2 + 5)

Compare 15 – 6 + 7 and 15 – (6 + 7)

5.OA.2 refers to expressions. Expressions are a series of numbers and symbols (+, -, x, ÷) without an equals sign. Equations result when two expressions are set equal to each other (2 + 3 = 4 + 1).

Example:
4(5 + 3) is an expression.
When we compute 4(5 + 3) we are evaluating the expression. The expression equals 32.
4(5 + 3) = 32 is an equation.

5.OA.2 calls for students to verbally describe the relationship between expressions without actually calculating them. This standard calls for students to apply their reasoning of the four operations as well as place value while describing the relationship between numbers. The standard does not include the use of variables, only numbers and signs for operations.

Students use their understanding of operations and grouping symbols to write expressions and interpret the meaning of a numerical expression.
Examples:

- Students write an expression for calculations given in words such as "divide 144 by 12, and then subtract 7/8.” They write (144 ÷ 12) – 7/8.

Students recognize that 0.5 x (300 ÷ 15) is ½ of (300 ÷ 15) without calculating the quotient.

Common Misconceptions

Students may believe the order in which a problem with mixed operations is written is the order to solve the problem. Allow students to use calculators to determine the value of the expression, and then discuss the order the calculator used to evaluate the expression. Do this with four-function and scientific calculators.

Check Out : http://wps.ablongman.com/ab_vandewalle_math_6/54/13858/3547873.cw/-/t/index.html for more information on Operation and Algebraic Thinking

Progressions Documents for the Common Core Math
http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_nbt_2011_04_073.pdf

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Domain	**Operations and Algebraic Thinking**
Cluster	***Analyze patterns and relationships.***
5th Grade Standards **5.OA.3**	**5.OA.3.** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

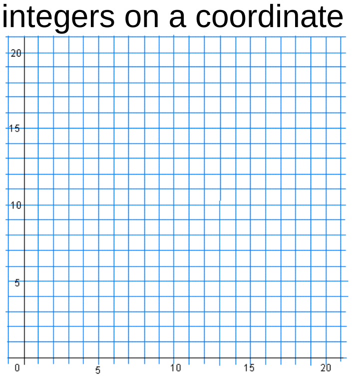
<u>Instructional Strategies</u>

Students should have experienced generating and analyzing numerical patterns using a given rule in Grade 4.

Given two rules with an apparent relationship, students should be able to identify the relationship between the resulting sequences of the terms in one sequence to the corresponding terms in the other sequence. For example, starting with 0, multiply by 4 and starting with 0, multiply by 8 and generate each sequence of numbers (0, 4, 8, 12, 16, …) and (0, 8, 16, 24, 32,…). Students should see that the terms in the second sequence are double the terms in the first sequence, or that the terms in the first sequence are half the terms in the second sequence.

Have students form ordered pairs and graph them on a coordinate plane. Patterns can be also discerned in graphs.

Graphing ordered pairs on a coordinate plane is introduced to students in the Geometry domain where students solve real-world and mathematical problems. For the purpose of this cluster, only use the first quadrant of the coordinate plane, which contains positive numbers only. Provide coordinate grids for the students, but also have them make coordinate grids. In Grade 6, students will position pairs of integers on a coordinate plane.



The graph of both sequences of numbers is a visual representation that will show the relationship between the two sequences of numbers.

Encourage students to represent the sequences in T-charts so that they can see a connection between the graph and the sequences.

0	0		0	0
1	4		1	8
2	8		2	16
3	12		3	24
4	16		4	32

5.OA.3 extends the work from Fourth Grade, where students generate numerical patterns when they are given one rule. In Fifth Grade, students are given two rules and generate two numerical patterns. The graphs that are created should be line graphs to represent the pattern. This is a linear function

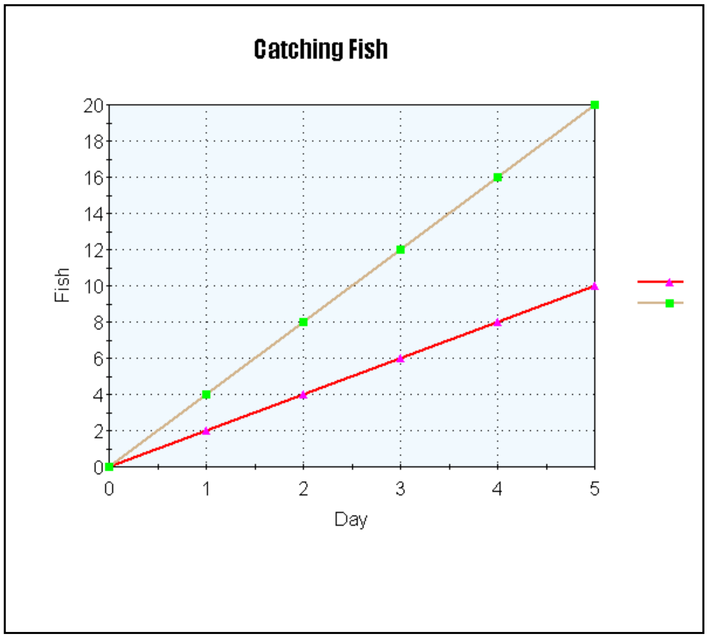
which is why we get the straight lines. The Days are the independent variable, Fish are the dependent variables, and the constant rate is what the rule identifies in the table.
Examples below:

Students make a chart (table0 to represent the number of fish that Sam and Terri catch.

Days	**Sam's Total Number of Fish**	**Terri's Total Number of Fish**
0	0	0
1	2	4
2	4	8
3	6	12
4	8	16
5	10	20

Student
Describe the pattern:
Since Terri catches 4 fish each day, and Sam catches 2 fish, the amount of Terri's fish is always greater. Terri's fish is also always twice as much as Sam's fish. Today, both Sam and Terri have no fish. They both go fishing each day. Sam catches 2 fish each day. Terri catches 4 fish each day. How many fish do they have after each of the five days? Make a graph of the number of fish.

Student
Plot the points on a coordinate plane and make a line graph, and then interpret the graph.
My graph shows that Terri always has more fish than Sam. Terri's fish increases at a higher rate since she catches 4 fish every day. Sam only catches 2 fish every day, so his number of fish increases at a smaller rate than Terri.
Important to note as well that the lines become increasingly further apart. Identify apparent relationships between corresponding terms. Additional relationships: The two lines will never intersect; there will not be a day in which boys have the same total of fish, explain the relationship between the number of days that



Use the rule "add 3" to write a sequence of numbers. Starting with a 0, students write 0, 3, 6, 9, 12, . .

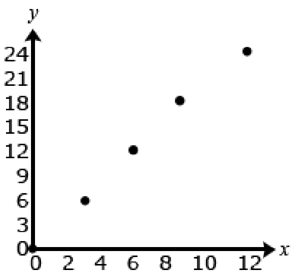
Use the rule "add 6" to write a sequence of numbers. Starting with 0, students write 0, 6, 12, 18, 24, . .

After comparing these two sequences, the students notice that each term in the second sequence is twice the corresponding terms of the first sequence. One way they justify this is by describing the patterns of the terms. Their justification may include some mathematical notation (See example below). A student may explain that both sequences start with zero and to generate each term of the second sequence he/she added 6, which is twice as much as was added to produce the terms in the first sequence. Students may also use the distributive property to describe the relationship between the two numerical patterns by reasoning that 6 + 6 + 6 = 2 (3 + 3 + 3).

0, $^{+3}$ 3, $^{+3}$ 6, $^{+3}$ 9, $^{+3}$12, . . .

0, $^{+6}$ 6, $^{+6}$ 12, $^{+6}$18, $^{+6}$ 24, . . .

Once students can describe that the second sequence of numbers is twice the corresponding terms of the first sequence, the terms can be written in ordered pairs and then graphed on a coordinate grid. They should recognize that each point on the graph represents two quantities in which the second quantity is twice the first quantity.



(0, 0)
(3, 6)
(6, 12)
(9, 18)
(12, 24)

Common Misconceptions

Students reverse the points when plotting them on a coordinate plane. They count up first on the *y*-axis and then count over on the *x*-axis. The location of every point in the plane has a specific place. Have students plot points where the numbers are reversed such as (4, 5) and (5, 4). Begin with students providing a verbal description of how to plot each point. Then, have them follow the verbal description and plot each point.

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Domain	**Number and Operations in Base Ten**
Cluster	***Understand the place value system.***
5th Grade Standards **5.NBT.1** **5.NBT.2**. **5.NBT.3.a** **5.NBT.3.b** **5.NBT.4**	***5.NBT.*1.** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. **5.NBT.2**. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **5.NBT.3.** Read, write, and compare decimals to thousandths. **a.** Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, *e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).* **b.** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **5.NBT.4.** Use place value understanding to round decimals to any place.

Instructional Strategies

In Grade 5, the concept of place value is extended to include decimal values to thousandths. The strategies for Grades 3 and 4 should be drawn upon and extended for whole numbers and decimal numbers. For example, students need to continue to represent, write and state the value of numbers including decimal numbers. For students who are not able to read, write and represent multi-digit numbers, working with decimals will be challenging.

Money is a good medium to compare decimals. Present contextual situations that require the comparison of the cost of two items to determine the lower or higher priced item. Students should also be able to identify how many pennies, dimes, dollars and ten dollars, etc., are in a given value. Help students make connections between the number of each type of coin and the value of each coin, and the expanded form of the number. Build on the understanding that it always takes ten of the number to the right to make the number to the left.

Number cards, number cubes, spinners and other manipulatives can be used to generate decimal numbers. For example, have students roll three number cubes, then create the largest and small number to the thousandths place. Ask students to represent the number with numerals and words.

5.NBT.1 calls for students to reason about the magnitude of numbers. Students should work with the idea that the tens place is ten times as much as the ones place, and the ones place is 1/10th the size of the tens place.

Example:

The 2 in the number 542 is different from the value of the 2 in 324. The 2 in 542 represents 2 ones or 2, while the 2 in 324 represents 2 tens or 20. Since the 2 in 324 is one place to the left of the 2 in 542 the value of the 2 is 10 times greater. Meanwhile, the 4 in 542 represents 4 tens or 40 and the 4 in 324 represents 4 ones or 4. Since the 4 in 324 is one place to the right of the 4 in 542 the value of the 4 in the number 324 is 1/10th of its value in the number 542.

Note the pattern in our base ten number system; all places to the right continue to be divided by ten and that places to the left of a digit are multiplied by ten.

In fourth grade, students examined the relationships of the digits in numbers for whole numbers only.

This standard extends this understanding to the relationship of decimal fractions. Students use base ten blocks, pictures of base ten blocks, and interactive images of base ten blocks to manipulate and investigate the place value relationships. They use their understanding of unit fractions to compare decimal places and fractional language to describe those comparisons.

Before considering the relationship of decimal fractions, students express their understanding that in multi-digit whole numbers, a digit in one place represents 10 times what it represents in the place to its right and 1/10 of what it represents in the place to its left.

A student thinks, “I know that in the number 5555, the 5 in the tens place (5555) represents 50 and the 5 in the hundreds place (5555) represents 500. So a 5 in the hundreds place is ten times as much as a 5 in the tens place or a 5 in the tens place is 1/10 of the value of a 5 in the hundreds place.

To extend this understanding of place value to their work with decimals, students use a model of one unit; they cut it into 10 equal pieces, shade in, or describe 1/10 of that model using fractional language (“This is 1 out of 10 equal parts. So it is 1/10”. I can write this using 1/10 or 0.1”). They repeat the process by finding 1/10 of a 1/10 (e.g., dividing 1/10 into 10 equal parts to arrive at 1/100 or 0.01) and can explain their reasoning, “0.01 is 1/10 of 1/10 thus is 1/100 of the whole unit.”

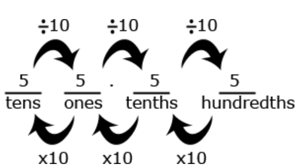
In the number 55.55, each digit is 5, but the value of the digits is different because of the placement.

5	**5**	**.**	**5**	**5**

The 5 that the arrow points to is 1/10 of the 5 to the left and 10 times the 5 to the right. The 5 in the ones place is 1/10 of 50 and 10 times five tenths.

5	**5**	**.**	**5**	**5**

The 5 that the arrow points to is 1/10 of the 5 to the left and 10 times the 5 to the right. The 5 in the tenths place is 10 times five hundredths.



5.NBT.2 includes multiplying by multiples of 10 and powers of 10, including 10^2 which is 10 x 10=100, and 103 which is 10 x 10 x 10=1,000. Students should have experiences working with connecting the pattern of the number of zeros in the product when you multiply by powers of 10.
Example:
2.5×10^3 = 2.5 x (10 x 10 x 10) = 2.5 x 1,000 = 2,500 Students should reason that the exponent above the 10 indicates how many places the decimal point is moving (not just that the decimal point is moving but that you are multiplying or making the number 10 times greater three times) when you multiply by a power of 10. Since we are multiplying by a power of 10 the decimal point moves to the right.
$350 \div 10^3$ = 350 ÷ 1,000 = 0.350 = 0.35 350/10 = 35, 35 /10 = 3.5 3.5 /10 =.0.35, or 350 x 1/10, 35 x 1/10, 3.5 x 1/10 this will relate well to subsequent work with operating with fractions. This example shows that when we divide by powers of 10, the exponent above the 10 indicates how many places the decimal point is moving (how many times we are dividing by 10 , the number becomes ten times smaller). Since we are dividing by powers of 10, the decimal point moves to the left.

Students need to be provided with opportunities to explore this concept and come to this understanding; this should not just be taught procedurally.

Examples:

Students might write:

- 36 x 10 = 36 x 10^1 = 360
- 36 x 10 x 10 = 36 x 10^2 = 3600
- 36 x 10 x 10 x 10 = 36 x 10^3 = 36,000
- 36 x 10 x 10 x 10 x 10 = 36 x 10^4 = 360,000

Students might think and/or say:

- I noticed that every time, I multiplied by 10 I added a zero to the end of the number. That makes sense because each digit's value became 10 times larger. To make a digit 10 times larger, I have to move it one place value to the left.
- When I multiplied 36 by 10, the 30 became 300. The 6 became 60 or the 36 became 360. So I had to add a zero at the end to have the 3 represent 3 one-hundreds (instead of 3 tens) and the 6 represents 6 tens (instead of 6 ones).

Students should be able to use the same type of reasoning as above to explain why the following multiplication and division problem by powers of 10 make sense.

- $523 \times 10^3 = 523{,}000$ The place value of 523 is increased by 3 places.
- $5.223 \times 10^2 = 522.3$ The place value of 5.223 is increased by 2 places.
- $52.3 \div 10^1 = 5.23$ The place value of 52.3 is decreased by one place.

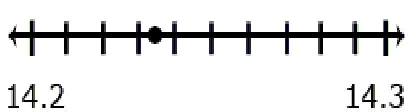
5.NBT.3a references expanded form of decimals with fractions included. Students should build on their work from Fourth Grade, where they worked with both decimals and fractions interchangeably. Expanded form is included to build upon work in 5.NBT.2 and deepen students' understanding of place value.

5.NBT.3b comparing decimals builds on work from fourth grade.

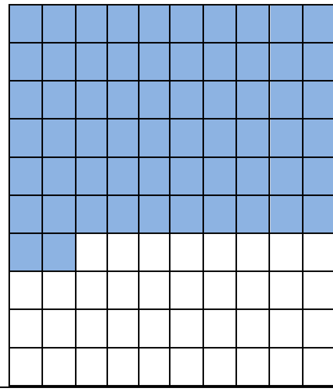
5.NBT.4 refers to rounding. Students should go beyond simply applying an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line to support their work with rounding.
When rounding a decimal to a given place, students may identify the two possible answers, and use their understanding of place value to compare the given number to the possible answers.
Example:

Round 14.235 to the nearest tenth.

- Students recognize that the possible answer must be in tenths thus, it is either 14.2 or 14.3. They then identify that 14.235 is closer to 14.2 (14.20) than to 14.3 (14.30).



5.NBT.4 references rounding. Students should use benchmark numbers to support this work. Benchmarks are convenient numbers for comparing and rounding numbers. 0., 0.5, 1, 1.5 are examples of benchmark numbers. Which benchmark number is the best estimate of the shaded amount in the model below? Explain your thinking.



Common Misconceptions

A common misconception that students have when trying to extend their understanding of whole number place value to decimal place value is that as you move to the left of the decimal point, the number increases in value. Reinforcing the concept of powers of ten is essential for addressing this issue.

A second misconception that is directly related to comparing whole numbers is the idea that the longer the number the greater the number. With whole numbers, a 5-digit number is always greater that a 1-, 2-, 3-, or 4-digit number. However, with decimals a number with one decimal place may be greater than a number with two or three decimal places. For example, 0.5 is greater than 0.12, 0.009 or 0.499. One method for comparing decimals it to make all numbers have the same number of digits to the right of the decimal point by adding zeros to the number, such as 0.500, 0.120, 0.009 and 0.499. A second method is to use a place-value chart to place the numerals for comparison.

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Domain	**Number and Operations – Base Ten**
Cluster	***Perform operations with multi-digit whole numbers and with decimals to hundredths.***
5th Grade Standards **5.NBT.5** **5.NBT.6** **5.NBT.7**	**5.NBT.5.** Fluently multiply multi-digit whole numbers using the standard algorithm. **5.NBT.6**. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **5.NBT.7**. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Instructional Strategies

Because students have used various models and strategies to solve problems involving multiplication with whole numbers, they should be able to transition to using standard algorithms effectively. With guidance from the teacher, they should understand the connection between the standard algorithm and their strategies.

Connections between the algorithm for multiplying multi-digit whole numbers and strategies such as partial products or lattice multiplication are necessary for students' understanding.
You can multiply by listing all the partial products. For example:

```
  234
 × 8
   32  Multiply the ones (8 × 4 ones = 32 ones)
  240  Multiply the tens (8 × 3 tens = 24 tens or 240
 1600  Multiply the hundreds (8 × 2 hundreds = 16 hundreds or 1600)
 1872 Add the partial products
```

The multiplication can also be done without listing the partial products by multiplying the value of each digit from one factor by the value of each digit from the other factor. Understanding of place value is vital in using the standard algorithm.

In using the standard algorithm for multiplication, when multiplying the ones, 32 ones is 3 tens and 2 ones. The 2 is written in the ones place. When multiplying the tens, the 24 tens is 2 hundreds and 4 tens. But, the 3 tens from the 32 ones need to be added to these 4 tens, for 7 tens. Multiplying the hundreds, the 16 hundreds is 1 thousand and 6 hundreds. But, the 2 hundreds from the 24 tens need to be added to these 6 hundreds, for 8 hundreds.

```
  234
 × 8
 1872
```

As students developed efficient strategies to do whole number operations, they should also develop efficient strategies with decimal operations.

Students should learn to estimate decimal computations before they compute with pencil and paper. The focus on estimation should be on the meaning of the numbers and the operations, not on how many decimal places are involved. For example, to estimate the product of 32.84 × 4.6, the estimate would be more than 120, closer to 150. Students should consider that 32.84 is closer to 30 and 4.6 is closer to 5. The product of 30 and 5 is 150. Therefore, the product of 32.84 × 4.6 should be close to 150.

Have students use estimation to find the product by using exactly the same digits in one of the factors with the decimal point in a different position each time. For example, have students estimate the product of 275 × 3.8; 27.5 × 3.8 and 2.75 × 3.8, and discuss why the estimates should or should not be the same.

5.NBT.5 refers to fluency which means accuracy (correct answer), efficiency (a reasonable amount of steps), and flexibility (using strategies such as the distributive property or breaking numbers apart also using strategies according to the numbers in the problem, 26 x 4 may lend itself to (25 x 4) + 4 where as another problem might lend itself to making an equivalent problem 32 x 4 = 64 x 8)). This standard builds upon students' work with multiplying numbers in Third and Fourth Grade. In Fourth Grade, students developed understanding of multiplication through using various strategies. While the standard algorithm is mentioned, alternative strategies are also appropriate to help students develop conceptual understanding. The size of the numbers should NOT exceed a three-digit factor by a two-digit factor.
Examples of alternative strategies:
There are 225 dozen cookies in the bakery. How many cookies are there?

Student 1
225 x 12
I broke 12 up into 10 and 2.
225 x 10 = 450
2,250 +450 = 2,700

Student 2
225 x 12
I broke up 225 into 200 and 25.
200 x 12 + 2,400
I broke 25 up into 5 x 5, so I had 5 x 5 x 12 or 5 x 12 x 5.
5 x 12 + 60. 60 x 5 + 30
2,400 + 300 = 2,700

Student 3
I doubled 225 and cut 12 in half to get 450 x 6. I then doubled 450 again and cut 6 in half to get 900 x 3. 900 x 3 + 2,700

Draw a array model for 225 x 12…. 200 x 10, 200 x 2, 20 x 10, 20 x 2, 5 x 10, 5 x 2
225 x 12

200	20	5	
2,000	200	50	10
400	40	10	2

2,000
400
200
40
50
+ 10
2,700

In prior grades, students used various strategies to multiply. Students can continue to use these different strategies as long as they are efficient, but must also understand and be able to use the standard algorithm. In applying the standard algorithm, students recognize the importance of place value.

Example:
123 x 34. When students apply the standard algorithm, they, decompose 34 into 30 + 4. Then they multiply 123 by 4, the value of the number in the ones place, and then multiply 123 by 30, the value of the 3 in the tens place, and add the two products.

5.NBT.6 references various strategies for division. Division problems can include remainders. Even though this standard leads more towards computation, the connection to story contexts is critical. Make sure students are exposed to problems where the divisor is the number of groups and where the divisor is the size of the groups.
Properties – **rules about how numbers work**
There are 1,716 students participating in Field Day. They are put into teams of 16 for the competition. How many teams get created? If you have left over students, what do you do with them?

Student 1
1,716 divided by 16
There are 100 16's in 1,716.
1,716 – 1,600 = 116
I know there are at least 6 16's.
116-96=4
There were 107 teams with 4 students left over.
If we put the extra students on different team, 4 teams will have 17 students.

Student 2
1,716 divided by 16.
There are 100 16's in 1,716.
Ten groups of 16 is 160.
That's too big.
Half of that is 80, which is 5 groups.
I know that 2 groups of 16's is 32.
I have 4 students left over.

Student 3
1,716 / 16 =
I want to get to 1,716
I know that 100 16/s equals 1,600
I know that 5 16/2 equals 80
1,600 + 80 = 1,680
Two more groups of 16/s equals 32, which gets us to 1,716
So we had 100 + 6 + 1 =107 teams
Those other 4 student can just hang out

Student 4
How many 16's are in 1,716?
We have an area of 1,716. I know that one side of my array is 16 units long. I used 16 as the height. I am trying to answer the question what is the width of my rectangle if the area is 1,716 and the height is 16.
100+ 7 = 107 R4

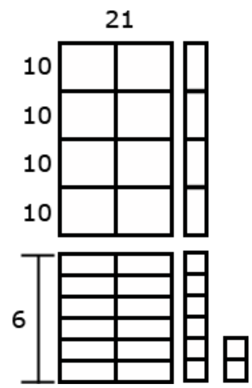
In fourth grade, students' experiences with division were limited to dividing by one-digit divisors. This standard extends students' prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a "familiar" number, a student might decompose the dividend using place value.

Example:
- Using expanded notation ~ 2682 ÷ 25 = (2000 + 600 + 80 + 2) ÷ 25
- Using his or her understanding of the relationship between 100 and 25, a student might think ~
  - I know that 100 divided by 25 is 4 so 200 divided by 25 is 8 and 2000 divided by 25 is 80.
  - 600 divided by 25 has to be 24.
  - Since 3 x 25 is 75, I know that 80 divided by 25 is 3 with a reminder of 5. (Note that a student might divide into 82 and not 80)
  - I can't divide 2 by 25 so 2 plus the 5 leaves a remainder of 7.
  - 80 + 24 + 3 = 107. So, the answer is 107 with a remainder of 7.

Using an equation that relates division to multiplication, 25 x *n* = 2682, a student might estimate the answer to be slightly larger than 100 because s/he recognizes that 25 x 100 = 2500.

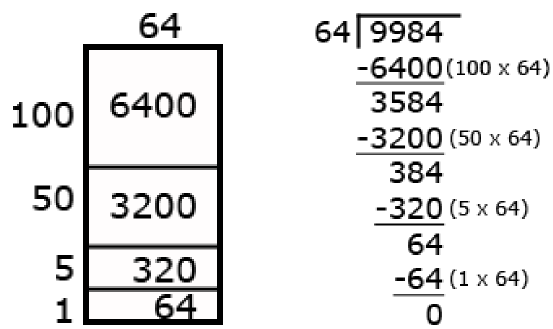
Example: 968 ÷ 21

- Using base ten models, a student can represent 962 and use the models to make an array with one dimension of 21. The student continues to make the array until no more groups of 21 can be made. Remainders are not part of the array.



Example: 9984 ÷ 64

- An area model for division is shown below. As the student uses the area model, s/he keeps track of how much of the 9984 is left to divide.



5.NBT.7 builds on the work from Fourth Grade where students are introduced to decimals and compare them. In Fifth Grade, students begin adding, subtracting, multiplying and dividing decimals. This work should focus on concrete models and pictorial representations, rather than relying solely on the algorithm. The use of symbolic notations involves having students record the answers to computations (2.25 x 3= 6.75), but this work should not be done without models or pictures. This standard includes students' reasoning and explanations of how they use models, pictures, and strategies.

Example:

A recipe for a cake requires 1.25 cups of milk, 0.40 cups of oil, and 0.75 cups of water. How much liquid is in the mixing bowl?

This standard requires students to extend the models and strategies they developed for whole numbers in grades 1-4 to decimal values. Before students are asked to give exact answers, they should estimate answers based on their understanding of operations and the value of the numbers.

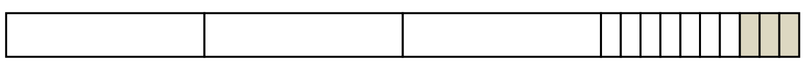
Examples:

- 3.6 + 1.7
  - A student might estimate the sum to be larger than 5 because 3.6 is more than 3 ½ and 1.7 is more than 1 ½.
- 5.4 – 0.8
  - A student might estimate the answer to be a little more than 4.4 because a number less than 1 is being subtracted.
- 6 x 2.4
  - A student might estimate an answer between 12 and 18 since 6 x 2 is 12 and 6 x 3 is 18. Another student might give an estimate of a little less than 15 because s/he figures the answer to be very close, but smaller than 6 x 2 ½ and think of 2 ½ groups of 6 as 12 (2 groups of 6) + 3 (½ of a group of 6).

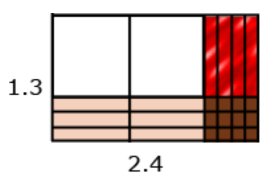
Students should be able to express that when they add decimals they add tenths to tenths and hundredths to hundredths. So, when they are adding in a vertical format (numbers beneath each other), it is important that they write numbers with the same place value beneath each other. This understanding can be reinforced by connecting addition of decimals to their understanding of addition of fractions. Adding fractions with denominators of 10 and 100 is a standard in fourth grade.

Example: 4 - 0.3

- 3 tenths subtracted from 4 wholes. The wholes must be divided into tenths.



The answer is 3 and 7/10 or 3.7.
Example: An area model can be useful for illustrating products.



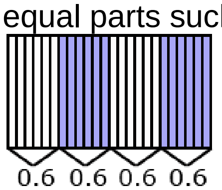

```
  2.4
x 1.3
  .12
  .60
  .40
+ 2.00
 3.12
```

Students should be able to describe the partial products displayed by the area model. For example,

"3/10 times 4/10 is 12/100.
3/10 times 2 is 6/10 or 60/100.
1 group of 4/10 is 4/10 or 40/100.
1 group of 2 is 2."

Example of division: finding the number in each group or share

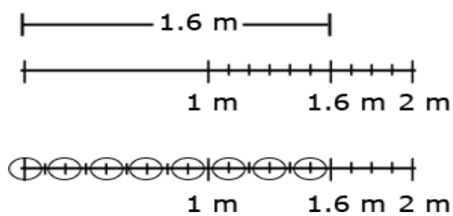
- Students should be encouraged to apply a fair sharing model separating decimal values into equal parts such as



Example of division: find the number of groups

- Joe has 1.6 meters of rope. He has to cut pieces of rope that are 0.2 meters long. How many can he cut?

- To divide to find the number of groups, a student might
  - draw a segment to represent 1.6 meters. In doing so, s/he would count in tenths to identify the 6 tenths, and be able identify the number of 2 tenths within the 6 tenths. The student can then extend the idea of counting by tenths to divide the one meter into tenths and determine that there are 5 more groups of 2 tenths.



  - count groups of 2 tenths without the use of models or diagrams. Knowing that 1 can be thought of as 10/10, a student might think of 1.6 as 16 tenths. Counting 2 tenths, 4 tenths, 6 tenths, . . .16 tenths, a student can count 8 groups of 2 tenths.
  - Use their understanding of multiplication and think, "8 groups of 2 is 16, so 8 groups of 2/10 is 16/10 or 1 6/10."

Technology Connections: Create models using Interactive Whiteboard software (such as SMART Notebook)

Common Misconceptions

Students might compute the sum or difference of decimals by lining up the right-hand digits as they would whole number. For example, in computing the sum of 15.34 + 12.9, students will write the problem in this manner:

15.34
+ 12.9
16.63

To help students add and subtract decimals correctly, have them first estimate the sum or difference. Providing students with a decimal-place value chart will enable them to place the digits in the proper place.

Check Out : http://wps.ablongman.com/ab_vandewalle_math_6/54/13858/3547873.cw/-/t/index.html for more information on Number and Operations in Base Ten

Progressions Documents for the Common Core Math
http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_nbt_2011_04_073.pdf

The Common Core Instructional Strategy documents are a compilation of research, "unpacked" standards from many states, instructional strategies and examples for each standard at each grade level. The intent is to help teachers understand what each standard means in terms of what students should know and be able to do. It provides only a sample of instructional strategies and examples. The goal of every teacher should be to guide students in understanding and making sense of mathematics.

Resources used for these documents: Common Core State Standards, *Elementary and Middle School Mathematics: Teaching Developmentally* by John A Van De Walle, Department of Education "unpacking" of the standards from Arizona, Ohio and North Carolina

Domain	**Number and Operations - Fractions**
Cluster	***Use equivalent fractions as a strategy to add and subtract fractions.***
5th Grade Standards **5.NF.1** **5.NF.2.**	**5.NF.1.** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* **5.NF.2.** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Instructional Strategies

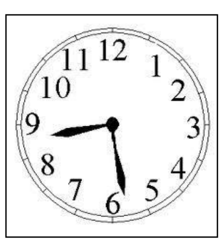
To add or subtract fractions with unlike denominators, students use their understanding of equivalent fractions to create fractions with the same denominators. Start with problems that require the changing of one of the fractions and progress to changing both fractions. Allow students to add and subtract fractions using different strategies such as number lines, area models, fraction bars or strips. Have students share their strategies and discuss commonalities in them.

Students need to develop the understanding that when adding or subtracting fractions, the fractions must refer to the same whole. Any models used must refer to the same whole. Students may find that a circular model might not be the best model when adding or subtracting fractions.

As with solving word problems with whole number operations, regularly present word problems involving addition or subtraction of fractions. The concept of adding or subtracting fractions with unlike denominators will develop through solving problems. Mental computations and estimation strategies should be used to determine the reasonableness of answers. Students need to prove or disprove whether an answer provided for a problem is reasonable.

Estimation is about getting useful answers, it is not about getting the right answer. It is important for students to learn which strategy to use for estimation. Students need to think about what might be a close answer.

5.NF.1 builds on the work in Fourth Grade where students add fractions with like denominators. In Fifth Grade, the example provided in the standard has students find a common denominator by finding the product of both denominators. For 1/3 + 1/6, a common denominator is 18, which is the product of 3 and 6. This process should be introduced using visual fraction models (area models, number lines, etc.) to build understanding before moving into the standard algorithm.
Example:
Present students with the problem 1/3 + 1/6. Encourage students to use the clock face as a model for solving the problem. Have students share their approaches with the class and demonstrate their thinking using the clock model.



Students should apply their understanding of equivalent fractions developed in fourth grade and their ability to rewrite fractions in an equivalent form to find common denominators. They should know that multiplying the denominators will always give a common denominator but may not result in the smallest denominator.

Examples:

- $\frac{2}{5}+\frac{7}{8}=\frac{16}{40}+\frac{35}{40}=\frac{51}{40}$

$3\frac{1}{4}-\frac{1}{6}=3\frac{3}{12}-\frac{2}{12}=3\frac{1}{12}$

5.NF.2 refers to number sense, which means students' understanding of fractions as numbers that lie between whole numbers on a number line. Number sense in fractions also includes moving between decimals and fractions to find equivalents, also being able to use reasoning such as 7/8 is greater than ¾ because 7/8 is missing only 1/8 and ¾ is missing ¼ so 7/8 is closer to a whole Also, students should use benchmark fractions to estimate and examine the reasonableness of their answers.
Example:
5/8 is greater than 6/10 because 5/8 is 1/8 larger than ½(4/8) and 6/10 is only 1/10 larger than ½ (5/10)

Example:
Your teacher gave you 1/7 of the bag of candy. She also gave your friend 1/3 of the bag of candy. If you and your friend combined your candy, what fraction of the bag would you have? Estimate your answer and then calculate. How reasonable was your estimate?

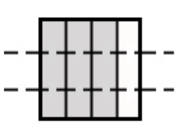
Student 1
1/7 is really close to 0. 1/3 is larger than 1/7, but still less than ½. If we put them together we might get close to ½.
1/7 + 1/3 = 3/21 + 7/21 = 10/21 the fraction does not simplify. I know that 10 is half of 20, so 10/21 is a little less that ½.

Student 2
1/7 is close to 1/6 but less than 1/6 and 1/3 is equivalent to 2/6, so I have a little less than 3/6 or 1/2

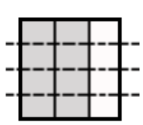
Examples:
Jerry was making two different types of cookies. One recipe needed ¾ cup of sugar and the other needed $\frac{2}{3}$ cup of sugar. How much sugar did he need to make both recipes?

- Mental estimation:
  - A student may say that Jerry needs more than 1 cup of sugar but less than 2 cups. An explanation may compare both fractions to ½ and state that both are larger than ½ so the total must be more than 1. In addition, both fractions are slightly less than 1 so the sum cannot be more than 2.

- Area model



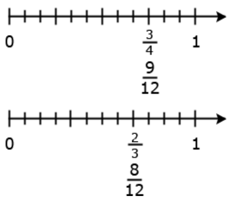
$\frac{3}{4}$ cup of sugar



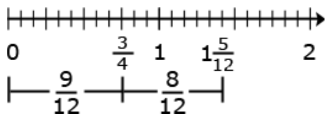
$\frac{2}{3}$ cup of sugar

$\frac{3}{4}=\frac{9}{12}$ $\frac{2}{3}=\frac{8}{12}$ $\frac{3}{4}+\frac{2}{3}=\frac{17}{12}=\frac{12}{12}+\frac{5}{12}=1\frac{5}{12}$

- Linear model



Solution:

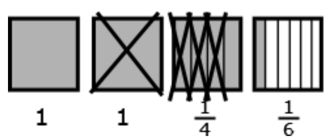


Example: Using a bar diagram

- Sonia had 2 1/3 candy bars. She promised her brother that she would give him ½ of a candy bar. How much will she have left after she gives her brother the amount she promised?
- If Mary ran 3 miles every week for 4 weeks, she would reach her goal for the month. The first day of the first week she ran 1 ¾ miles. How many miles does she still need to run the first week?
    - Using addition to find the answer:1 ¾ + n = 3
    - A student might add 1 ¼ to 1 ¾ to get to 3 miles. Then he or she would add 1/6 more. Thus 1 ¼ miles + 1/6 of a mile is what Mary needs to run during that week.

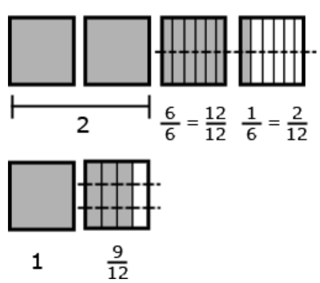
Example: Using an area model to subtract

- This model shows 1 ¾ subtracted from 3 1/6 leaving 1 + ¼ + 1/6 which a student can then change to 1 + 3/12 + 2/12 = 1 5/12.



3 1/6 and 1 ¾ can be expressed with a denominator of 12. Once this is done a student can complete the problem, 2 14/12 – 1 9/12 = 1 5/12.

- This diagram models a way to show how 3 1/6 and 1 ¾ can be expressed with a denominator of 12. Once this is accomplished, a student can complete the problem, 2 14/12 – 1 9/12 = 1 5/12.



Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies for calculations with fractions extend from students' work with whole number operations and can be supported through the use of physical models.

Example:

- Elli drank 3/5 quart of milk and Javier drank $\frac{1}{10}$ of a quart less than Ellie. How much milk did

  they drink all together?

Solution:

- $\frac{3}{5} - \frac{1}{10} = \frac{6}{10} - \frac{1}{10} = \frac{5}{10}$ This is how much milk Javier drank
- $\frac{3}{5} + \frac{5}{10} = \frac{6}{10} + \frac{5}{10} = \frac{11}{10}$ Together they drank $1\frac{1}{10}$ quarts of milk

This solution is reasonable because Ellie drank more than ½ quart and Javier drank ½ quart so together they drank slightly more than one quart.

Common Misconceptions

Students often mix models when adding, subtracting or comparing fractions. Students will use a circle for thirds and a rectangle for fourths when comparing fractions with thirds and fourths. Remind students that the representations need to be from the same whole models with the same shape and size.

Check Out : http://wps.ablongman.com/ab_vandewalle_math_6/54/13858/3547873.cw/-/t/index.html for more information on fractions

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Domain	**Number and Operations - Fractions**
Cluster	***Apply and extend previous understandings of multiplication and division to multiply and divide fractions.***
5th Grade Standards ***5.NF.3*** ***5.NF .4.a*** ***5.NF .4.b*** ***5.NF .5.a*** ***5.NF .5.b*** ***5.NF .6*** ***5.NF .7.a*** ***5.NF .7.b*** ***5.NF .7.c***	**5.NF.3**. Interpret a fraction as division of the numerator by the denominator $\frac{a}{b}$ = a ÷ b. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* **5.NF 4.** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **a.** Interpret the product $\frac{a}{b}$ × q as a parts of a partition of q into b equal parts equivalently, as the result of a sequence of operations a × q ÷ b. *For example, use a visual fraction model to show $\frac{2}{3}$ × 4 = $\frac{8}{3}$ and create a story context for this equation. Do the same with $\frac{2}{3}$ × $\frac{4}{5}$ $\frac{8}{15}$.* *(In general, $\frac{a}{b}$ × $\frac{c}{d}$= $\frac{ac}{bd}$.)* **b.** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. **5.NF .5**. Interpret multiplication as scaling (resizing), by: **a.** Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. **b.** Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{nXa}{nXb}$ to the effect of multiplying $\frac{a}{b}$ by 1. **5.NF 6**. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **5.NF 7**. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.1 **a**. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for $\frac{1}{3}$ ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $\frac{1}{3}$ ÷ 4 = $\frac{1}{12}$ because $\frac{1}{12}$ × 4 = $\frac{1}{3}$.* **b.** Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ $\frac{1}{5}$ and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ $\frac{1}{5}$ = 20 because 20 × $\frac{1}{5}$= 4.* **c.** Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share $\frac{1}{2}$ lb of chocolate equally? How many $\frac{1}{3}$-cup servings are in 2 cups of raisins?*

Instructional Strategies

Connect the meaning of multiplication and division of fractions with whole-number multiplication and division. Consider area models of multiplication and both sharing and measuring models for division.

Ask questions such as, "What does 2 × 3 mean?" and "What does 12 ÷ 3 mean?" Then, follow with questions for multiplication with fractions, such as, "What does $\frac{3}{4} \times \frac{1}{3}$ mean?" "What does $\frac{3}{4} \times 7$ mean?" (7 sets of $\frac{3}{4}$) and What does $7 \times \frac{3}{4}$ mean?" ($\frac{3}{4}$ of a set of 7)

The meaning of $4 \div \frac{1}{2}$ (how many $\frac{1}{2}$ are in 4) and $\frac{1}{2} \div 4$ (how many groups of 4 are in $\frac{1}{2}$) also should be illustrated with models or drawings. Encourage students to use models or drawings to multiply or divide with fractions. Begin with students modeling multiplication and division with whole numbers. Have them explain how they used the model or drawing to arrive at the solution.

Models to consider when multiplying or dividing fractions include, but are not limited to: area models using rectangles or squares, fraction strips/bars and sets of counters.

Models to consider when multiplying or dividing fractions include, but are not limited to: area models using rectangles or squares, fraction strips/bars and sets of counters.

Use calculators or models to explain what happens to the result of multiplying a whole number by a fraction (3 × 12, 4 × 12, $5 \times \frac{1}{2}$ …and $4 \times \frac{1}{2}$, $4 \times \frac{1}{3}$, $4 \times \frac{1}{4}$,…) and when multiplying a fraction by a number greater than 1.

Use calculators or models to explain what happens to the result when dividing a unit fraction by a non-zero whole number ($\frac{1}{8} \div 4$, $\frac{1}{8} \div 8$, $\frac{1}{8} \div 16$…) and what happens to the result when dividing a whole number by a unit fraction ($4 \div \frac{1}{4}$, $8 \div \frac{1}{4}$, $12 \div \frac{1}{4}$).

Present problem situations and have students use models and equations to solve the problem. It is important for students to develop understanding of multiplication and division of fractions through contextual situations.

5.NF.3 calls for students to extend their work of partitioning a number line from Third and Fourth Grade. Students need ample experiences to explore the concept that a fraction is a way to represent the division of two quantities. Students need to develop a fundamental understanding that the multiplication of a fraction by a whole number could be represented as repeated addition of a unit fraction (e.g., $2 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4}$)

Example:

Your teacher gives 7 packs of paper to your group of 4 students. If you share the paper equally, how much paper does each student get?

student 1	Student 2	Student 3	Student 4	1	2	3	4	1	2	4	4	1	2	3	4
pack 1	Pack 2	Pack 3	Pack 4	Pack 5				Pack 6				Pack 7			

Each student receives 1 whole pack of paper and $\frac{1}{4}$ of the each of the 3 packs of paper. So each student gets 1 $\frac{3}{4}$ packs of paper.

Students are expected to demonstrate their understanding using concrete materials, drawing models, and explaining their thinking when working with fractions in multiple contexts. They read $\frac{3}{5}$as "three fifths" and after many experiences with sharing problems, learn that $\frac{3}{5}$ can also be interpreted as "3 divided by 5."

Examples:

- Ten team members are sharing 3 boxes of cookies. How much of a box will each student get?
  When working this problem a student should recognize that the 3 boxes are being divided into 10 groups, so s/he is seeing the solution to the following equation, 10 x n = 3 (10 groups of some amount is 3 boxes) which can also be written as n = 3 ÷ 10. Using models or diagram, they divide each box into 10 groups, resulting in each team member getting $\frac{3}{10}$ *of a box.*
- Two afterschool clubs are having pizza parties. For the Math Club, the teacher will order 3 pizzas for every 5 students. For the student council, the teacher will order 5 pizzas for every 8 students. Since you are in both groups, you need to decide which party to attend. How much pizza would you get at each party? If you want to have the most pizza, which party should you attend?
- The six fifth grade classrooms have a total of 27 boxes of pencils. How many boxes will each classroom receive?

Students may recognize this as a whole number division problem but should also express this equal sharing problem as $\frac{27}{6}$. They explain that each classroom gets $\frac{27}{6}$ boxes of pencils and can further determine that each classroom get $4\frac{3}{6}$ or $4\frac{1}{2}$ boxes of pencils.

5.NF.4 extends student's work of multiplication from earlier grades. In Fourth Grade, students worked with recognizing that a fraction such as $\frac{3}{5}$ actually could be represented as 3 pieces that are each one-fifth (3 x $\frac{1}{5}$). In Fifth Grade, students are only multiplying fractions less than one. They are not multiplying mixed numbers until Sixth Grade.

Students are expected to multiply fractions including proper fractions, improper fractions, and mixed numbers. They multiply fractions efficiently and accurately as well as solve problems in both contextual and non-contextual situations.

As they multiply fractions such as $\frac{3}{5}$ x 6, they can think of the operation in more than one way.

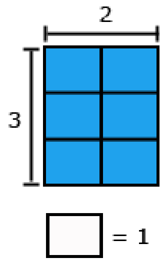
- 3 x (6 ÷ 5) or (3 x $\frac{6}{5}$)
- (3 x 6) ÷ 5 or 18 ÷ 5 or $\frac{18}{5}$

Students create a story problem for $\frac{3}{5}$ x 6 such as,

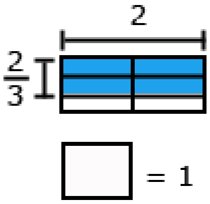
- Isabel had 6 feet of wrapping paper. She used $\frac{3}{5}$ of the paper to wrap some presents. How much does she have left?
- Every day Tim ran $\frac{3}{5}$ of mile. How far did he run after 6 days? (Interpreting this as 6 x $\frac{3}{5}$)

Examples: Building on previous understandings of multiplication

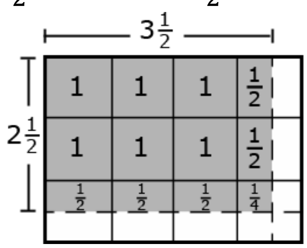
- Rectangle with dimensions of 2 and 3 showing that 2 x 3 = 6.



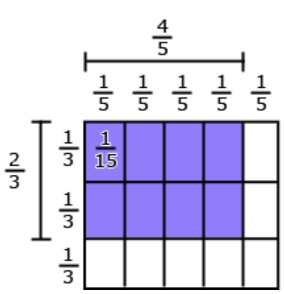
- Rectangle with dimensions of 2 and $\frac{2}{3}$ showing that 2 x $\frac{2}{3}$ = $\frac{4}{3}$



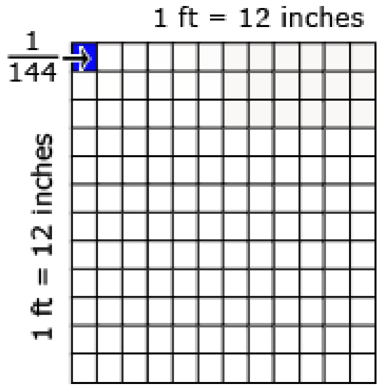
- $2\frac{1}{2}$ groups of $3\frac{1}{2}$



- In solving the problem $\frac{2}{3}$ x $\frac{4}{5}$, students use an area model to visualize it as a 2 by 4 array of small rectangles each of which has side lengths $\frac{1}{3}$ and $\frac{1}{5}$. They reason that $\frac{1}{3}$ x $\frac{1}{5}$ = $\frac{1}{3X5}$ by counting squares in the entire rectangle, so the area of the shaded area is (2 x 4) x $\frac{1}{3X5}$ = $\frac{2 \text{ x } 4}{3 \text{ x } 5}$. They can explain that the product is less than $\frac{4}{5}$ because they are finding $\frac{2}{3}$ of $\frac{4}{5}$, . They can further estimate that the answer must be between $\frac{2}{5}$ and $\frac{4}{5}$ because $\frac{2}{3}$ of $\frac{4}{5}$, is more than $\frac{1}{2}$ of $\frac{4}{5}$ and less than one group of $\frac{4}{5}$.



- Larry knows that $\frac{1}{12}$x $\frac{1}{12}$ is $\frac{1}{144}$. To prove this he makes the following array.

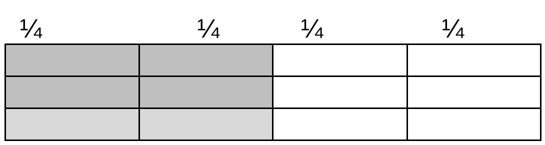


5.NF.4a references both the multiplication of a fraction by a whole number and the multiplication of two fractions. Visual fraction models (area models, tape diagrams, number lines) should be used and created by students during their work with this standard.

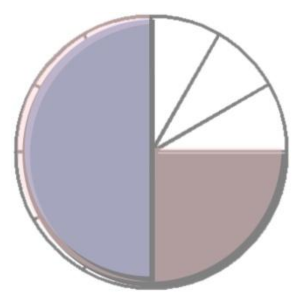
Example:
Three-fourths of the class is boys. Two-thirds of the boys are wearing tennis shoes. What fraction of the class are boys with tennis shoes?

This question is asking what $\frac{2}{3}$ of $\frac{3}{4}$ is, or what is $\frac{2}{3}$ x $\frac{3}{4}$. What is $\frac{2}{3}$ x $\frac{3}{4}$: in this case you have $\frac{2}{3}$ groups of size $\frac{3}{4}$ (a way to think about it in terms of the language for whole numbers is 4 x 5 you have 4 groups of size 5. The array model is very transferable from whole number work and then to binomials.

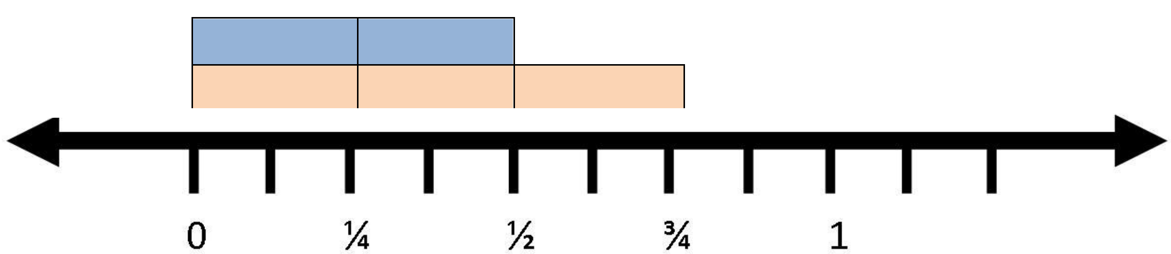
Student Example 1
I drew a rectangle to represent the whole class. I shaded 3 columns to represent the fraction that are boys. I then split the rectangle with horizontal lines into thirds. The dark area represents the fraction of the boys in the class wearing tennis shoes, which is 6 out of 12. That is $\frac{6}{12}$, which equals $\frac{1}{2}$.



Student Example 2
A fraction circle could be used to model student thinking. First I shade the fraction circle to show the ¾ and then overlay with $\frac{2}{3}$of that?

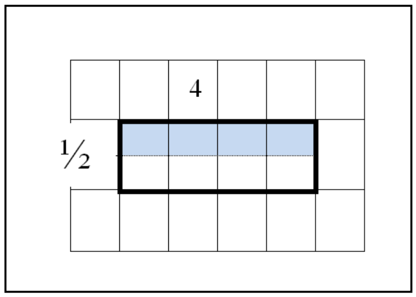


Student Example 3



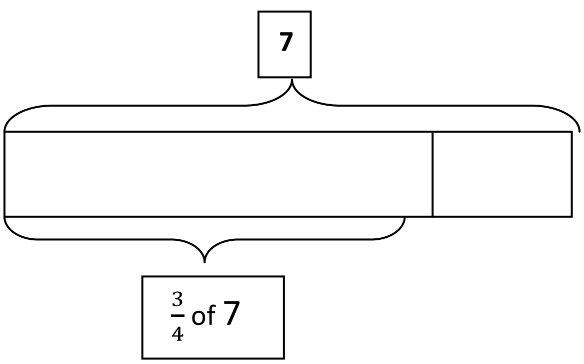
5.NF.4b extends students' work with area. In Third Grade students determine the area of rectangles and composite rectangles. In Fourth Grade students continue this work. The Fifth Grade standard calls students to continue the process of covering (with tiles). Grids (see picture) below can be used to support this work.

Example:
The home builder needs to cover a small storage room floor with carpet. The storage room is 4 meters long and half of a meter wide. How much carpet do you need to cover the floor of the storage room? Use a grid to show your work and explain your answer.
In the grid below I shaded the top half of 4 boxes. When I added them together, I added ½ four times, which equals 2. I could also think about this with multiplication $\frac{1}{2}$ x 4 is equal to $\frac{4}{2}$ which is equal to 2.



5.NF.5 Interpret multiplication as scaling (resizing)
Examples:

- $\frac{3}{4} \times 7$ is less than 7 because 7 is multiplied by a factor less than 1 so the product must be less than 7.



- $2\frac{2}{3}$ x 8 must be more than 8 because 2 groups of 8 is 16 and $2\frac{2}{3}$ is almost 3 groups of 8. So the answer must be close to, but less than 24.

- $\frac{3}{4} = \frac{5X3}{5X4}$ because multiplying $\frac{3}{4}$ by $\frac{5}{5}$ is the same as multiplying by 1

5.NF.5a calls for students to examine the magnitude of products in terms of the relationship between two types of problems. This extends the work with 5.OA.1.

Example 1:	Example 2:
Mrs. Jones teaches in a room that is 60 feet wide and 40 feet long. Mr. Thomas teaches in a room that is half as wide, but has the same length. How do the dimensions and area of Mr. Thomas' classroom compare to Mrs. Jones' room? Draw a picture to prove our answer	How does the product of 225 x 60 compare to the product of 225 x 30? How do you know? Since 30 is half of 60, the product of 225 X 60 will be double or twice as large as the product of 225 X 30

5.NF.5b asks students to examine how numbers change when we multiply by fractions. Students should have ample opportunities to examine both cases in the standard: a) when multiplying by a fraction greater than 1, the number increases and b) when multiplying by a fraction less the one, the number decreases. This standard should be explored and discussed while students are working with 5.NF.4, and should not be taught in isolation.

Example:
Mrs. Bennett is planting two flower beds. The first flower bed is 5 meters long and 6/5 meters wide. The second flower bed is 5 meters long and $\frac{5}{6}$ meters wide. How do the areas of these two flower beds compare? Is the value of the area larger or smaller than 5 square meters? Draw pictures to prove your answer.

5.NF.6 builds on all of the work done in this cluster. Students should be given ample opportunities to use various strategies to solve word problems involving the multiplication of a fraction by a mixed number. This standard could include fraction by a fraction, fraction by a mixed number or mixed number by a mixed number.

Example:
There are $2\frac{1}{2}$ bus loads of students standing in the parking lot. The students are getting ready to go on a field trip. $\frac{2}{5}$ of the students on each bus are girls. How many busses would it take to carry ***only*** the girls?

Student1	**Student 2**
I drew 3 grids and 1 grid represents 1 bus. I cut the third grid in half and I marked out the right half of the third grid, leaving $2\frac{1}{2}$ grids. I then cut each grid into fifths, and shaded two-fifths of each grid to represent the number of girls. When I added up the shaded prices, $\frac{2}{5}$ of the 1st and 2nd bus were both shaded, and $\frac{1}{5}$ of the last bus was shaded	$2\frac{1}{2}$ X $\frac{2}{5}$ = I split the $2\frac{1}{2}$ into 2 and $\frac{1}{2}$ 2 X $\frac{2}{5}$ = $\frac{4}{5}$ $\frac{1}{2}$ X $\frac{2}{5}$ =$\frac{2}{10}$ I then added $\frac{4}{5}$ and $\frac{2}{10}$ That equals 1 whole bus load

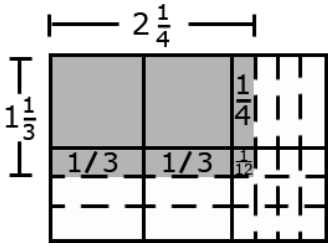
Examples:

Evan bought 6 roses for his mother. $\frac{2}{3}$ of them were red. How many red roses were there?
Using a visual, a student divides the 6 roses into 3 groups and counts how many are in 2 of the 3 groups.

A student can use an equation to solve. $\frac{2}{3} \times 6 = \frac{12}{3} = 4$ *red roses.*

Mary and Joe determined that the dimensions of their school flag needed to be $1\frac{1}{3}$ ft. by $2\frac{1}{4}$ ft. What will be the area of the school flag?

A student can draw an array to find this product and can also use his or her understanding of decomposing numbers to explain the multiplication. Thinking ahead a student may decide to multiply by $1\frac{1}{3}$ instead of $2\frac{1}{4}$.



The explanation may include the following:

- First, I am going to multiply $2\frac{1}{4}$ by 1 and then by $\frac{1}{3}$.
- When I multiply $2\frac{1}{4}$ by 1, it equals $2\frac{1}{4}$.
- Now I have to multiply $2\frac{1}{4}$ by $\frac{1}{3}$.
- $\frac{1}{3}$ times 2 is $\frac{2}{3}$.
- $\frac{1}{3}$ times $\frac{1}{4}$ is $\frac{1}{12}$.
- So the answer is $2\frac{1}{4} + \frac{2}{3} + \frac{1}{12}$ or $2\frac{3}{12} + \frac{8}{12} + \frac{1}{12} = 2\frac{12}{12} = 3$

5.NF.7 is the first time that students are dividing with fractions. In Fourth Grade students divided whole numbers, and multiplied a whole number by a fraction. The concept *unit fraction* is a fraction that has a one in the numerator. For example, the fraction $\frac{3}{5}$ is 3 copies of the unit fraction $\frac{1}{5}$.

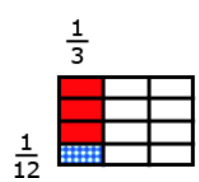
$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$ or $\frac{1}{5} \times 3$ or $3 \times \frac{1}{5}$

In fifth grade, students experience division problems with whole number divisors and unit fraction dividends (fractions with a numerator of 1) or with unit fraction divisors and whole number dividends. Students extend their understanding of the meaning of fractions, how many unit fractions are in a whole, and their understanding of multiplication and division as involving equal groups or shares and the number of objects in each group/share. In sixth grade, they will use this foundational understanding to divide into and by more complex fractions and develop abstract methods of dividing by fractions.

Division Example: Knowing the number of groups/shares and finding how many/much in each group/share

- Four students sitting at a table were given $\frac{1}{3}$ of a pan of brownies to share. How much of a pan will each student get if they share the pan of brownies equally?

The diagram shows the $\frac{1}{3}$ pan divided into 4 equal shares with each share equaling $\frac{1}{12}$ of the pan.

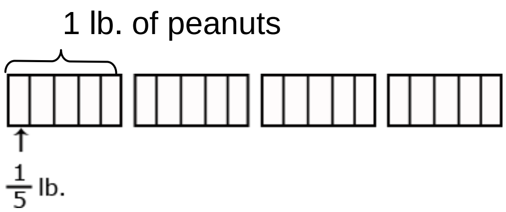


Examples:

Knowing how many in each group/share and finding how many groups/shares

- Angelo has 4 lbs of peanuts. He wants to give each of his friends $\frac{1}{5}$ lb. How many friends can receive $\frac{1}{5}$ lb of peanuts?

A diagram for $4 \div \frac{1}{5}$ is shown below. Students explain that since there are five fifths in one whole, there must be 20 fifths in 4 lbs.



- How much rice will each person get if 3 people share $\frac{1}{2}$ lb of rice equally?

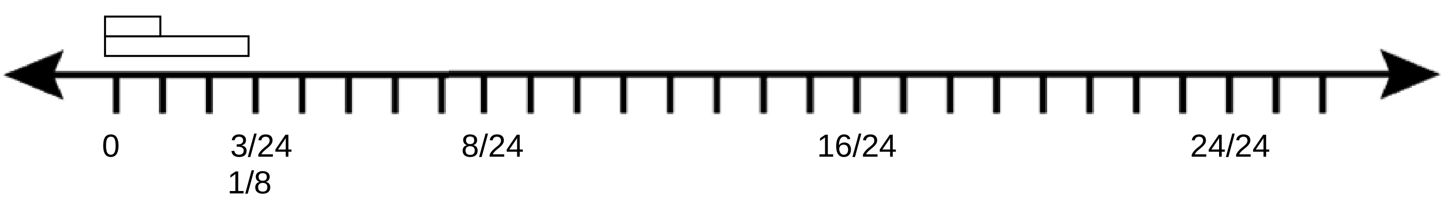
  $\frac{1}{2} \div 3 = \frac{3}{6} \div 3 = \frac{1}{6}$

A student may think or draw $\frac{1}{2}$ and cut it into 3 equal groups then determine that each of those parts is $\frac{1}{6}$.
A student may think of $\frac{1}{2}$ as equivalent to $\frac{3}{6}$. $\frac{3}{6}$ divided by 3 is $\frac{1}{6}$.

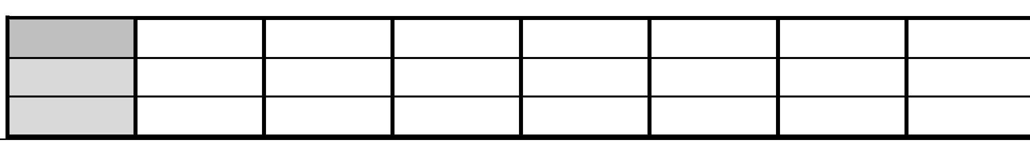
5.NF.7a asks students to work with story contexts where a unit fraction is divided by a non-zero whole number. Students should use various fraction models and reasoning about fractions.

Example:
You have $\frac{1}{8}$ of a bag of pens and you need to share them among 3 people. How much of the bag does each person get?

Student 1
Expression $\frac{1}{8}$ divided by 3



Student 2
I drew a rectangle and divided it into 8 columns to represent my $\frac{1}{8}$. I shaded the first column. I then needed to divide the shaded region into 3 parts to represent sharing among 3 people. I shaded one-third of the first column even darker. The dark shade is $\frac{1}{24}$ of the grid or $\frac{1}{24}$ of the bag of pens.



Student 3

$\frac{1}{8}$ of a bag of pens divided by 3 people . I know that my answer will be less than $\frac{1}{8}$ since I am sharing $\frac{1}{8}$ into 3 groups. I multiplied 8 by 3 and got 24, so my answer is $\frac{1}{24}$ of the bag of pens. I know that my answer is correct because $\frac{1}{24}$ X 3 = $\frac{3}{24}$ which equals $\frac{1}{8}$.

5.NF.7b calls for students to create story contexts and visual fraction models for division situations where a whole number is being divided by a unit fraction.

Example: Create a story context for 5 ÷ 1/6. Find your answer and then draw a picture to prove your answer and use multiplication to reason about whether your answer makes sense. How many 1/6 are there in 5?

Student

The bowl holds 5 Liters of water. If we use a scoop that holds $\frac{1}{5}$ of a liter, how many scoops will we need in order to fill the entire bowl?

I created 5 boxes. Each box represents 1 liter of water. I then divided each box into fifths to represent the size of the scoop. My answer is the number of small boxes, which is 25. That makes sense since 5 x 5 = 25.

1 = $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$ a whole has $\frac{5}{5}$ so five wholes would be $\frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{5}{5} = \frac{25}{5}$

5.NF.7c extends students' work from other standards in 5.NF.7. Student should continue to use visual fraction models and reasoning to solve these real-world problems.

Example:
How many $\frac{1}{3}$ cup servings are in 2 cups of raisins?

Student

I know that there are three $\frac{1}{3}$ cup servings in 1 cup of raisins. Therefore, there are 6 servings in 2 cups of raisins. I can also show this since 2 divided by $\frac{1}{3}$ = 2 X 3 = 6 servings of raisins.

Common Misconceptions

Students may believe that multiplication always results in a larger number. Using models when multiplying with fractions will enable students to see that the results will be smaller.
Additionally, students may believe that division always results in a smaller number. Using models when dividing with fractions will enable students to see that the results will be larger.

Check Out : http://wps.ablongman.com/ab_vandewalle_math_6/54/13858/3547873.cw/-/t/index.html for more information on Number and Operations in Fractions

Progressions Documents for the Common Core Math
http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_nbt_2011_04_073.pdf

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Resources used for these documents: Common Core State Standards, *Elementary and Middle School Mathematics: Teaching Developmentally* by John A Van De Walle, Department of Education "unpacking" of the standards from Arizona, Ohio and North Carolina

Domain	**Measurement and Data**
Cluster	**Convert like measurement units within a given measurement system.**
5th Grade Standards **5.MD.1**	**5.MD.1.** Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

<u>Instructional Strategies</u>

Students should gain ease in converting units of measures in equivalent forms within the same system. To convert from one unit to another unit, the relationship between the units must be known. In order for students to have a better understanding of the relationships between units, they need to use measuring tools in class. The number of units must relate to the size of the unit. For example, students have discovered that there are 12 inches in 1 foot and 3 feet in 1 yard. This understanding is needed to convert inches to yards. Using 12-inch rulers and yardsticks, students can see that three of the 12-inch rulers are equivalent to one yardstick (3 × 12 inches = 36 inches; 36 inches = 1 yard). Using this knowledge, students can decide whether to multiply or divide when making conversions.

Once students have an understanding of the relationships between units and how to do conversions, they are ready to solve multi-step problems that require conversions within the same system. Allow students to discuss methods used in solving the problems.

5.MD.1 calls for students to convert measurements within the same system of measurement in the context of multi-step, real-world problems. Both customary and standard measurement systems are included; students worked with both metric and customary units of length in Second Grade. In Third Grade, students work with metric units of mass and liquid volume. In Fourth Grade, students work with both systems and begin conversions within systems in length, mass and volume.
Students should explore how the base-ten system supports conversions within the metric system.
Example: 100 cm = 1 meter.

From previous grades: relative size, liquid volume, mass, length, kilometer (km), meter (m), centimeter (cm), kilogram (kg), gram (g), liter (L), milliliter (mL), inch (in), foot (ft), yard (yd), mile (mi), ounce (oz), pound (lb), cup (c), pint (pt), quart (qt), gallon (gal), hour, minute, second

In fifth grade, students build on their prior knowledge of related measurement units to determine equivalent measurements. Prior to making actual conversions, they examine the units to be converted, determine if the converted amount will be more or less units than the original unit, and explain their reasoning. They use several strategies to convert measurements. When converting metric measurement, students apply their understanding of place value and decimals.

<u>Common Misconceptions</u>

When solving problems that require renaming units, students use their knowledge of renaming the numbers as with whole numbers. Students need to pay attention to the unit of measurement which dictates the renaming and the number to use. The same procedures used in renaming whole numbers should not be taught when solving problems involving measurement conversions. For example, when subtracting 5 inches from 2 feet, students may take one foot from the 2 feet and use it as 10 inches. Since there were no inches with the 2 feet, they put 1 with 0 inches and make it 10 inches.

2 feet - 5 inches ______	is thought of as	2 feet 0 inches - 5 inches ______	becomes	1 foot 10 inches - 5 inches ______ 1 foot 5 inches

Check Out : http://wps.ablongman.com/ab_vandewalle_math_6/54/13858/3547873.cw/-/t/index.html for more information on Measurement and Data

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Domain	**Measurement and Data**
Cluster	***Represent and interpret data.***
5th Grade Standards **5.MD.2**	**5.MD.2**. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

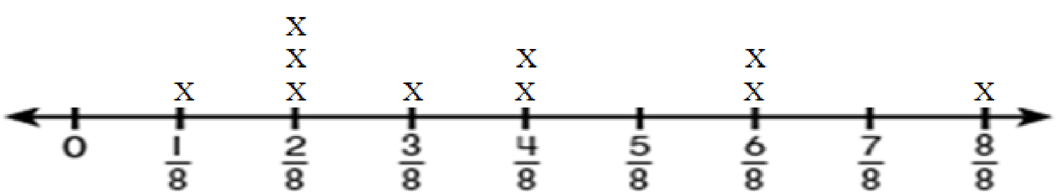
Instructional Strategies

Using a line plot to solve problems involving operations with unit fractions now includes multiplication and division. Revisit using a number line to solve multiplication and division problems with whole numbers. In addition to knowing how to use a number line to solve problems, students also need to know which operation to use to solve problems.

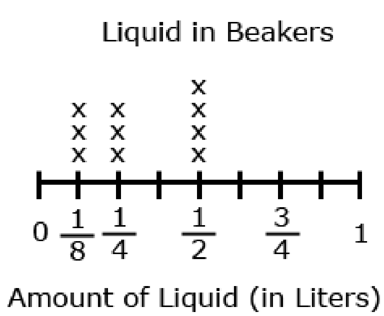
Use the tables for common addition, subtraction, multiplication and division situations (Table 1 and Table 2 in the Common Core State Standards for Mathematics) as a guide to the types of problems students need to solve without specifying the type of problem. Allow students to share methods used to solve the problems. Also have students create problems to show their understanding of the meaning of each operation and how they pertain to line plots, data, and fractions.

5.MD.2 This standard provides a context for students to work with fractions by measuring objects to one-eighth of a unit. This includes length, mass, and liquid volume. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot.

Example:
Students measured objects in their desk to the nearest ½, ¼, or 1/8 of an inch then displayed data collected on a line plot. How many object measured ¼? ½? If you put all the objects together end to end what would be the total length of **all** the objects?



Ten beakers, measured in liters, are filled with a liquid.



The line plot shows the amount of liquid in liters in 10 beakers. If the liquid is redistributed equally, how much liquid would each beaker have? (This amount is the mean.)

Students apply their understanding of operations with fractions. They use either addition and/or multiplication to determine the total number of liters in the beakers. Then the sum of the liters is shared evenly among the ten beakers.

Check Out : http://wps.ablongman.com/ab_vandewalle_math_6/54/13858/3547873.cw/-/t/index.html for more information on Measurement and Data

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Domain	**Measurement and Data**
Cluster	***Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.***
5th Grade Standards **5.MD.3.a** **5.MD.3.b** **5.MD.4** **5.MD.5.a** **5.MD.5.b** **5.MD.5.c**	**5.MD.3**. Recognize volume as an attribute of solid figures and understand concepts of volume measurement. **a**. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. **b.** A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. **5.MD 4.** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. **5.MD 5**. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. **a.** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. **b.** Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems. **c.** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

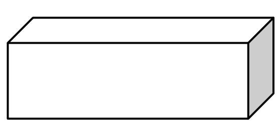
<u>Instructional Strategies</u>

Volume refers to the amount of space that an object takes up and is measured in cubic units such as cubic inches or cubic centimeters.

Students need to experience finding the volume of rectangular prisms by counting unit cubes, in metric and standard units of measure, before the formula is presented. Provide multiple opportunities for students to develop the formula for the volume of a rectangular prism with activities similar to the one described below.

Give students one block (a I- or 2- cubic centimeter or cubic-inch cube), a ruler with the appropriate measure based on the type of cube, and a small rectangular box. Ask students to determine the number of cubes needed to fill the box. Have students share their strategies with the class using words, drawings or numbers. Allow them to confirm the volume of the box by filling the box with cubes of the same size.

By stacking geometric solids with cubic units in layers, students can begin understanding the concept of how addition plays a part in finding volume. This will lead to an understanding of the formula for the volume of a right rectangular prism, b x h, where b is the area of the base. A right rectangular prism has three pairs of parallel faces that are all rectangles.



Have students build a prism in layers. Then, have students determine the number of cubes in the bottom layer and share their strategies. Students should use multiplication based on their knowledge of arrays and its use in multiplying two whole numbers.

Ask what strategies can be used to determine the volume of the prism based on the number of cubes in the bottom layer. Expect responses such as “adding the same number of cubes in each layer as were on the bottom layer” or multiply the number of cubes in one layer times the number of layers.

5.MD.3
Students’ prior experiences with volume were restricted to liquid volume. As students develop their understanding volume they understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. This cube has a length of 1 unit, a width of 1 unit and a height of 1 unit and is called a cubic unit. This cubic unit is written with an exponent of 3 (e.g., in^3, m^3). Students connect this notation to their understanding of powers of 10 in our place value system. Models of cubic inches, centimeters, cubic feet, etc are helpful in developing an image of a cubic unit. Students estimate how many cubic yards would be needed to fill the classroom or how many cubic centimeters would be needed to fill a pencil box.

5.MD.4
Students understand that same sized cubic units are used to measure volume. They select appropriate units to measure volume. For example, they make a distinction between which units are more appropriate for measuring the volume of a gym and the volume of a box of books. They can also improvise a cubic unit using any unit as a length (e.g., the length of their pencil). Students can apply these ideas by filling containers with cubic units (wooden cubes) to find the volume. They may also use drawings or interactive computer software to simulate the same filling process.

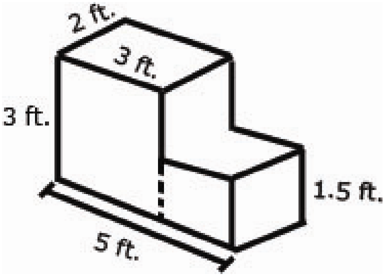
5.MD.5
Students need multiple opportunities to measure volume by filling rectangular prisms with cubes and looking at the relationship between the total volume and the area of the base. They derive the volume formula (volume equals the area of the base times the height) and explore how this idea would apply to other prisms. Students use the associative property of multiplication and decomposition of numbers using factors to investigate rectangular prisms with a given number of cubic units.

Examples:

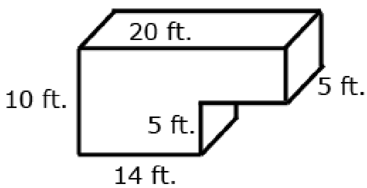
- When given 24 cubes, students make as many rectangular prisms as possible with a volume of 24 cubic units. Students build the prisms and record possible dimensions.

Length	Width	Height
1	2	12
2	2	6
4	2	3
8	3	1

- Students determine the volume of concrete needed to build the steps in the diagram below.



- A homeowner is building a swimming pool and needs to calculate the volume of water needed to fill the pool. The design of the pool is shown in the illustration below.



5. MD.5a & b involves finding the volume of right rectangular prisms (see picture above). Students should have experiences to describe and reason about why the formula is true. Specifically, that they are covering the bottom of a right rectangular prism (length x width) with multiple layers (height). Therefore, the formula (length x width x height) is an extension of the formula for the area of a rectangle.

5.MD.5c calls for students to extend their work with the area of composite figures into the context of volume. Students should be given concrete experiences of breaking apart (decomposing) 3-dimensional figures into right rectangular prisms in order to find the volume of the entire 3-dimensional figure.

Check Out : http://wps.ablongman.com/ab_vandewalle_math_6/54/13858/3547873.cw/-/t/index.html for more information on Measurement and Data

Progressions Documents for the Common Core Math
http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_nbt_2011_04_073.pdf

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Resources used for these documents: Common Core State Standards, *Elementary and Middle School Mathematics: Teaching Developmentally* by John A Van De Walle, Department of Education "unpacking" of the standards from Arizona, Ohio and North Carolina

Domain	**Geometry**
Cluster	***Graph points on the coordinate plane to solve real-world and mathematical problems.***
5th Grade Standards **5.G.1** **5.G.2**	**5.G.1**. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). **5.G.2**. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation

<u>Instructional Strategies</u>

Students need to understand the underlying structure of the coordinate system and see how axes make it possible to locate points anywhere on a coordinate plane. This is the first time students are working with coordinate planes, and only in the first quadrant. It is important that students create the coordinate grid themselves. This can be related to two number lines and reliance on previous experiences with moving along a number line.

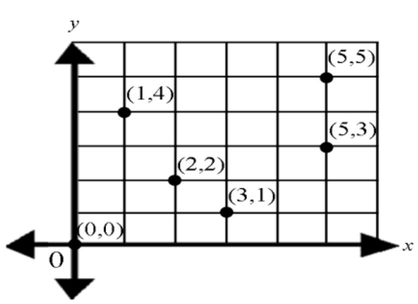
Multiple experiences with plotting points are needed. Provide points plotted on a grid and have students name and write the ordered pair. Have students describe how to get to the location. Encourage students to articulate directions as they plot points.

Present real-world and mathematical problems and have students graph points in the first quadrant of the coordinate plane. Gathering and graphing data is a valuable experience for students. It helps them to develop an understanding of coordinates and what the overall graph represents. Students also need to analyze the graph by interpreting the coordinate values in the context of the situation.

5.G.1

Examples:

- Students can use a classroom size coordinate system to physically locate the coordinate point (5, 3) by starting at the origin point (0,0), walking 5 units along the x axis to find the first number in the pair (5), and then walking up 3 units for the second number in the pair (3). The ordered pair names a point in the plane.



- Graph and label the points below in a coordinate system.
    - A (0, 0)

- B (5, 1)
- C (0, 6)
- D (2.5, 6)
- E (6, 2)
- F (4, 1)

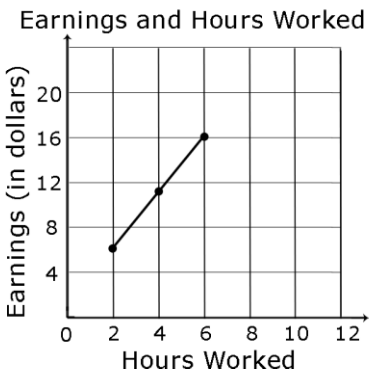
G (3, 0)

5.G.1 and **5.G.2** deal with only the first quadrant (positive numbers) in the coordinate plane.

5.G.2 references real-world and mathematical problems, including the traveling from one point to another and identifying the coordinates of missing points in geometric figures, such as squares, rectangles, and parallelograms.
Examples:

- Sara has saved $20. She earns $8 for each hour she works.
  - If Sara saves all of her money, how much will she have after working 3 hours? 5 hours? 10 hours?
  - Create a graph that shows the relationship between the hours Sara worked and the amount of money she has saved.
  - What other information do you know from analyzing the graph?

- Use the graph below to determine how much money Jack makes after working exactly 9 hours.



Common Misconceptions
When playing games with coordinates or looking at maps, students may think the order in plotting a coordinate point is not important. Have students plot points so that the position of the coordinates is switched. For example, have students plot (3, 4) and (4, 3) and discuss the order used to plot the points. Have students create directions for others to follow so that they become aware of the importance of direction and distance.

Check Out :
http://wps.ablongman.com/ab_vandewalle_math_6/54/13858/3547873.cw/-/t/index.html for more information on Geometry
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Domain	Geometry
Cluster	***Classify two-dimensional figures into categories based on their properties.***
5th Grade Standards 5.G.3 5.G.4	**5.G.3.** Understand that attributes belonging to a category of two dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. **5.G.4.** Classify two-dimensional figures in a hierarchy based on properties.

Instructional Strategies

This cluster builds from Grade 3 when students described, analyzed and compared properties of two-dimensional shapes. They compared and classified shapes by their sides and angles, and connected these with definitions of shapes. In Grade 4 students built, drew and analyzed two-dimensional shapes to deepen their understanding of the properties of two-dimensional shapes. They looked at the presence or absence of parallel and perpendicular lines or the presence or absence of angles of a specified size to classify two-dimensional shapes. Now, students classify two-dimensional shapes in a hierarchy based on properties. Details learned in earlier grades need to be used in the descriptions of the attributes of shapes. The more ways that students can classify and discriminate shapes, the better they can understand them. The shapes are not limited to quadrilaterals.

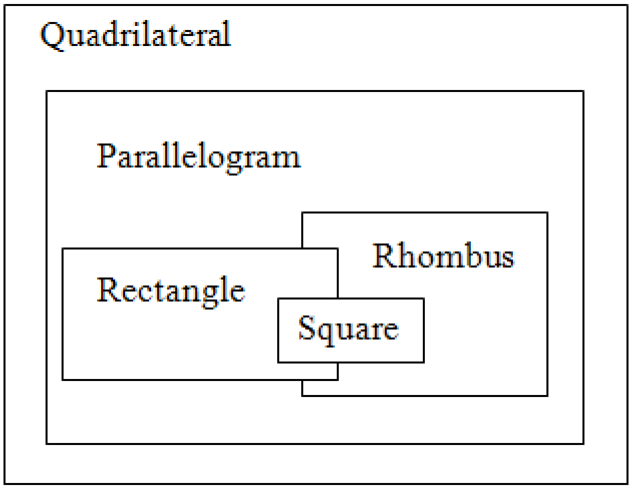
Students can use graphic organizers such as flow charts or T-charts to compare and contrast the attributes of geometric figures. Have students create a T-chart with a shape on each side. Have them list attributes of the shapes, such as number of sides, number of angles, types of lines, etc. they need to determine what's alike or different about the two shapes to get a larger classification for the shapes. Pose questions such as, "Why is a square always a rectangle?" and "Why is a rectangle not always a square?"

5.G.3 calls for students to reason about the attributes (properties) of shapes. Student should have experiences discussing the property of shapes and reasoning.
Example:
Examine whether all quadrilaterals have right angles. Give examples and non-examples.
Geometric properties include properties of sides (parallel, perpendicular, congruent), properties of angles (type, measurement, congruent), and properties of symmetry (point and line).

Example:
- If the opposite sides on a parallelogram are parallel and congruent, then rectangles are parallelograms.

A sample of questions that might be posed to students include:
- A parallelogram has 4 sides with both sets of opposite sides parallel. What types of quadrilaterals are parallelograms?
- Regular polygons have all of their sides and angles congruent. Name or draw some regular polygons.
- All rectangles have 4 right angles. Squares have 4 right angles so they are also rectangles. True or False?
- A trapezoid has 2 sides parallel so it must be a parallelogram. True or False?

5.G.4 this standard builds on what was done in 4th grade.
Figures from previous grades: **polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle**



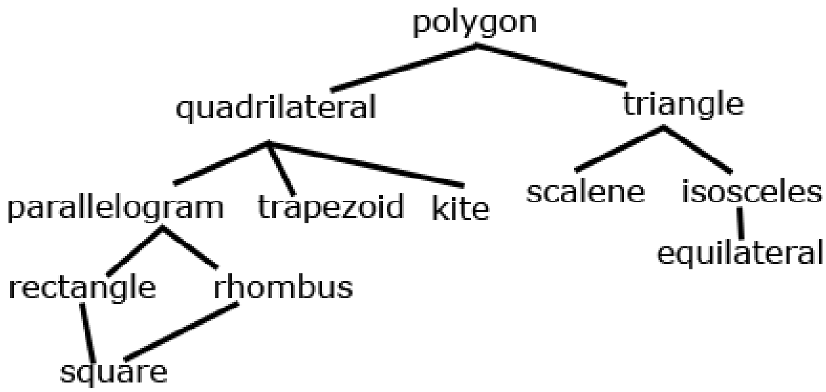
Student should be able to reason about the attributes of shapes by examining:
What are ways to classify triangles?
Why can't trapezoids and kites be classified as parallelograms?
Which quadrilaterals have opposite angles congruent and why is this true of certain quadrilaterals?
How many lines of symmetry does a regular polygon have?

Properties of figure may include:
- Properties of sides—parallel, perpendicular, congruent, number of sides
- Properties of angles—types of angles, congruent

Examples:
- A right triangle can be both scalene and isosceles, but not equilateral.
- A scalene triangle can be right, acute and obtuse.

Triangles can be classified by:
- Angles
  - Right: The triangle has one angle that measures 90º.
  - Acute: The triangle has exactly three angles that measure between 0º and 90º.
  - Obtuse: The triangle has exactly one angle that measures greater than 90º and less than 180º.

- Sides
  - Equilateral: All sides of the triangle are the same length.
  - Isosceles: At least two sides of the triangle are the same length.
  - Scalene: No sides of the triangle are the same length.



Common Misconceptions

Students think that when describing geometric shapes and placing them in subcategories, the last category is the only classification that can be used.

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